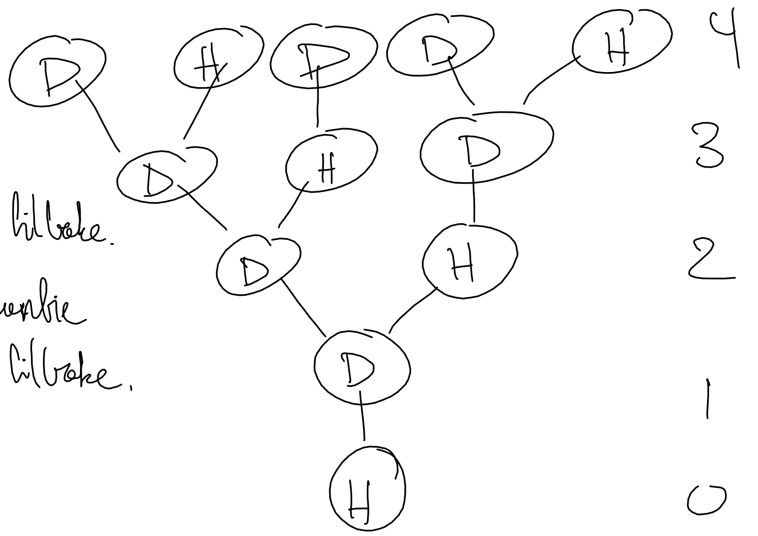
A8: 2, 5, 10, 12, 14

B.2: 2, 7, 9, 11

8.5



x_k: Antall forgjengere for en hanbie k generasjoner tilbake.

y_k: Antall forgjengere en hunbie har k generasjoner tilbake.

$$\begin{cases} x_{k+1} = y_k \\ y_{k+1} = x_k + y_k \end{cases}$$

$$y_k = x_{k-1} + y_{k-1} \implies x_{k+1} = x_{k-1} + \underbrace{y_{k-1}}_{= x_k}$$

$$x_{k+1} = x_{k-1} + x_k$$

Dette medfører at x_k er fibonaccitallene.

8.10

a) Løs andregradslikningen

$$z^2+2z+4=0$$

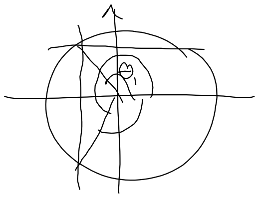
og skriv løsningen på eksponentialform.

$$z=\frac{-2\pm\sqrt{4-4\cdot 1\cdot 4}}{2}=\frac{-2\pm 2\sqrt{3}i}{2}=-1\pm\sqrt{3}i$$

$z_1=-1+\sqrt{3}i$, $z_2=-1-\sqrt{3}i$.

$\rho_1=\sqrt{(-1)^2+(\sqrt{3})^2}=2$ $\qquad z_1=\rho_1(\cos\theta_1+i\sin\theta_1)$

$$\left.\begin{aligned}\cos\theta_1&=-\frac{1}{2}\\ \sin\theta_1&=\frac{\sqrt{3}}{2}\end{aligned}\right\}\Rightarrow \theta_1=\frac{\pi}{2}+\frac{\pi}{6}=\frac{2\pi}{3}\Rightarrow z_1=2e^{i\frac{2\pi}{3}}.$$



$$\theta_2=2\pi-\frac{2\pi}{3}=\frac{6\pi}{3}-\frac{2\pi}{3}=\frac{4\pi}{3}$$

$$z_2=2e^{i\frac{4\pi}{3}}.$$

b) Finn den generelle løsningen til

$$x_{n+2}+2x_{n+1}+4x_n=0$$

Skriv løsningen uten bruk av komplekse tall.

Kar. likn.: $r^2+2r+4=0$

$r_1=2e^{i\frac{2\pi}{3}}$, $r_2=2e^{i\frac{4\pi}{3}}$

$$x_n=C2^n\cos\left(n\frac{2\pi}{3}\right)+D2^n\sin\left(n\frac{2\pi}{3}\right)$$

c) Finn den generelle løsningen til

$$x_{n+2}+2x_{n+1}+4x_n=21n$$

$x_n=x_n^h+x_n^s$. Anta at $x_n^s=An+B$

$x_{n+2}^s=A(n+2)+B=An+2A+B$

$x_{n+1}^s=A(n+1)+B=An+A+B$

$$An+2A+B+2(An+A+B)+4(An+B)=21n$$

$$7An+4A+7B=21n$$

$$\begin{cases}7A=21 & \Rightarrow A=3\\ 4A+7B=0 & \Rightarrow 7B=-4\cdot A=-12,\ B=-\frac{12}{7}\end{cases}$$

$$x_n^s=3n-\frac{12}{7}.$$

$$x_n=C2^n\cos\left(\frac{2n\pi}{3}\right)+D2^n\sin\left(\frac{2n\pi}{3}\right)+3n-\frac{12}{7}$$

okt 15-12:33

8.12 Vi har gitt diff. likn.

$$x_{n+2} - x_n = 0 \qquad (x_{n+2} = x_n)$$

a) Vis at likningen ikke har noen løsning for $x_0 = 0$ og $x_2 = 1$.

