Oppgaver Mat1001
A1: 4, 7, 8, 10
A2: 4, 13,
B : 1.17, 1.22

1.4) Løs følgende likningssystemer

a) $L_1: 7x - 5x = 3$

$$2x = 3$$
$$x = \frac{3}{2}$$

b) $L_1: 7x - 5y = 3$

Løser for y: $7x - 3 = 5y$

$$y = \frac{7}{5}x - \frac{3}{5}$$

sett $x = s$

$$(x,y) \in \left\{ \left(s, \frac{7}{5}s - \frac{3}{5}\right) : s \in \mathbb{R} \right\}$$

c) $\begin{cases} L_1: 7x - 5y = 3 \\ L_2: x + 5y = 1 \end{cases}$

$L_1 + L_2 \Rightarrow 8x = 4 \Leftrightarrow x = \frac{1}{2}$

Setter inn i L_2: $\frac{1}{2} + 5y = 1$

$$5y = \frac{1}{2}$$
$$y = \frac{1}{10}$$

$$\mathcal{L} = \left\{ \left(\frac{1}{2}, \frac{1}{10}\right) \right\}$$

d) $\begin{cases} L_1: & x+2y-z=5 \\ L_2: & 2x-y=2 \\ L_3: & x+y+z=3 \end{cases}$

$L_1+L_3: \quad 2x+3y=8$

$\begin{cases} 2x-y=2 & : L_2 \\ 2x+3y=8 & : L_4 \end{cases}$

$L_4-L_2: \quad 4y=\underbrace{8-2}_{6} \iff y=\frac{3}{2}$

Setter inn i $L_2 \Rightarrow$ $2x-\frac{3}{2}=2$

$2x=\frac{4}{2}+\frac{3}{2}=\frac{7}{2}$

$x=\frac{7}{4}$

Setter inn i L_3

$z=3-\frac{7}{4}-\frac{3}{2}=\frac{12}{4}-\frac{7}{4}-\frac{6}{4}=\underline{\underline{-\frac{1}{4}}}$

$L=\left\{\left(\frac{7}{4},\frac{3}{2},-\frac{1}{4}\right)\right\}$

1.7) Løs likningssystemene både algebraisk og geometrisk.

a) $\begin{cases} L_1: 3x - \frac{1}{2}y = 0 \\ L_2: -6x + y = 0 \end{cases}$

$L_2 \Rightarrow y = 6x$, setter inn i L_1

$3x - \frac{1}{2}(6x) = 0.$

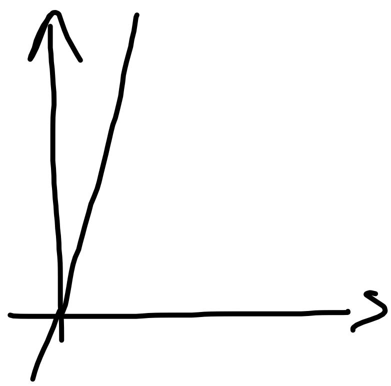
Likningene er like!

Parameter fremstilling:

$x = t, \quad y = 6t$

$\mathcal{L} = \{(t, 6t) : t \in \mathbb{R}\}$

Geometrisk:



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1.7

b) $\begin{cases} L_1 : x + 2y = 1 \\ L_2 : x + 2y = 2 \end{cases}$

$L_1 - L_2 \Rightarrow \quad 0 = 1 - 2$

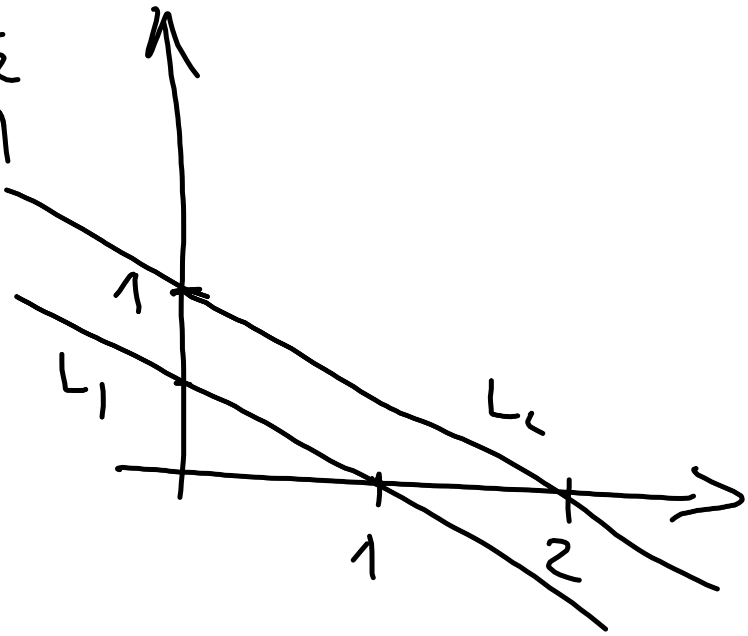
$\qquad\qquad\quad 0 = -1$

Vi har motsigelse. Finnes ikke (x, y) s.a L_1 og L_2 er samme.

