A10

Grenseverdien (når $x \to \infty$) for en rasjonal funksjon styres av høyeste grad i teller/nevner.

3 a) $\lim_{x\to\infty} \frac{7x^2+4x^4}{3x^3-2x^2} = \lim_{x\to\infty} \frac{\frac{7x^2}{x^4}+4\frac{x^4}{x^4}}{\frac{3x^3}{x^4}-\frac{2x^2}{x^4}} = \lim_{x\to\infty} \frac{\frac{7}{x^2}+4}{\frac{3}{x}-\frac{2}{x^2}} \text{ "=" } \frac{0+4}{0-0} \text{ "=" } \infty$

b) $\lim_{x\to\infty} \frac{8x^2+2x+7}{\sqrt{x}-4x^2} = \lim_{x\to\infty} \frac{\frac{8x^2}{x^2}+\frac{2x}{x^2}+\frac{7}{x^2}}{\frac{\sqrt{x}}{x^2}-\frac{4x^2}{x^2}}$

$= \lim_{x\to\infty} \frac{8+\frac{2}{x}+\frac{7}{x^2}}{\frac{1}{x^{3/2}}-4} = \frac{8}{-4} = \underline{\underline{-2}}$

$\boxed{\frac{\sqrt{x}}{x^2} = \frac{x^{\frac{1}{2}}}{x^2} = x^{\frac{1}{2}}x^{-2} = x^{-\frac{3}{2}}}$

c) $\lim_{x\to\infty} \frac{x^4+\sqrt{x}+e^{x^2}}{7+\sin(\sqrt{x})}$

$= \lim_{x\to\infty} \frac{\frac{x^4}{x^4}+\frac{\sqrt{x}}{x^4}+\frac{e^{x^2}}{x^4}}{\frac{7}{x^4}+\frac{\sin\sqrt{x}}{x^4}}$

$= \lim_{x\to\infty} \frac{1+\frac{1}{x^{\frac{7}{2}}}+\frac{e^{x^2}}{x^4}}{\frac{7}{x^4}+\frac{\sin\sqrt{x}}{x^4}} = \infty$

Hvilke funksjoner blir dominerende når $x \to \infty$?

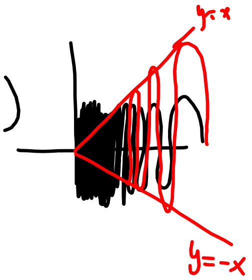
Eksp. funksjoner er sterkt!!

d) $\lim_{x\to\infty} \frac{x^2-4x^3}{8+7x^2} = \lim_{x\to\infty} \frac{\frac{1}{x}-4}{\frac{8}{x^3}+\frac{7}{x}} \text{ "=" } \frac{-4}{0} = \underline{\underline{-\infty}}$

$\frac{x^2-4x^3}{8+7x^2} = \frac{\frac{1}{x}-4}{\frac{8}{x^3}+\frac{7}{x}} \xrightarrow[x\to\infty]{} -\infty$

5.

a) $\lim\limits_{x\to 0} \sin\frac{1}{x}$ eksisterer ikke (fluktuerer)



b) $\lim\limits_{x\to 0} \cos\frac{1}{x}$ —— " ——

c) $\lim\limits_{x\to 0} x\cdot\cos\frac{1}{x} = \underline{\underline{0}}$

d) $\lim\limits_{x\to 0} \mathrm{tg}\, x = \lim\limits_{x\to 0} \frac{\sin x}{\cos x} = \frac{0}{1} = \underline{\underline{0}}$

