Oppgaver: All: 3, 4, 5

B: 1.12, 1.16

11.3 Deriver funksjonene:

a) $f(x) = \ln|\cos(x)| + \sin\left(\frac{x}{2}\right) \qquad (\cos(x) \neq 0)$

$$\Big(\ln|\underbrace{\cos(x)}_{u(x)}|\Big)' = \frac{1}{\cos(x)} \cdot (\cos(x))' = -\frac{\sin x}{\cos x} = -\tan x$$

$$\left(\sin\left(\frac{x}{2}\right)\right)' = \cos\left(\frac{x}{2}\right) \cdot \frac{1}{2} = \frac{1}{2}\cos\left(\frac{x}{2}\right)$$

$$f'(x) = -\tan x + \frac{1}{2}\cos\left(\frac{x}{2}\right)$$

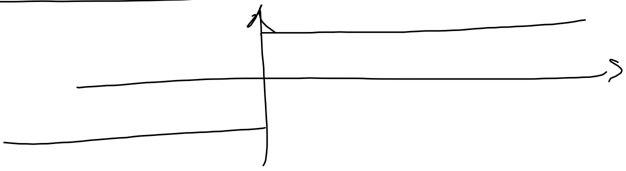
b) $f(x) = \dfrac{4x^3 + 5x}{\sin(x+2)} \qquad u(x) = 4x^3 + 5x, \; v(x) = \sin(x+2)$

$$\left[\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}\right]$$

$u'(x) = 4 \cdot 3x^2 + 5 \quad , \quad v'(x) = \cos(x+2) \cdot 1$

$$f'(x) = \frac{(12x^2 + 5)\sin(x+2) - (4x^3 + 5x)\cos(x+2)}{(\sin(x+2))^2}$$

c) $f(x) = \dfrac{x}{|x|}$



$f'(x) = 0 \qquad x \neq 0.$

$$f'(x) = \frac{1 \cdot |x| - x\frac{|x|}{x}}{|x|^2} = \frac{|x| - |x|}{|x|^2} = 0$$

e) $f(x) = e^{2x^2 + x - 1} \qquad u(x) = 2x^2 + x - 1$

$u'(x) = 4x + 1$

$$f'(x) = e^{u(x)}u'(x) = e^{2x^2+x-1}(4x+1)$$

11.4 Deriver funksjonene:

a) $f(x) = \sin x \cos x + x^2$ $\quad (uv)' = u'v + uv'$

$(\sin x \cos x)' = \cos x \cos x + \sin x(-\sin x)$

$= (\cos x)^2 - (\sin x)^2$

$f'(x) = \cos^2 x - \sin^2 x + 2x$

b) $f(x) = \cos(\underbrace{\sqrt{x}}_{u(x)}) - \sqrt{x}$ $\quad u'(x) = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f'(x) = -\sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}} = \frac{-1}{2\sqrt{x}}\left(\sin(x) + 1\right)$.

c) $f(x) = \frac{\sin x^2}{3x} = \underbrace{\sin x^2}_{u(x)} \cdot \underbrace{\frac{1}{3x}}_{v(x)}$

$f'(x) = 2x \cos x^2 \frac{1}{3x} + \sin x^2 \left(-\frac{1}{3x^2}\right)$

$= \frac{2}{3}\cos x^2 - \frac{\sin x^2}{3x^2}$

$(uv)' = u'v + uv'$

$u'(x) = \cos x^2 \cdot 2x$

$v'(x) = \left(\frac{1}{3}x^{-1}\right)' = \frac{1}{3}\left(-\frac{1}{x^2}\right)$

$= -\frac{1}{3x^2}$

11.5 Deriver:

a) $f(x) = \cos x^2 = -\sin(x^2) \cdot 2x = -2x\sin(x^2)$.

b) $f(x) = 4x \ln x$ $\quad (x > 0)$

$= 4\ln x + 4$

c) $f'(x) = (e+2)3x^2$ $\quad f(x) \overset{?}{=} (e+2)x^3$

d) $f(x) = e^{\overbrace{2x^2 + \ln(x+2)}^{u(x)}}$ $\quad u'(x) = 4x + \frac{1}{x+2}$

$f'(x) = e^{2x^2 + \ln(x+2)}\left(4x + \frac{1}{x+2}\right)$.

