A: 7.11, 7.12, 7.14 (8.2, 8.5, 8.10)

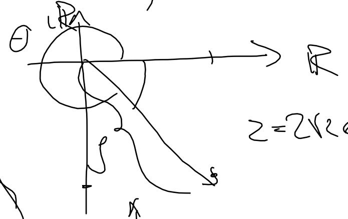
B: 2.6, 2.10, 2.11

7.11 Skriv tallene på polarform.

$z = \rho(\cos\theta + i\sin\theta)$

a) $z = 2 - 2i$

$\rho = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

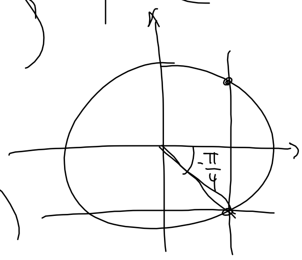


$z = 2\sqrt{2}e^{i\frac{7\pi}{4}}$

$\rho\cos\theta = 2 \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

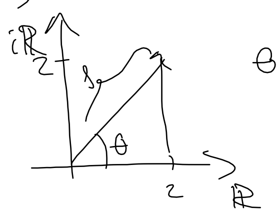
$\rho\sin\theta = -2 \Rightarrow \sin\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\theta = 2\pi + \left(-\frac{\pi}{4}\right) = \frac{7\pi}{4}$



$z = 2\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right)$

b) $z = 2 + 2i$ $\quad z = \rho(\cos\theta + i\sin\theta)$

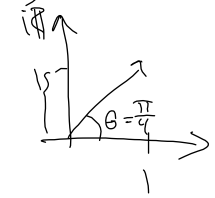


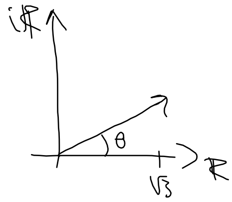
$\theta = \frac{\pi}{4}$ $\quad \rho = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ $\quad z = 2\sqrt{2}e^{i\frac{\pi}{4}}$

$z = 2\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

c) $z = 1 + i$ $\quad \rho = \sqrt{1^2 + 1^2} = \sqrt{2}$ $\quad z = \sqrt{2}e^{i\frac{\pi}{4}}$

$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$





$z = 2e^{i\frac{\pi}{6}}$

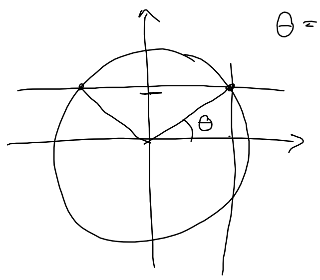
d) $z = \sqrt{3} + i$

$\rho = \sqrt{3+1} = \sqrt{4} = 2$

$z = \rho(\cos\theta + i\sin\theta) \Rightarrow$ $\quad \rho\cos\theta = \sqrt{3} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\rho\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2}$

$= 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ $\quad \theta = \frac{\pi}{6}$



e) $z = i$ $\quad z = e^{i\frac{\pi}{2}}$

$\rho = 1$

$\left.\begin{array}{l}\cos\theta = 0 \\ \sin\theta = 1\end{array}\right\} \Rightarrow \theta = \frac{\pi}{2}$

$i = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

7.12 Skriv tallene fra ~~opp~~ oppg. 7.11 på eksponentialform.

$z = \rho e^{i\theta}$

okt 1-12:08

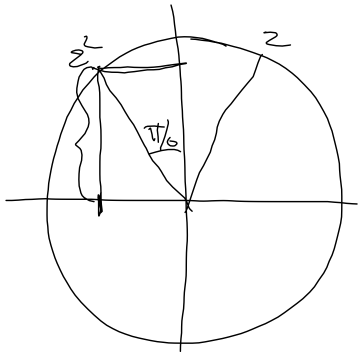
7.14 La $z = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$

Bruk Moivres formel til å finne z^2, z^3, z^5 og z^{10}.

de Moivres formel:

$$\boxed{(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)}$$

$$z^2 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^2 = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$$

