Oppgaver: A11: 8, 9, 10
A12: 2, 3, 5
B3: 11, 12

11.9 Finn de ubestemte integralene:

a) $\int e^x \sin x\,dx$ $u'(x) = e^x$ $v(x) = \sin x$

$\left(\int u'v = uv - \int uv'\right)$

$\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx$

$u' = e^x$ $v = \cos x$

$\int e^x \cos x\,dx = e^x \cos x + \int e^x \sin x\,dx$

$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$

$\int e^x \sin x\,dx = \frac{1}{2} e^x (\sin x - \cos x) + C$

Alternativ: $e^{ix} = \cos x + i \sin x$

b) $\int \frac{\ln x}{x}\,dx$ substitusjon: $u(x) = \ln x$ $u'(x) = \frac{1}{x}$

$\int \frac{1}{x} \ln x\,dx = \int u\,du = \frac{1}{2}u^2 + C$ $\frac{du}{dx} = \frac{1}{x}$ $du = \frac{1}{x}dx$

$= \frac{1}{2}(\ln x)^2 + C$

c) $\int \frac{1}{x^2-4}\,dx$ $(x^2-4) = (x-2)(x+2)$

$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2} \Rightarrow 1 = A(x+2) + B(x-2)$

$x = 2 \Rightarrow 1 = 4A \Leftrightarrow A = \frac{1}{4}$

$x = -2 \Rightarrow 1 = -4B$ $B = -\frac{1}{4}$

$\int \frac{1}{x^2-4}\,dx = \int \frac{1}{4(x-2)}\,dx - \int \frac{1}{4(x+2)}\,dx$

$= \frac{1}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| + C$

$= \frac{1}{4} \ln \frac{|x-2|}{|x+2|} + C$

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11.10

a) $\int 3\ln x\,dx \qquad u'=3 \qquad v=\ln x$

$$\int 3\ln x\,dx = 3x\ln x - \underbrace{\int 3x\cdot\frac{1}{x}\,dx}_{3x+C}$$

$$= \underline{\underline{3(x\ln x - x) + C}}$$

b) $\int \cos^2 x\,dx$

$$\left.\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ 1 &= \cos^2 x + \sin^2 x \end{aligned}\right\} = 2\cos^2 x = \cos 2x + 1$$

$$\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2}$$

$$\int \cos^2 x\,dx = \frac{1}{2}\int \cos 2x\,dx + \frac{1}{2}x$$

$$u(x) = 2x \quad \frac{du}{dx} = 2 \quad du = 2dx$$

$$\frac{1}{2}\int \cos 2x\,dx = \frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin(2x)$$

$$\int \cos^2 x\,dx = \underline{\underline{\frac{1}{4}\sin(2x) + \frac{1}{2}x + C}}$$

d) $\int x\sin\sqrt{x}\,dx \qquad u(x) = \sqrt{x} \quad u'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2u}$

$$\frac{du}{dx} = \frac{1}{2u} \quad \boxed{2u\,du = dx}$$

$$x = u^2$$

$$\int u^2\sin(u)2u\,du = 2\underbrace{\int u^3\sin(u)\,du}_{I_1}$$

$f = -\cos u \quad g' = 3u^2$

$f' = \sin u \quad g = u^3$

$$\int f'g = fg - \int fg'$$

$$I_1 = -\cos(u)u^3 + 3\underbrace{\int u^2\cos u\,du}_{I_2}$$

$f' = \cos u \quad g = u^2$

$f = \sin u \quad g' = 2u$

$$I_2 = u^2\sin u - 2\underbrace{\int u\sin u\,du}_{I_3}$$

$f' = \sin u \quad g = u$

$f = -\cos u \quad g' = 1$

$$I_3 = -u\cos u + \int \cos u\,du$$

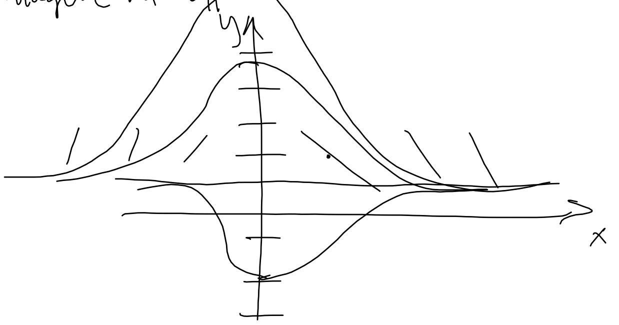
$$-u\cos u + \sin u$$

$$I = -2u^3\cos u + 6\left(u^2\sin u - 2I_3\right)$$

$$= -2u^3\cos u + 6u^2\sin u + 12u\cos u - 12\sin u + C$$

$$= \underline{\underline{-2x^{3/2}\cos\sqrt{x} + 6x\sin\sqrt{x} + 12\sqrt{x}\cos\sqrt{x} - 12\sin\sqrt{x} + C}}$$