$\mathcal{L} = \emptyset$ (tomme mengden)

$L_1 : y = -\frac{1}{2}x + \frac{1}{2}$

$L_2 : y = -\frac{1}{2}x + 1$



c) $\begin{cases} L_1 : 3x + y = 4 \\ L_2 : 6x + y = 8 \end{cases}$

$2 \cdot L_1 - L_2 \Rightarrow y = 0$

Vs : $2 \cdot (3x + y) - (6x + y) = 6x - 6x + 2y - y$
$= y$

Hs : $2 \cdot 4 - 8 = 0$

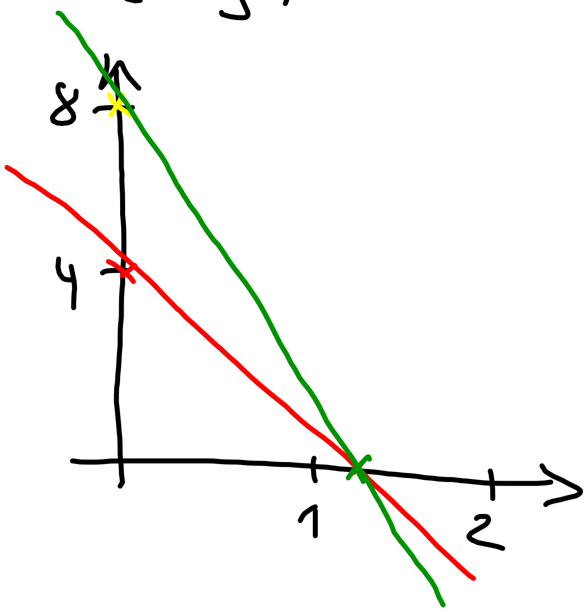
Setter inn i L_1 $\quad 3x + 0 = 4 \Leftrightarrow x = \frac{4}{3}$

$$\mathcal{L} = \left\{ \left(\frac{4}{3}, 0 \right) \right\}$$

Geometrisk:

$L_1 : y = -3x + 4$

$L_2 : y = -6x + 8$



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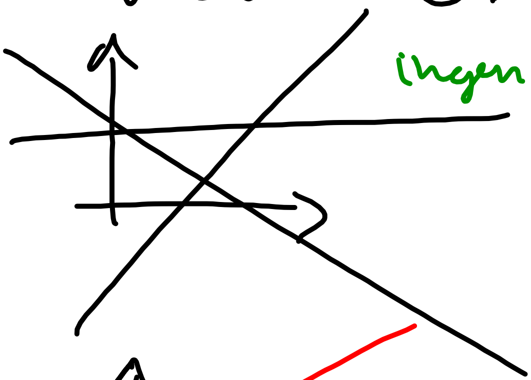
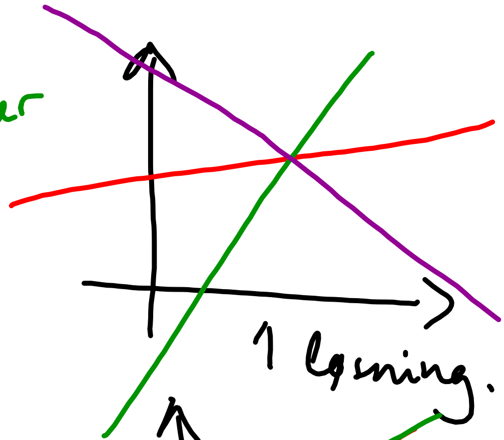
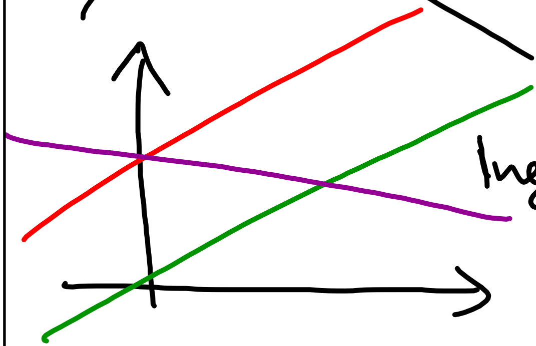
1.8 Gjør rede for de geometriske fenomenene som fremkommer når vi har et system med tre likninger i to variable.



ingen løsninger



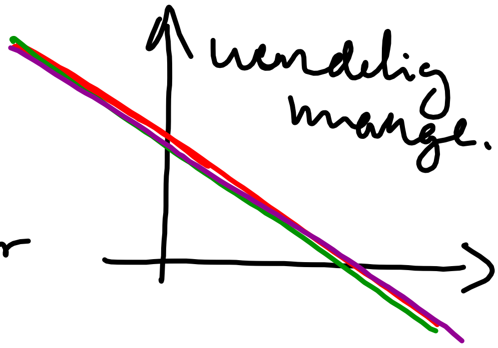
1 løsning.



