A8: 20

A10: 3, 5, 9, 10

B.1.11

10.3 Finn grenseverdiene

a) $\lim_{x \to \infty} \frac{7x^2 + 4x^4}{3x^3 - 2x^2} = \lim_{x \to \infty} \frac{\overset{\to 0}{7x^{-1}} + \overset{\to \infty}{4x}}{3 - \underset{\to 0}{2x^{-1}}} = \infty$

b) $\lim_{x \to \infty} \frac{8x^2 + 2x + 7}{\sqrt{x} - 4x^2} = \lim_{x \to \infty} \frac{8 + \frac{2}{x} + \frac{7}{x^2}}{\frac{1}{x^{3/2}} - 4} = -2$

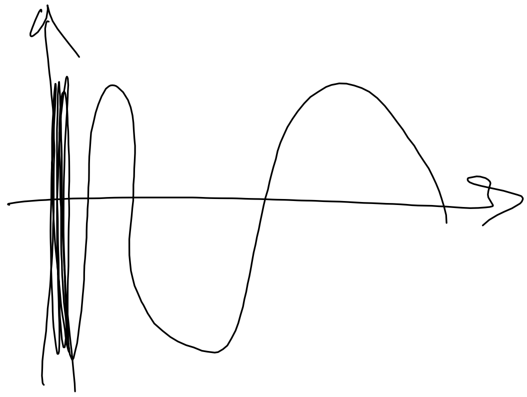
c) $\lim_{x \to \infty} \frac{x^4 + \sqrt{x} + e^{x^2}}{7 + \sin(\sqrt{x})} = \infty$

d) $\lim_{x \to \infty} \frac{x^2 - 4x^3}{8 + 7x^2} = \lim_{x \to \infty} \frac{1 - 4x}{\frac{8}{x^2} + 7} = -\infty$

okt 22-12:09

10.5 Eksisterer følgende grenseverdier?

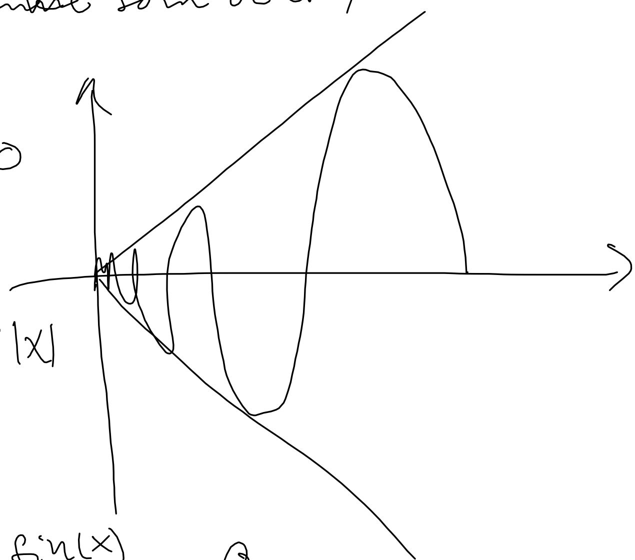
a) $\lim\limits_{x \to 0} \sin\left(\frac{1}{x}\right)$



Funksjonen oscillerer fortere og fortere mellom -1 og 1 når vi nærmer oss 0.

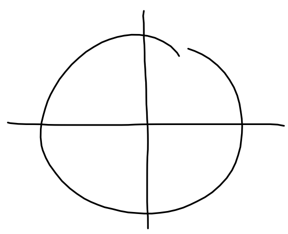
b) $\lim\limits_{x \to 0} \cos\left(\frac{1}{x}\right)$ (samme som over)

c) $\lim\limits_{x \to 0} x \cos\left(\frac{1}{x}\right) = 0$



$$\left|x \cos\left(\frac{1}{x}\right)\right| = |x| \underbrace{\left|\cos\left(\frac{1}{x}\right)\right|} \leq |x|$$

d) $\lim\limits_{x \to 0} \tan(x) = \lim\limits_{x \to 0} \frac{\sin(x)}{\cos(x)} = \underline{\underline{0}}$



