Oppgaver: A12: 6, 9, 10, 12
B3: 1, 3

12.6 Finn den generelle løsningen:

a) $y'' + 2y' - 5y = 0$

Kar. likn: $r^2 + 2r - 5 = 0$

$$r = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-5)}}{2} = -1 \pm \sqrt{1-(-5)} = -1 \pm \sqrt{6}$$

$r_1 = -1 + \sqrt{6} \quad r_2 = -1 - \sqrt{6}$

$$\underline{y(x) = Ce^{(-1+\sqrt{6})x} + De^{(-1-\sqrt{6})x}}$$

c) $y'' + 4y' + 4y = 0$

Kar. likn: $r^2 + 4r + 4 = 0 \Leftrightarrow$

$(r+2)^2 = r^2 + 4r + 4$

$r_1 = r_2 = -2$

$$\underline{y(x) = (C + Dx)e^{-2x}}$$

12.10 Løs initialverdiproblemene.

a) $y' + \frac{x}{2}y = 2x \quad , \quad y(2) = 5 \qquad y' + f(x)y = g(x)$

$$y' = 2x - \frac{x}{2}y$$

$f(x) = \frac{x}{2} \quad F'(x) = f(x) \quad F(x) = \frac{1}{4}x^2$

Integrerende faktor $e^{F(x)} = e^{\frac{1}{4}x^2}$

$$e^{\frac{1}{4}x^2}y'(x) + \frac{x}{2}e^{\frac{1}{4}x^2}y(x) = 2xe^{\frac{1}{4}x^2}$$

$$(u v' + u' v = (uv)')$$

$$\left(e^{\frac{1}{4}x^2}y(x)\right)' = 2xe^{\frac{1}{4}x^2}$$

$$e^{\frac{1}{4}x^2}y(x) = \int 2xe^{\frac{1}{4}x^2}dx + C$$

$$\left(\begin{array}{l} u(x) = x^2 \quad u'(x) = 2x \quad du = u'(x)dx \\ \int 2xe^{\frac{1}{4}x^2}dx = \int e^{\frac{1}{4}u}du = 4e^{\frac{1}{4}u} + C \\ \qquad = 4e^{\frac{1}{4}x^2} + C \end{array} \right)$$

$$e^{\frac{1}{4}x^2}y(x) = 4e^{\frac{1}{4}x^2} + C$$

$$\underline{y(x) = 4 + Ce^{-\frac{1}{4}x^2}}$$

Initialverdi: $y(2) = 5$

$y(2) = 4 + Ce^{-1} \qquad Ce^{-1} = 1 \Rightarrow C = e$

$$y(x) = 4 + e^1e^{-\frac{1}{4}x^2} = \underline{\underline{4 + e^{1-\frac{1}{4}x^2}}}$$

b) $y\cos x + y' = \cos x \qquad y(0) = \pi.$

$y' + \cos x\, y = \cos x \quad \Rightarrow f(x) = \cos x \quad g(x) = \cos x.$

$F'(x) = f(x) \quad F(x) = \sin x$

$$e^{\sin x}y'(x) + \cos x\, e^{\sin x}y(x) = \cos x\, e^{\sin x}$$

$$\left(e^{\sin x}y(x)\right)' = \cos x\, e^{\sin x}$$

$$e^{\sin x}y(x) = e^{\sin x} + C$$

$$\underline{y(x) = 1 + Ce^{-\sin x}}$$

$y(0) = \pi \qquad y(0) = 1 + C \Rightarrow 1 + C = \pi$

$C = \pi - 1$

$$y(x) = 1 + (\pi - 1)e^{-\sin x}.$$

c) $\frac{y'}{y} - 2x - 1 = 0 \quad , \quad y(1) = e$

$\frac{y'}{y} = 2x + 1 \Rightarrow \int \frac{1}{y}dy = \int 2x + 1\, dx$

$$\ln|y| = x^2 + x + C$$

$$|y| = e^{x^2 + x + C}$$

$$y = \pm e^{x^2 + x + C}$$

$y(1) = e \qquad e^{1+1+C} = e^1 \Rightarrow C = -1$

$$\underline{\underline{y(x) = e^{x^2 + x - 1}}}$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x - 1 = -\cos^2 x$

d) $y\underbrace{(\sin^2 x - 1)}_{-\cos^2 x} = y' \quad , \quad y(0) = 2.$

$-y\cos^2 x - y' = 0 \quad , \quad \boxed{y' + \cos^2 x\, y = 0}$

$y' + f(x)y = g(x)$

$$\int \cos^2 x\, dx = \int \cos x \cos x\, dx$$

$u' = \cos x \quad v = \cos x$

$u = \sin x \quad v' = -\sin x$

$$\int \cos^2 x\, dx = \sin x \cos x + \int \sin^2 x\, dx$$

$$= \sin x \cos x + x - \int \cos^2 x\, dx$$

$$2\int \cos^2 x\, dx = \sin x \cos x + x$$

$$\int \cos^2 x\, dx = \frac{1}{2}\sin x \cos x + \frac{1}{2}x + C$$

$$y'(x)e^{\frac{1}{2}\sin x \cos x + \frac{1}{2}x} + \cos^2 x\, e^{\frac{1}{2}\sin x \cos x + \frac{1}{2}x}y(x) = 0$$

$$\left(e^{\frac{1}{2}\sin x \cos x + \frac{1}{2}x}y(x)\right)' = 0$$

$$y(x) = Ce^{-\frac{1}{2}\sin x \cos x - \frac{1}{2}x}$$

$y(0) = 2 \Rightarrow C = 2 \Rightarrow \underline{\underline{y(x) = 2e^{-\frac{1}{2}\sin x \cos x - \frac{1}{2}x}}}.$