For $n = 0 \Rightarrow x_2 = x_0$. Dette gir $1 = 0$ for $x_2 = 1$ og $x_0 = 0$.

b) Finn løsning for $x_0 = 0, x_1 = 1$.

Kar. likn. $r^2 - 1 = 0 \qquad (r+1)(r-1) = r^2 - 1$

$r_1 = 1, \; r_2 = -1.$

$$\underline{\underline{x_n = C + D(-1)^n}}$$

$x_0 = 0 \Rightarrow C + D = 0$
$x_1 = 1 \Rightarrow C - D = 1$ $\qquad 2C = 1, \; C = \frac{1}{2}, \; D = -\frac{1}{2}$

$$\underline{\underline{x_n = \frac{1}{2} - \frac{1}{2}(-1)^n}}$$

$\left(x_0 = 0, \; x_2 = 1 \Rightarrow \begin{matrix} C + D = 0 \\ C + D = 1 \end{matrix} \right)$

8.14

x_n: Sannsynligheten for at Mia vinner når hun har n kroner.

Gitt at Mia har n kroner og skal vinne kan enten

(i) Hun vinner runden og etterpå spillet.

(ii) Hun taper runden, og etterpå vinner hun spillet.

$$x_n = \frac{3}{5}x_{n+1} + \frac{2}{5}x_{n-1}$$

$$\Rightarrow \quad \frac{3}{5}x_{n+2} - x_{n+1} + \frac{2}{5}x_n = 0$$

Kar likn: $\frac{3}{5}r^2 - r + \frac{2}{5}$

$$r = \frac{1 \pm \sqrt{1^2 - 4\frac{3}{5}\cdot\frac{2}{5}}}{2\cdot\frac{3}{5}} = \frac{1 \pm \sqrt{\frac{1}{25}}}{\frac{6}{5}} = \frac{5\left(1 \pm \frac{1}{5}\right)}{6}$$

$$= \frac{5 \pm 1}{6}, \quad r_1 = 1, \quad r_2 = \frac{2}{3}$$

$$x_n = C + D\left(\frac{2}{3}\right)^n$$

($x_0 = 0$, $x_{20} = 1$ Initial betingelser.)

$$x_0 = 0 \Rightarrow C + D = 0 \qquad : L_1$$

$$x_{20} = 1 \Rightarrow C + D\left(\frac{2}{3}\right)^{20} = 1 \qquad : L_2$$

$$L_1 - L_2 \Rightarrow D - D\left(\frac{2}{3}\right)^{20} = -1$$

$$D\left(1 - \left(\frac{2}{3}\right)^{20}\right) = -1$$

$$D = \frac{1}{\left(\frac{2}{3}\right)^{20} - 1}, \quad L_1 \Rightarrow C = \frac{1}{1 - \left(\frac{2}{3}\right)^{20}}$$

$$x_n = \frac{1}{1 - \left(\frac{2}{3}\right)^{20}} - \frac{1}{1 - \left(\frac{2}{3}\right)^{20}}\left(\frac{2}{3}\right)^n$$

$$= \frac{1}{1 - \left(\frac{2}{3}\right)^{20}}\left(1 - \left(\frac{2}{3}\right)^n\right)$$

$\underline{\underline{x_5 \approx 0.87}}$

okt 15-13:28

B:2.7

a) Finn den generelle løsningen til

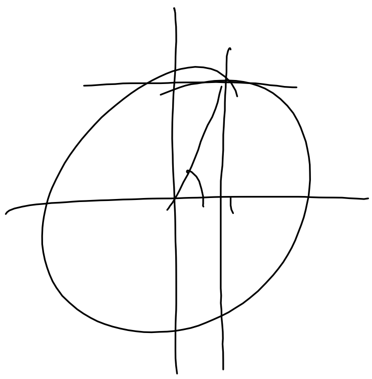
$$x_{n+2} - x_{n+1} + x_n = 0$$

og skriv på både kompleks og reell form.

Kar. likn: $r^2 - r + 1 = 0$

$$r = \frac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot 1}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$x_n = E \Big(\underbrace{\frac{1}{2} + \frac{\sqrt{3}}{2} i}_{r_1}\Big)^n + \bar{E} \Big(\underbrace{\frac{1}{2} - \frac{\sqrt{3}}{2} i}_{r_2}\Big)^n$$



$$\rho_1 = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

$$\left.\begin{aligned} \cos\theta &= \frac{1}{2} \\ \sin\theta &= \frac{\sqrt{3}}{2} \end{aligned}\right\} \quad \theta = \frac{\pi}{3}$$

$$\underline{\underline{x_n = C\cos\left(n\frac{\pi}{3}\right) + D\sin\left(n\frac{\pi}{3}\right)}}$$