10.9 Finn middelverdi, amplitude, periode og akrofase. Skisser funksjonene.

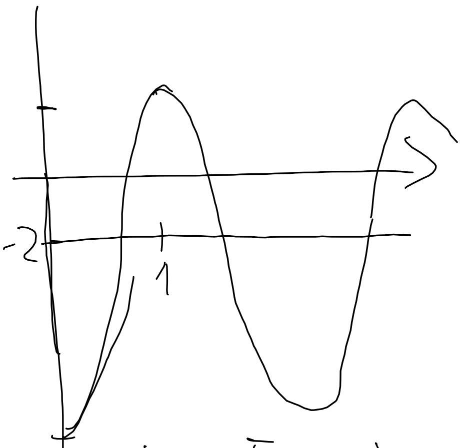
a) $f(t) = -2 + 4\cos(\pi t - \pi)$

$$f(t) = A_0 + A\cos\left(\frac{2\pi}{T}(t - t_0)\right)$$

A_0: middelverdi

A: amplitude

T: periode



$A_0 = -2$, $A = 4$ $\quad \pi t - \pi = \pi(t-1) = \frac{2\pi}{2}(t-1)$

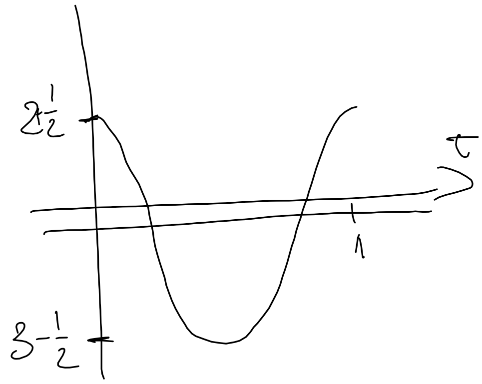
$T = 2$. $\quad t_0 = 1 \quad$ akrofase $= 1$.

$f(0) = -2 + 4\cos(-\pi) = \underline{-6}$

b) $f(t) = -\frac{1}{2} + 3\cos(\underbrace{\pi(2t-2)})$

$A_0 = -\frac{1}{2}$, $A = 3$ $\quad \frac{2\pi}{1}(t-1)$

$T = 1$ $\quad$ akrofase $= 0$.



c)

10.10 Skriv uttrykket på formen

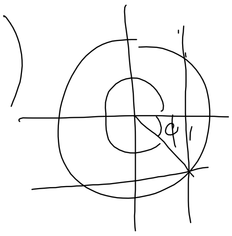
$A\cos(bx-\phi)$

a) $\cos x - \sin x = A\cos(bx-\phi)$

$$\left[\begin{array}{l} C\cos(bx)+D\sin(bx) = A\cos(bx-\phi) \\ A=\sqrt{C^2+D^2} \quad \cos\phi = \frac{C}{A},\ \sin\phi = \frac{D}{A} \end{array}\right]$$

$C=1,\ D=-1,\ b=1$

$A = \sqrt{1+1} = \sqrt{2}$ $\qquad \left(\phi = \frac{7\pi}{4}\right)$



$$\left.\begin{array}{l} \cos\phi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \sin\phi = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \end{array}\right\} \phi = -\frac{\pi}{4}$$

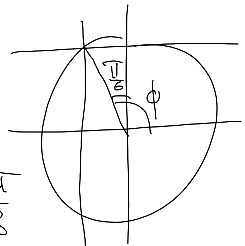
$$\cos x - \sin x = \underline{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}$$

b) $-\sqrt{3}\cos(3x)+3\sin(3x) = A\cos(bx-\phi)$

$$\left[\begin{array}{l} C\cos(bx)+D\sin(bx) = A\cos(bx-\phi) \\ A=\sqrt{C^2+D^2} \quad \cos\phi = \frac{C}{A},\ \sin\phi = \frac{D}{A} \end{array}\right]$$

$C=-\sqrt{3},\ D=3,\ b=3$

$A = \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$



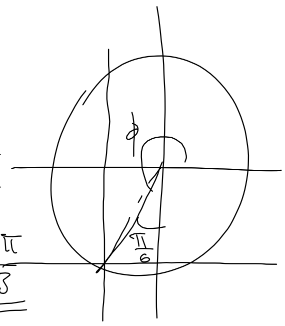
$$\left.\begin{array}{l} \cos\phi = -\frac{\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2} \\ \sin\phi = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \end{array}\right\} \begin{array}{l} \phi = \frac{\pi}{2}+\frac{\pi}{6} \\ \quad = \frac{4\pi}{6} = \frac{2\pi}{3} \end{array}$$

$$\underline{2\sqrt{3}\cos\left(3x-\frac{2\pi}{3}\right)}$$

c) $-\cos\left(\frac{x}{4}\right)-\sqrt{3}\sin\left(\frac{x}{4}\right) = A\cos(bx-\phi)$