12.2 Finn alle løsningene til diff.lign.



a) $y' + 2xy = x$

$y' = x - 2xy$

$= x(1-2y)$

$e^{x^2} y' + 2x e^{x^2} y = x e^{x^2}$

$(e^{x^2} y)' = x e^{x^2}$ $\quad \frac{du}{dx} = 2x \quad du = 2x\,dx$

$e^{x^2} y = \frac{1}{2}\int 2x e^{x^2}\,dx + C$

$= \frac{1}{2}\int e^u\,du + C = \frac{1}{2} e^{x^2} + C$

$$\underline{\underline{y(x) = \frac{1}{2} + C e^{-x^2}}}$$

d) $(x^2+1)y' + 3xy = 6x$ $\qquad y' + f(x) y = g(x)$

$$y' + \underbrace{\frac{3x}{x^2+1}}_{f(x)} y = \underbrace{\frac{6x}{x^2+1}}_{g(x)}$$

$\int f(x)\,dx = \int \frac{3x}{\underbrace{x^2+1}_{u(x)}}\,dx = \frac{3}{2}\int \frac{1}{u}\,du = \frac{3}{2}\ln(x^2+1) + C$

$u'(x) = 2x \quad du = 2x\,dx$

Integrerende faktor $\exp\left(\frac{3}{2}\ln(x^2+1)\right) = \left(e^{\ln(x^2+1)}\right)^{3/2} = \underline{\underline{(x^2+1)^{3/2}}}$

$(x^2+1)^{3/2} y' + 3x\sqrt{x^2+1}\, y = 6x\sqrt{x^2+1}$

$\left((x^2+1)^{3/2} y\right)' = 6x\sqrt{x^2+1}$ $\qquad \frac{1}{\frac{1}{2}+1} u^{\frac{1}{2}+1} = \frac{2}{3} u^{\frac{3}{2}}$

$\Rightarrow (x^2+1)^{3/2} y = \int 6x\sqrt{\underbrace{x^2+1}_{u(x)}}\,dx = 3\int \sqrt{u}\,du$

$u'(x) = 2x \quad du = 2x\,dx$

$(x^2+1)^{3/2} y = \frac{2}{3}(x^2+1)^{3/2} + C$

$$\underline{\underline{y(x) = \frac{2}{3} + \frac{C}{(x^2+1)^{3/2}}}}$$

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12.5

$$\frac{dy}{dx} = y'(x), \quad dy = y'(x)dx$$

a) $y^2 y' = 5x$

$$\int y^2(x) y'(x) dx = \int 5x \, dx + C$$

$$\int y^2 dy = \frac{5}{2}x^2 + C$$

$$\frac{1}{3}y^3 = \frac{5}{2}x^2 + C, \quad y^3 = \frac{15}{2}x^2 + C$$

$$y = \sqrt[3]{\frac{15}{2}x^2 + C}$$

b) $\frac{1}{x} y' = \cos x \qquad x \neq 0$

$$y' = x \cos x$$

$$y(x) = \int x \cos x \, dx + C$$

$u(x) = x \quad v'(x) = \cos x$

$u'(x) = 1 \quad v(x) = \sin x$

$$\int x \cos x \, dx = x \sin x + \cos x + C$$

$$y(x) = x \sin x + \cos x + C$$

c) $e^y y' = e^{2x}$

$$\int e^y y' \, dx = \int e^{2x} dx + C$$

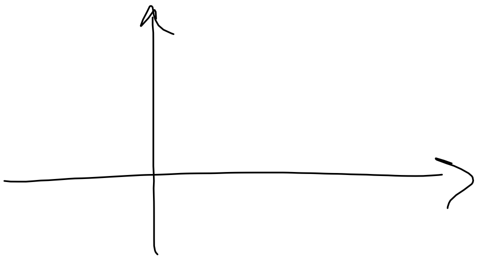
$$\int e^y dy = \frac{1}{2}e^{2x} + C$$

$$e^y = \frac{1}{2}e^{2x} + C \qquad y(x) = \ln\left(\frac{1}{2}e^{2x} + C\right)$$

12.3

$$xy' + (2x-3)y = 4x^4$$

$$y' + f(x)y = g(x)$$



Anta $x \neq 0$. Deler på x

$$y' + \underbrace{\left(2-\frac{3}{x}\right)}_{f} y = \underbrace{4x^3}_{g}$$

$$\int f(x)\,dx = \int 2 - \frac{3}{x}\,dx = 2x - 3\ln|x| + C$$

Integrerende faktor $e^{2x-3\ln|x|} = e^{2x}\left(e^{\ln|x|}\right)^{-3} = e^{2x}\frac{1}{|x|^3}$

$$\frac{1}{|x|^3} = \begin{cases} \frac{1}{x^3} & x > 0 \\ -\frac{1}{x^3} & x < 0 \end{cases}$$

$x > 0$

$$\frac{e^{2x}}{x^3}y' + \left(2-\frac{3}{x}\right)\frac{e^{2x}}{x^3}y = 4e^{2x}$$

$$\left(\frac{e^{2x}}{x^3}y\right)' = 4e^{2x}$$

$$\frac{e^{2x}}{x^3}y = 2e^{2x} + C$$

$$y(x) = 2x^3 + Cx^3e^{-2x}$$

$x < 0$

$$-\frac{e^{2x}}{x^3}y' - \left(2-\frac{3}{x}\right)\frac{e^{2x}}{x^3} =$$