B 1.16

a) Finn egenverdiene og egenvektorene til

$$M = \begin{bmatrix} \frac{1}{2} & \frac{5}{9} \\ \frac{5}{9} & \frac{1}{2} \end{bmatrix}.$$

$Mv = \lambda v$

$(M - \lambda I)v = 0$

$$\det(M - \lambda I) = 0$$

Kar. likn.:

$$M - \lambda I = \begin{bmatrix} \frac{1}{2}-\lambda & \frac{5}{9} \\ \frac{5}{9} & \frac{1}{2}-\lambda \end{bmatrix} \Rightarrow \left(\frac{1}{2}-\lambda\right)^2 - \left(\frac{5}{9}\right)^2 = 0.$$

$$\frac{1}{4} - \lambda + \lambda^2 - \frac{25}{81} = 0$$

$$\frac{1}{4} - \frac{25}{81} = \frac{81-100}{4\cdot 81} = -\underbrace{\frac{19}{4\cdot 81}}_{324}$$

$$\lambda^2 - \lambda - \frac{19}{324} = 0$$

$$\lambda = \frac{1 \pm \sqrt{1 + 4\cdot\frac{19}{4\cdot 81}}}{2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{81+19}{81}}$$

$$= \frac{1}{2} \pm \frac{1}{2}\cdot\frac{10}{9} = \frac{1}{2} \pm \frac{5}{9}.$$

Egenverdiene er $\left\{ \frac{1}{2} + \frac{5}{9}, \frac{1}{2} - \frac{5}{9} \right\}$

$\lambda = \frac{1}{2} + \frac{5}{9}$

$Mx = \lambda x \Leftrightarrow (M - \lambda I)x = 0$ $\left(M = \begin{bmatrix} \frac{1}{2} & \frac{5}{9} \\ \frac{5}{9} & \frac{1}{2} \end{bmatrix} \right)$

$$\left[\begin{array}{cc|c} -\frac{5}{9} & \frac{5}{9} & 0 \\ \frac{5}{9} & -\frac{5}{9} & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 1 & -1 & 0 \end{array}\right] \cdot(-1)$$

$$\sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 - x_2 = 0 \\ x_1 = x_2, \quad x_2 = t \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix} = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$L = \left\{ t\begin{bmatrix} 1 \\ 1 \end{bmatrix} : t \in \mathbb{R} \right\}$$

$\lambda = \frac{1}{2} - \frac{5}{9}$

$(M - \lambda I)x = 0$ $\quad M = \begin{bmatrix} \frac{1}{2} & \frac{5}{9} \\ \frac{5}{9} & \frac{1}{2} \end{bmatrix}$

$$\left[\begin{array}{cc|c} \frac{5}{9} & \frac{5}{9} & 0 \\ \frac{5}{9} & \frac{5}{9} & 0 \end{array}\right] \begin{array}{l} -1 \;\cdot\frac{9}{5} \end{array} \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow x_1 + x_2 = 0$$

$x_2 = t$

$x_1 = -x_2 = -t$

$$L = \left\{ t\begin{bmatrix} -1 \\ 1 \end{bmatrix} : t \in \mathbb{R} \right\}$$