$C=-1,\ D=-\sqrt{3},\ b=\frac{1}{4}$

$A=\sqrt{1+3} = 2$



$$\left.\begin{array}{l} \cos(\phi) = -\frac{1}{2} \\ \sin(\phi) = -\frac{\sqrt{3}}{2} \end{array}\right\} \begin{array}{l} \phi = \frac{3\pi}{2} - \frac{\pi}{6} = \frac{9\pi-\pi}{6} \\ \quad = \frac{8\pi}{6} = \underline{\frac{4\pi}{3}} \end{array}$$

$$-\cos\left(\frac{x}{4}\right)-\sqrt{3}\sin\left(\frac{x}{4}\right) = \underline{2\cos\left(\frac{x}{4}-\frac{4\pi}{3}\right)}$$

B.1.11

a) For hvilke verdier av a vil likningssystemet

$$\begin{cases} 2x + a^2 y = a \\ x + 2y = 1 \end{cases}$$

ha én løsning? uendelig mange løsninger? eller ingen.

Koeffisientmatrisen er gitt ved $\begin{bmatrix} 2 & a^2 \\ 1 & 2 \end{bmatrix} =: A$

$$\det(A) = \begin{vmatrix} 2 & a^2 \\ 1 & 2 \end{vmatrix} = 2 \cdot 2 - a^2 = 4 - a^2 = (2-a)(2+a)$$

Systemet har én løsning for $a \notin \{2, -2\}$

$a = 2$

$$\begin{cases} 2x + 4y = 2 & : L_1 \\ x + 2y = 1 & : L_2 \end{cases}$$

$L_1 = 2L_2 \Rightarrow$ uendelig mange løsninger.

$a = -2$

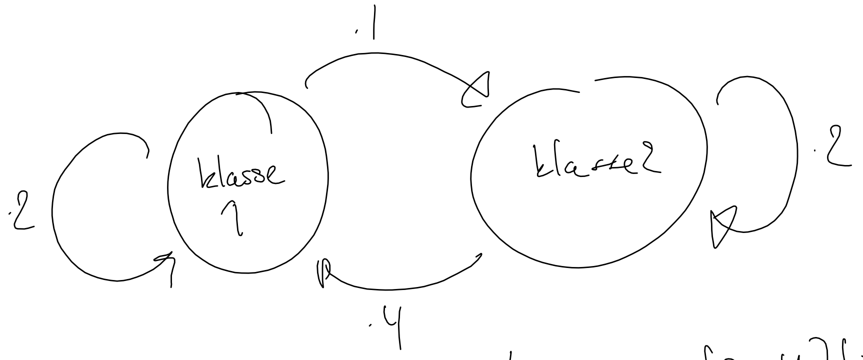
$$\begin{cases} 2x + 4y = -2 \\ x + 2y = 1 \end{cases}$$

$$\begin{bmatrix} 2 & 4 & -2 \\ 1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & -4 \end{bmatrix}$$

$\Rightarrow$ ingen løsning.

b) X_n : Antall planter i klasse 1 (unge)

y_n : Antall planter i klasse 2 (voksne)



$$x_{n+1} = 2x_n + 4y_n \qquad \left(\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}}_{A} \begin{bmatrix} x_n \\ y_n \end{bmatrix} \right)$$
$$y_{n+1} = x_n + 2y_n$$

$$X_0 + y_0 = 100$$

Anta $x_2 = 1200$ og $y_2 = 600$. Finn x_0.

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = A \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = AA \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = A^2 \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 16 \\ 4 & 8 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 8 & 16 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 8x_0 + 16y_0 \\ 4x_0 + 8y_0 \end{bmatrix}$$

$$\overset{\shortparallel}{\begin{bmatrix} 1200 \\ 600 \end{bmatrix}} \qquad \left\{ \begin{array}{l} 8x_0 + 16y_0 = 1200 \\ 4x_0 + 8y_0 = 600 \end{array} \right\}$$

uendelig mange løsn.

$\Rightarrow x_0 + 2y_0 = 150$

Vi har også $x_0 + y_0 = 100$

$$\left\{ \begin{array}{l} x_0 + y_0 = 100 \\ x_0 + 2y_0 = 150 \end{array} \right. \qquad \underline{x_0 = 50 \ , \ y_0 = 50}$$