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12.12 $L(t)$: Antall laks i en insjø ved tiden t.
(t måles i uker)

$$L'(t) = -k\sqrt{L(t)} \qquad k>0$$

$$L(0) = 1000 \qquad L(4) = 720$$

Løser likning: $\frac{L'(t)}{\sqrt{L(t)}} = -k$

$$\int \frac{L'(t)}{\sqrt{L(t)}}\,dt = \underbrace{\int -k\,dt}_{-kt+C} \qquad \frac{dL}{dt} = L'(t) \qquad dL = L'(t)\,dt$$

$$\int \frac{L'(t)}{\sqrt{L(t)}}\,dt = \frac{4}{2}\int \frac{1}{2\sqrt{L(t)}}\,dL = 2\sqrt{L(t)} + C$$

$$2\sqrt{L(t)} = -kt + C$$

$$\sqrt{L(t)} = -\frac{k}{2}t + C$$

$$\underline{\underline{L(t) = \left(C - \frac{k}{2}t\right)^2}}$$

$L(0)=1000 \Rightarrow \quad C^2 = 1000 \quad C = \sqrt{1000}$

$L(4) = 720 \Rightarrow \left(C - \frac{k}{2}4\right)^2 = 720$

$$C - \frac{k}{2}4 = \pm\sqrt{720}$$

$$2k = C \pm \sqrt{720}$$

$$k = \frac{1}{2}\left(C \pm \sqrt{720}\right)$$

$k_1 \simeq 2.4 \qquad k_2 \simeq 29.2$

Når er fisken død? $L(t_0) = 0$

$$C - \frac{k}{2}t_0 = 0 \qquad \frac{k}{2}t_0 = C \qquad t_0 = \frac{2}{k}C$$

k_1 gir $t_0 = \frac{2}{2.4}\sqrt{1000} \simeq 26.4$

k_2 gir $t_0 = \frac{2}{29.2}\sqrt{1000} \simeq 2.2$

$L(4) = 720 \Rightarrow L(2.2) \neq 0$. Dermed er k_1 riktig parameter. All fisken er død etter 26.4 uker.

B 3.3 $x(t)$: Antall friske personer ved tiden t.

$y(t)$: Antall smittede ved tiden t.

$x(t)+y(t) = N+1$, $y(0) = 1$.

Anta $\frac{dy}{dt} = \beta x y$ $\beta > 0$, konst.

a) Løs differensialligningen. Bruker $x(t) = N+1 - y(t)$

$$y'(t) = \beta(N+1-y(t))y(t)$$

$$\int \frac{y'(t)}{(N+1-y(t))y(t)}dt = \underbrace{\int \beta\, dt}_{\beta t + C}$$

$$\overset{\shortparallel}{\int \frac{1}{(N+1-y)y}dy}$$ Vi bruker delbrøksoppspalting.

$$\frac{1}{(N+1-y)y} = \frac{A}{y} + \frac{B}{(N+1-y)} \quad \Big|\, y(N+1-y)$$

$$1 = A(N+1-y) + By$$

$y = N+1 \Rightarrow 1 = B(N+1) \Rightarrow B = \frac{1}{N+1}$

$y = 0 \Rightarrow 1 = A(N+1) \Rightarrow A = \frac{1}{N+1}$

$$\frac{1}{(N+1-y)y} = \frac{1}{N+1}\left(\frac{1}{y} + \frac{1}{N+1-y}\right)$$

$$\int \frac{1}{(N+1-y)y}dy = \frac{1}{N+1}\Big(\underbrace{\ln y + (-1)\ln(N+1-y)}_{\ln\left(\frac{y}{N+1-y}\right)}\Big) + C$$

$$\frac{1}{N+1}\ln\left(\frac{y(t)}{N+1-y(t)}\right) = \beta t + C \qquad e^{a+b} = e^a e^b$$

$$\ln\left(\frac{y}{N+1-y}\right) = (N+1)\beta t + C$$

$$\frac{y}{N+1-y} = Ce^{(N+1)\beta t}$$

$$\overset{\shortparallel}{\frac{1}{(N+1)y^{-1}-1}} = Ce^{(N+1)\beta t}$$

$$1 = Ce^{(N+1)\beta t}((N+1)y^{-1}-1)$$

$$1 = (N+1)Ce^{(N+1)\beta t}y^{-1} - Ce^{(N+1)\beta t} \quad \Big|\, y$$

$$y + yCe^{(N+1)\beta t} = (N+1)Ce^{(N+1)\beta t}$$

$$y = \frac{(N+1)Ce^{(N+1)\beta t}}{1+Ce^{(N+1)\beta t}}$$

$$y(t) = \frac{(N+1)C}{e^{-(N+1)\beta t} + C}$$

$y(0) = 1$ $\quad 1 = \frac{(N+1)C}{1+C} \Rightarrow C = \frac{1}{N}$

$$y(t) = \frac{1+\frac{1}{N}}{e^{-(N+1)\beta t} + \frac{1}{N}}.$$

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