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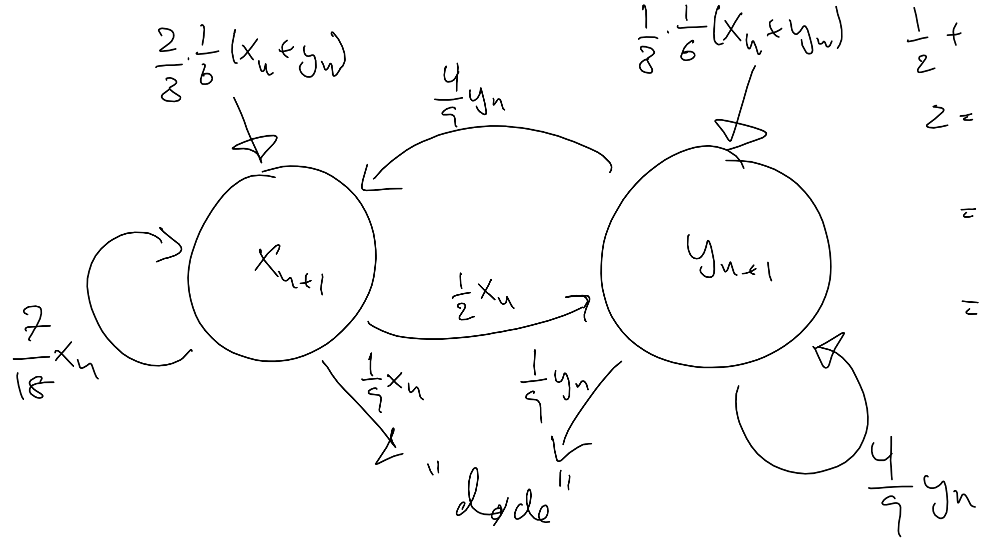
y_n: Antall immune etter $10n$ år.

x_n: Antall mottakelig for smitte etter $10n$ år.



$$\frac{1}{2}+\frac{1}{9}+z=1$$

$$z=1-\frac{1}{2}-\frac{1}{9}=\frac{18-9-2}{18}=\frac{7}{18}$$

$$x_{n+1}=\frac{7}{18}x_n+\frac{2}{3}\cdot\frac{1}{6}(x_n+y_n)+\frac{4}{9}y_n$$

$$y_{n+1}=\frac{4}{9}y_n+\frac{1}{3}\cdot\frac{1}{6}(x_n+y_n)+\frac{1}{2}x_n$$

$$x_{n+1}=\left(\frac{7}{18}+\frac{2}{18}\right)x_n+\left(\frac{4}{9}+\frac{1}{9}\right)y_n$$

$$=\frac{1}{2}x_n+\frac{5}{9}y_n$$

$$y_{n+1}=\left(\frac{4}{9}+\frac{1}{18}\right)y_n+\left(\frac{1}{2}+\frac{1}{18}\right)x_n$$

$$=\frac{5}{9}x_n+\frac{1}{2}y_n$$

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix}=\underbrace{\begin{bmatrix} \frac{1}{2} & \frac{5}{9} \\ \frac{5}{9} & \frac{1}{2} \end{bmatrix}}_{M}\begin{bmatrix} x_n \\ y_n \end{bmatrix}$$

11.8 Finn de ubestemte integralene.

a) $\int (x^5 - 3x)\,dx = \frac{1}{6}x^6 - \frac{3}{2}x^2 + C$

b) $\int (x^{5/6} - 1)\,dx = \frac{1}{\frac{5}{6}+1} x^{\frac{5}{6}+1} - x + C$

$= \frac{6}{11} x^{\frac{11}{6}} - x + C$

c) $\int 3x e^{x^2}\,dx =$

$u(x) = x^2 \quad \frac{du}{dx} = 2x \quad du = 2x\,dx$

$u'(x) = 2x$

$\frac{du}{dx} = u'(x)$

$du = u'(x)\,dx$

$\int 3x e^{x^2}\,dx = \frac{3}{2}\int 2x e^{x^2}\,dx$

$= \frac{3}{2}\int u'(x) e^{u(x)}\,dx$

$= \frac{3}{2}\int e^u\,du = \frac{3}{2}e^u + C$

$= \frac{3}{2}e^{x^2} + C$

d) $\int x^3 \sin(x^4)\,dx \qquad u(x) = x^4 \quad u'(x) = 4x^3$

$du = u'(x)\,dx$

$= \frac{1}{4}\int 4x^3 \sin(x^4)\,dx = \frac{1}{4}\int u'(x)\sin(u(x))\,dx$

$= \frac{1}{4}\int \sin(u)\,du = -\frac{1}{4}\cos u + C$

$= -\frac{1}{4} \cancel{\cos} \cos x^4 + C.$