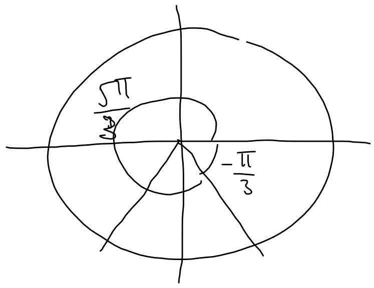


$$z^2 = -\frac{1}{2} + i\,\frac{\sqrt{3}}{2}$$

$$z^3 = \cos(\pi) + i\sin(\pi)$$

$$= -1 + i\cdot 0 = -1$$

$$z^5 = \cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)$$

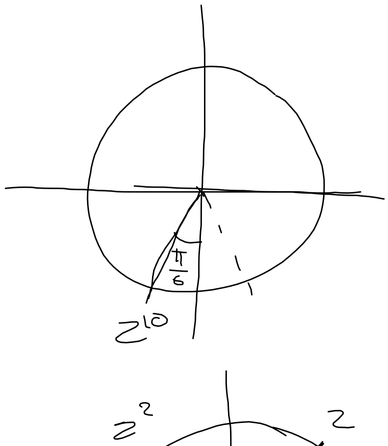


$$= \frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{2} - i\,\frac{\sqrt{3}}{2}$$

$$z^{10} = \cos\left(\frac{10\pi}{3}\right) + i\sin\left(\frac{10\pi}{3}\right)$$

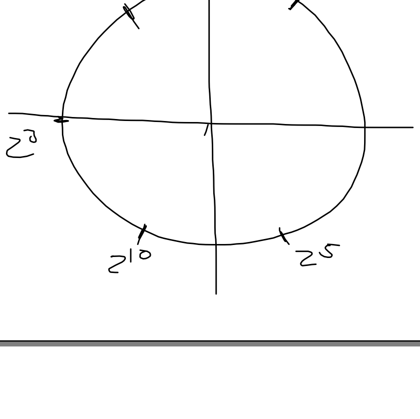
$$\frac{10\pi}{3} - 2\pi = \frac{4\pi}{3} = \pi + \frac{\pi}{3}$$



$$z^{10} = -\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)$$

$$= -\frac{1}{2} - i\,\frac{\sqrt{3}}{2}$$

$$z^6 = 1$$



B:2.6 $z = 1+i \quad w = 2-i$

$$\frac{z}{w} = \frac{(1+i)(2+i)}{(2-i)(2+i)} = \frac{2+2i+i+i^2}{2^2-i^2} = \frac{1+3i}{5} = \underline{\underline{\frac{1}{5}+\frac{3}{5}i}}$$

$$\bar{z} - \frac{w}{z} = 1-i-\left(\frac{1}{2}-\frac{3}{2}i\right) = \underline{\underline{\frac{1}{2}+\frac{1}{2}i}}$$

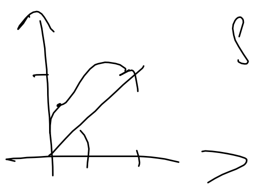
$$\frac{w}{z} = \frac{2-i}{1+i} = \frac{(2-i)(1-i)}{(1+i)(1-i)} = \frac{2-i-2i+i^2}{1^2-i^2} = \frac{1-3i}{2} = \frac{1}{2}-\frac{3}{2}i$$

B:2.10

$$z = \frac{1}{\sqrt{2}}(1+i)e^{-i\frac{\pi}{4}}$$

a) 1 b) $\frac{1}{\sqrt{2}}$ c) $1-i$

$1+i$



$\rho = \sqrt{2}, \ \theta = \frac{\pi}{4} \qquad 1+i = \sqrt{2}e^{i\frac{\pi}{4}}$

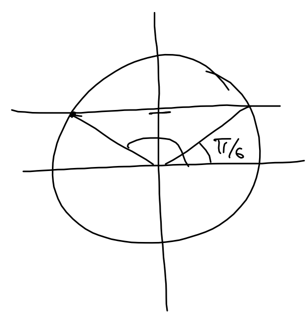
$$\Rightarrow \quad z = \frac{1}{\cancel{\sqrt{2}}}\cancel{\sqrt{2}}\underbrace{e^{i\frac{\pi}{4}}e^{-i\frac{\pi}{4}}}_{e^0} = \underline{\underline{1}}$$

