MAT1001, Fall 2012

Oblig 2

Deadline: Thursday, November 8, 1430 pm

You may work on these exercises together with your fellow students, but each of you has to hand in his/her own version of the solution to the assignment for approvement. Please put your name and course code (MAT 1001) on your answer sheet. The assignments must be handed in to the Mathematics Department's reception by the given date and time. The time limit is absolute and assignments will not be accepted after the deadline unless an agreement has been made with the administration beforehand.

Each exercise counts 0-5 points, all together a maximum of 60 points. Your Oblig 2 is approved if your score is 30 points or more and if you have made an attempt to solve all exercises.

Exercise 1

In the first exercise we consider a second order inhomogenous difference equation given by

$$x_{n+1} - \tfrac{3}{2}x_n - x_{n-1} = -\tfrac{3}{2}n^2 + \tfrac{7}{3}, \qquad n \geq 1$$

a) Find the general solution x_n^h of the associated homogenous difference equation.

b) Find a particular solution x_n^s of the inhomogenous equation. Use this to give the general solution $x_n = x_n^h + x_n^s$ of the inhomogenous equation.

c) Suppose that we have given the initial conditions $x_0 = 3$, $x_1 = \frac{31}{6}$. Find the particular solution of the equation which satisfy the given initial conditions.

d) What happens to x_n when $n \to \infty$?

Exercise 2

A sound wave is modelled by the function

$$f(t) = A\cos(t - t_0)$$

where A is the amplitude. The frequency of the sound is given by $\frac{1}{2\pi}$. The acrophase t_0 is the least positive value of t which gives a maximum for $f(t)$.

a) Use the summation formula for cosinus to write $f(t)$ as a linear combination of $\sin t$ and $\cos t$.

b) We add two sound waves of the same frequency. For simplicity we assume $A = 1$. Write the sum

$$\cos(t - t_0) + \cos(t - t_1)$$

as $C\cos(t - t_2)$. Show that $C = \sqrt{2 + 2\cos(t_0 - t_1)}$.

c) Consider two special cases, i) $t_0 = t_1$ og ii) $t_1 - t_0 = \pi$. Explain what happens to the two sound waves in the two cases.

Exercise 3

In this exercise we will use the following formula for e

$$e = \lim_{n\to\infty}(1+\tfrac{1}{n})^n$$

a) We need to know that

$$\lim_{h\to 0}\frac{e^h-1}{h} = 1$$

Show that this is true for all h of the form $h = \frac{1}{n}$ by using the formula for e as given above.

b) Let $f(x) = e^x$. The derivative of the function f is given by the formula

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Use a) and the definition of the derivative to show that $f'(x) = e^x$.

Exercise 4

a) Compute the integral

$$\int \tan x\, dx$$

In this exercise and in exercise b) we assume that $\cos x > 0$.

b) Solve the differential equation

$$y' + (2\tan x)y = 2\tan x$$

where $y(0) = 3$.

c) The addition formula for sinus gives $\sin 2x = 2\sin x\cos x$. Compute

$$\int \sin 2x\, dx \qquad 2\int \sin x \cos x\, dx$$

by the method of substitution, respectively by integration by parts. The answers seem to be different. Can you comment on this fact?

THE END