Ingen løsninger

1 løsning

Ingen løsninger



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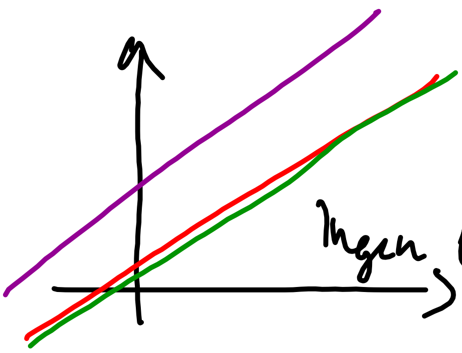

Ingen løsninger

1.10
$$\begin{cases} L_1: x+4y-2z=1 \\ L_2: x-y+z=1 \\ L_3: 2x+3y-z=2 \end{cases}$$

a) Substitusjon:

L_2': $x=1+y-z$

L_2 inni $L_1 \Rightarrow (1+y-z)+4y-2z=1$

$5y-3z=0$

L_2 inni $L_3 \Rightarrow 2(1+y-z)+3y-z=2$

$5y-3z=0$

$$\begin{cases} x-y+z=1 \\ 5y-3z=0 \end{cases}$$

Parameterfremstilling:

$z=t \qquad 5y-3t=0 \Leftrightarrow y=\frac{3}{5}t$

Fra L_2: $x\underbrace{-\frac{3}{5}t+\frac{5}{5}t}_{\frac{2}{5}t}=1 \Leftrightarrow x=-\frac{2}{5}t+1$

$$\ell=\left\{\left(-\frac{2}{5}t+1, \frac{3}{5}t, t\right) : t\in\mathbb{R}\right\}$$

(Addisjonsmetode)

b) Bruk (a) til å finne en løsning for

1) $x=10 \qquad x(t)=-\frac{2}{5}t+1$

$10=-\frac{2}{5}t+1 \Leftrightarrow 9=-\frac{2}{5}t \Leftrightarrow -\frac{5}{2}\cdot 9=t$

$t=-\frac{45}{2}$

$y\left(-\frac{45}{2}\right)=\frac{3}{5}\cdot\left(-\frac{45}{2}\right)=-\frac{3\cdot 9}{2}=-\frac{27}{2}$

$z\left(-\frac{45}{2}\right)=-\frac{45}{2}$

Punktet hvor $x=10$ er

$$\left(10, -\frac{27}{2}, -\frac{45}{2}\right)$$

2) $z=5 \Rightarrow t=5$

$$\left\{\left(-\frac{2}{5}t+1, \frac{3}{5}t, t\right) : t\in\mathbb{R}\right\}$$

$$\Rightarrow \left(-\frac{2}{5}5+1, \frac{3}{5}\cdot 5, 5\right)=\underline{\underline{(-1,3,5)}}$$

d) Hva skjer når vi bytter ut 2 på høyresiden i L_3 med 3 ?

L_2 inn i $L_1 \Rightarrow 5y-3z=0$

$L_3' : 2x+3y-z=3$

$L_2 \Rightarrow x = 1+y-z$

L_2 inn i $L_3' \Rightarrow 2(1+y-z)+3y-z=3$

$5y-3z=1$

Dette gir Ingen løsninger.

e) Bytter ut L_3 med

$L_3' : ax+3y-z=2$

L_2 inn i L_3' $\quad (x=1+y-z)$

$a(1+y-z)+3y-z=2$

Husk at $5y-3z=0 \Leftrightarrow y=\frac{3}{5}z$

$(3+a)y-(1+a)z=2-a$

$(3+a)\frac{3}{5}z-\frac{5}{5}(1+a)z=2-a$

$\frac{1}{5}(3\cdot 3+3a-5-5a)z=2-a$

$\frac{1}{5}(4-2a)z=(2-a)$ $\quad a\neq 2$

$z=\frac{5\cancel{(2-a)}}{2\cancel{(2-a)}}=\frac{5}{2}$

Dette gir løsning $\left(0,\frac{3}{2},\frac{5}{2}\right)$

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