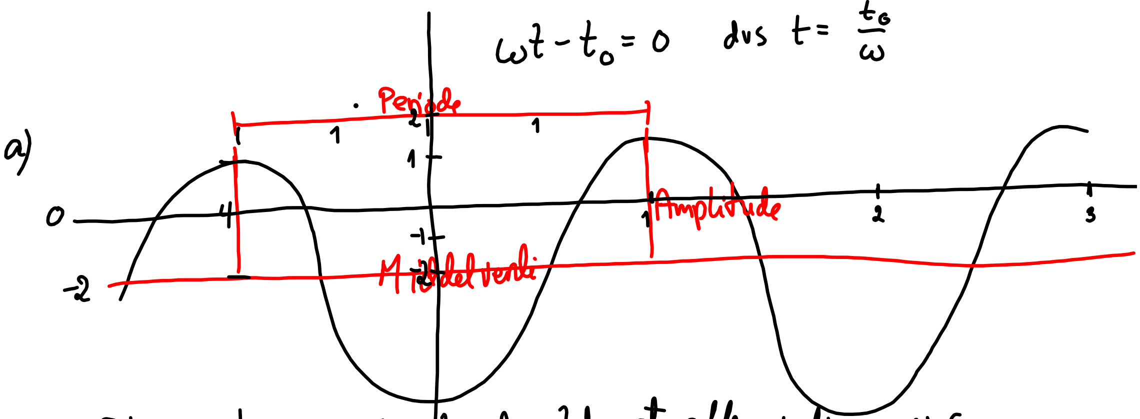
9.

	Middelverdi	Amplitude	Periode	Akrof.
$-2+4\cos(\pi t-\pi)$	-2	4	2	1
$-\frac{1}{2}+3\cos(\pi(2t-2))$	$-\frac{1}{2}$	3	1	0
$7\cos\left(\frac{\pi}{7}t-7\right)$	0	7	14	$\frac{49}{\pi}-14$
$A_0+A\cos(\omega t-t_0)$	A_0	A	$\frac{2\pi}{\omega}$	$0\le\frac{t_0}{\omega}-m\cdot\frac{2\pi}{\omega}$ $m\in\mathbb{Z}$

$\frac{\pi}{7}t-7=0$ dvs $t=\frac{49}{\pi}\approx 15{,}6$

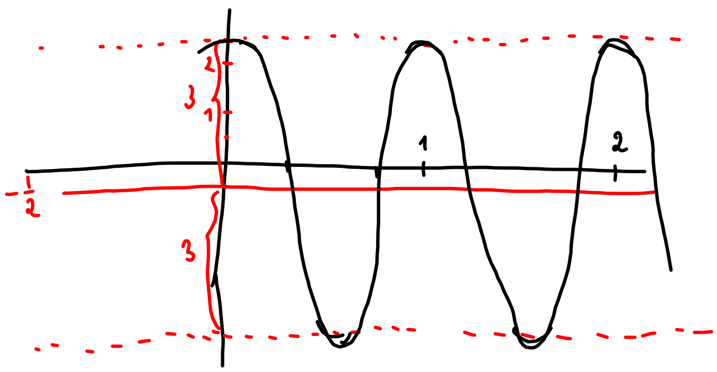
$\omega t-t_0=0$ dvs $t=\frac{t_0}{\omega}$

a)



Plasser koordinatsystemet slik at alle verdier passer.

b)



10. $A\cos(bx-\varphi) = A\cos bx\cos\varphi + A\sin bx\sin\varphi$

$= \underbrace{A\cos\varphi}_{B} \cdot \cos(bx) + \underbrace{A\sin\varphi}_{C} \sin(bx)$

$$\boxed{\begin{aligned} &\sqrt{B^2+C^2} = A \\ &\cos\varphi = \frac{B}{A} \\ &\sin\varphi = \frac{C}{A} \end{aligned}}$$

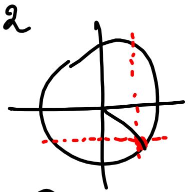
a) $\cos x - \sin x$ $\quad b=1$

$B=1 \quad C=-1$

$A = \sqrt{1^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$

$\cos\varphi = \frac{1}{\sqrt{2}}$

$\sin\varphi = -\frac{1}{\sqrt{2}}$



$\varphi = -\frac{\pi}{4}$ eller $\varphi = \frac{7\pi}{4}$

$\cos x - \sin x = \sqrt{2}\cos\left(x - \frac{7\pi}{4}\right)$

b) $-\sqrt{3}\cos(3x) + 3\sin(3x) = \underline{\underline{2\sqrt{3}\cos\left(3x - \frac{2\pi}{3}\right)}}$