B:2.11

$$z = \frac{1}{2}(\sqrt{3}+i)e^{i\frac{\pi}{3}}$$

a) i b) 1 c) $\frac{1}{2}\sqrt{3}+2i$

$\sqrt{3}+i = \rho(\cos\theta + i\sin\theta)$



$\rho = \sqrt{(\sqrt{3})^2+1^2} = 2$

$$\left.\begin{array}{l}\cos\theta = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{1}{2}\end{array}\right\} \Rightarrow \theta = \frac{\pi}{6}$$

$$z = \frac{1}{\cancel{2}}\cancel{2}e^{i\pi/6}e^{i\frac{\pi}{3}} = e^{i\left(\frac{\pi}{6}+\frac{\pi}{3}\right)} = e^{i\frac{\pi}{2}} = \underline{\underline{i}}$$

$\underline{\underline{a}}$

$$e^{i\frac{\pi}{2}} = 1\left(\overbrace{\cos\frac{\pi}{2}}^{0} + \overbrace{i\sin\frac{\pi}{2}}^{1}\right) = i$$

8.2 Finn løsningen til differenslikningen

$$x_{n+2} - \frac{1}{2}x_{n+1} = \frac{1}{2}x_n \quad , \; n \geq 0$$

$$x_0 = 2 \;, \; x_1 = \frac{1}{2}$$

standardform:

$$x_{n+2} - \frac{1}{2}x_{n+1} - \frac{1}{2}x_n = 0$$

Kar. likn: $r^2 - \frac{1}{2}r - \frac{1}{2} = 0$

$$r = \frac{\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - 4\cdot 1\cdot\left(-\frac{1}{2}\right)}}{2} = \frac{\frac{1}{2} \pm \sqrt{\frac{9}{4}}}{2} = \frac{1}{4} \pm \frac{3}{4}$$

$$r_1 = 1 \;, \; r_2 = -\frac{1}{2}.$$

Generell løsning: $\underline{x_n = C + D\left(-\frac{1}{2}\right)^n}$

$$x_0 = 2 \Rightarrow 2 = C + D \quad : \mathrm{I}$$

$$x_1 = \frac{1}{2} \Rightarrow \frac{1}{2} = C + D\left(-\frac{1}{2}\right)$$

$$= C - \frac{1}{2}D \quad : \mathrm{II}$$

$$2 - \frac{1}{2} = \cancel{C - C} + D + \frac{1}{2}D \qquad (\mathrm{I} - \mathrm{II})$$

$$\frac{3}{2} = \frac{3}{2}D \Rightarrow D = 1 \;, \; C = 1$$

$$\underline{\underline{x_n = 1 + \left(-\frac{1}{2}\right)^n}}$$

b) $\lim_{n\to\infty} x_n = \lim_{n\to\infty}\left(1 + \left(-\frac{1}{2}\right)^n\right) =$

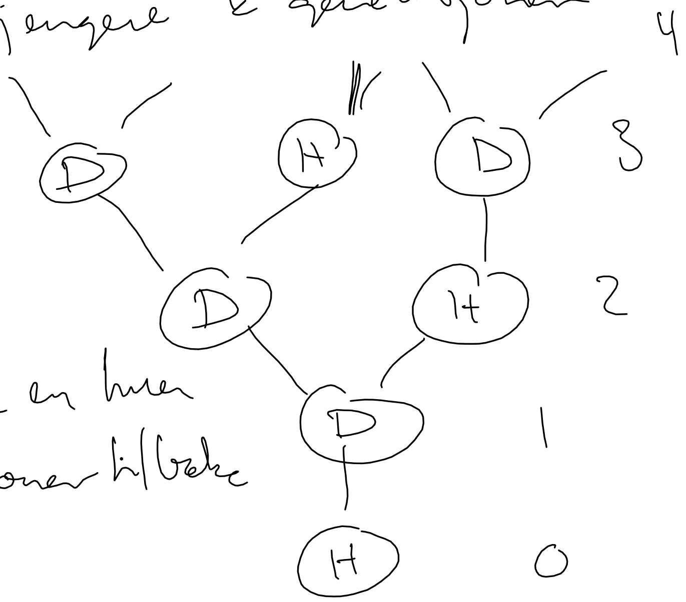
$$= \underbrace{\lim_{n\to\infty} 1}_{=1} + \underbrace{\lim_{n\to\infty}\left(-\frac{1}{2}\right)^n}_{=0} = \underline{\underline{1}}$$

8.5

x_k = Antall forgjengere k generasjoner tilbake.

$x_0 = 1$

$x_1 = 1$



y_k: Antall forgjengere en hunn har k generasjoner tilbake

$$\begin{cases} x_{k+1} = y_k \\ y_{k+1} = x_k + y_k \end{cases}$$

okt 1-13:57