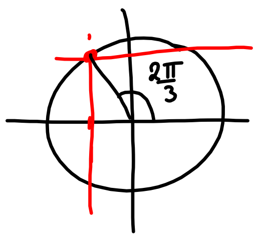
$b=3$

$A = \sqrt{(-\sqrt{3})^2 + 3^2} = \sqrt{3+9} = \sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}$

$\cos\varphi = \frac{-\sqrt{3}}{2\sqrt{3}} = -\frac{1}{2}$

$\sin\varphi = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}\cancel{\sqrt{3}}}{2\cancel{\sqrt{3}}} = \frac{\sqrt{3}}{2}$

$\varphi = \frac{2\pi}{3}$



c) $-\cos\left(\frac{x}{4}\right) - \sqrt{3}\sin\left(\frac{x}{4}\right) = \underline{\underline{2\cos\left(\frac{x}{4} - \frac{4\pi}{3}\right)}}$

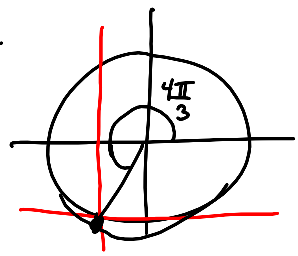
$b = \frac{1}{4} \quad B=-1, \; C=-\sqrt{3}$

$A = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\cos\varphi = \frac{B}{A} = \frac{-1}{2} = -\frac{1}{2}$

$\sin\varphi = \frac{C}{A} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

$\varphi = \frac{4\pi}{3}$



Bl. 11

a) $2x + a^2 y = a$
$x + 2y = 1$

Koeff.matrise $\begin{pmatrix} 2 & a^2 \\ 1 & 2 \end{pmatrix}$

$$\det\begin{pmatrix} 2 & a^2 \\ 1 & 2 \end{pmatrix} = 2\cdot 2 - a^2\cdot 1 = 4 - a^2 = 0$$

dvs $a = \pm 2$

$a \neq \pm 2$ Én løsning

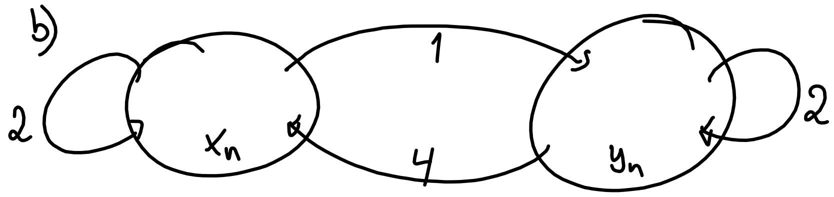
$a = 2$ $\quad 2x + 4y = 2$
$\quad x + 2y = 1$

Egentlig kun én likning
dvs. ∞ mange løsninger

$a = -2$ $\quad 2x + 4y = -2$
$\quad x + 2y = 1 \quad | \cdot 2$
$\quad 2x + 4y = 2$

Ingen løsninger

b)



$$x_{n+1} = 2x_n + 4y_n$$
$$y_{n+1} = x_n + 2y_n$$

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

$$\begin{pmatrix} 1200 \\ 600 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}}_{= \begin{pmatrix} 8 & 16 \\ 4 & 8 \end{pmatrix}}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

$x_0 + y_0 = 100$

$$1200 = 8x_0 + 16y_0$$
$$600 = 4x_0 + 8y_0$$

gir $150 = x_0 + 2y_0$

Må løse

$$x_0 + y_0 = 100$$
$$x_0 + 2y_0 = 150$$

gir $\underline{\underline{y_0 = 50 \quad x_0 = 50}}$