A.3

4)

(a) Løs 1.6 c) - e) ved Gauss-Jordan

c)
$$\begin{aligned} 2x-3y &= -2 \\ 2x+y &= 1 \\ 3x+2y &= 1 \end{aligned} \qquad \begin{bmatrix} 2 & -3 & -2 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \tfrac{1}{2}R_1 \sim$$

$$\begin{bmatrix} 1 & -\frac{3}{2} & -1 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{matrix} -2R_1 \text{ til } R_2 \\ -3R_1 \text{ til } R_3 \end{matrix} \sim \begin{bmatrix} 1 & -\frac{3}{2} & -1 \\ 0 & 4 & 3 \\ 0 & \frac{13}{2} & 4 \end{bmatrix} \tfrac{1}{4}R_2 \sim$$

$$\begin{bmatrix} 1 & -\frac{3}{2} & -1 \\ 0 & 1 & \frac{3}{4} \\ 0 & \frac{13}{2} & 4 \end{bmatrix} -\tfrac{13}{2}R_2 \text{ til } R_3 \sim \begin{bmatrix} 1 & -\frac{3}{2} & -1 \\ 0 & 1 & \frac{3}{4} \\ 0 & 0 & -\frac{7}{8} \end{bmatrix}$$

Siste likning blir da $0 \cdot y = -\frac{7}{8}$ dvs $0 = -\frac{7}{8}$ umulig

Ingen løsning

d)

$$\begin{aligned} 2x_1 + 2x_2 + 2x_3 &= 0 \\ -2x_1 + 5x_2 + 2x_3 &= 1 \\ 8x_1 + x_2 + 4x_3 &= -1 \end{aligned} \qquad \begin{bmatrix} 2 & 2 & 2 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix} \begin{array}{l} \frac{1}{2}R_1 \\ \\ \\ \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ -2 & 5 & 2 & 1 \\ 8 & 1 & 4 & -1 \end{bmatrix} \begin{array}{l} \\ 2R_1 \text{ til } R_2 \\ -8R_1 \text{ til } R_3 \end{array} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & -7 & -4 & -1 \end{bmatrix} \begin{array}{l} \\ \\ R_2 \text{ til } R \end{array} \sim$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 7 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \frac{1}{7}R_2 \\ \\ \end{array} \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ -R_2 \text{ til } R_1 \\ \\ \end{array} \sim$$

$$\sim \begin{bmatrix} 1 & 0 & \frac{3}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{4}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Setter $x_3 = t$, $x_2 = \frac{1}{7} - \frac{4}{7}t$

$x_1 = -\frac{1}{7} - \frac{3}{7}t$

$$\{(x_1, x_2, x_3)\} = \left\{\left(-\frac{1}{7} - \frac{3}{7}t,\ \frac{1}{7} - \frac{4}{7}t,\ t\right) : t \in \mathbb{R}\right\}$$

e) $x - y + 2z - w = -1$
$2x + y - 2z - 2w = -2$
$-x + 2y - 4z + w = 1$
$3x \quad - 3w = -3$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix} \begin{array}{l} \\ -2R_1 \text{ til } R_2 \\ R_1 + R_3 \\ -3R_1 \text{ til } R_4 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \frac{1}{3}R_2 \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \\ -R_2 \text{ til } R_3 \\ -3R_2 \text{ til } R_3 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} +R_2 \text{ til } R_1 \\ \\ \\ \\ \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Setter $z = t$, $w = s$

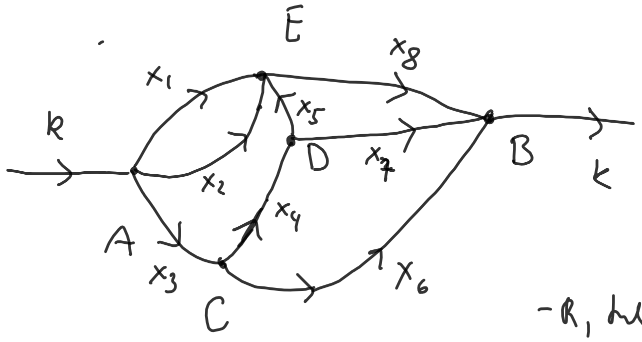
$x = -1 + s$, $y = 2t$

$\{(x, y, z, w)\} = \{(-1+s, 2t, t, s) : t, s \in \mathbb{R}\}$

A.3.4

b) Står løst i detalj i fasiten.
Se fasit

6) Veinett i London:



$-R_1$ till R_8

Gir oss et likningssystem:

A $\quad x_1 + x_2 + x_3 = k$

B $\quad x_6 + x_7 + x_8 = k$

C $\quad x_3 = x_4 + x_6$

D $\quad x_4 = x_5 + x_7$

E $\quad x_1 + x_2 + x_5 = x_8$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & k \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & k \\ 0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$\sim$ fortsetter radreduksjon. Detaljer se fasit

$$\sim \cdots\cdots \sim \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & k \\ 0 & 0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & k \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Kan velge x_2, x_5, x_7 i x_8 fritt

(svarer ikke til ledende enere i matrisen)

Setter $x_2 = s$, $x_5 = t$, $x_7 = u$, $x_8 = v$

$x_1 = -s - t + v$, $\quad x_3 = t - v + k$

$x_4 = t + u$, $\quad x_6 = -u - v + k$

Må ha $x_i \geq 0 \quad i = 1,2,3,4,5,6,7,8$

Må ha: $v \geq s + t \quad (x_1 \geq 0) \qquad k \geq v - t \quad (x_3 \geq 0)$

$k \geq u + v \quad (x_6 \geq 0)$

A.5

Skriv ut leddene

1) a) $\left\{ \frac{1}{n+1} \right\}_{n=0}^{10}$ $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$
$\frac{1}{9}, \frac{1}{10}, \frac{1}{11}$

b) $\{2n-3\}_{n=2}^{7}$ $1, 3, 5, 7, 9, 11$

c) $\{2n+1\}_{n=0}^{5}$ $1, 3, 5, 7, 9, 11$

d) $\{2n\}_{n=0}^{5}$ $0, 2, 4, 6, 8, 10$

e) $\left\{ \frac{1}{n^2+n+2} \right\}_{n=0}^{5}$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{14}, \frac{1}{22}$ $\frac{1}{32}$

f) $\{1000\}_{n=0}^{5}$ $1000, 1000, 1000, 1000, 1000, 1000$

g) $\left\{ \frac{(-1)^n}{2n} \right\}_{n=1}^{5}$ $\frac{-1}{2}, \frac{1}{4}, \frac{-1}{6}, \frac{1}{8}, \frac{-1}{10}$

h) $\{(-1)^n\}_{n=0}^{5}$ $1, -1, 1, -1, 1, -1$

i) $\{(0.5)^n\}_{n=0}^{5}$ $1, 0.5, 0.25, 0.125, 0.0625$
0.03125.

A.5.2

a) $\left\{ \frac{1}{n+1} \right\}_{n=0}^{\infty}$ konvergerer mot 0
siden $n+1 \to \infty$, når $n \to \infty$

b) $\{2n-3\}_{n=2}^{\infty}$ konvergerer ikke
(går mot ∞)

c) $\{2n+1\}_{n=0}^{\infty}$ ~~mot~~ Som b)

d) $\{2n\}_{n=0}^{\infty}$ som b)

e) $\left\{ \frac{1}{n^2+n+2} \right\}_{n=0}^{\infty}$ konvergerer mot 0
siden $n^2+n+2 \to \infty$

(Hvis a_n går mot ∞ vil alltid $\frac{1}{a_n}$
gå mot 0. N.B.)

f) $\{1000\}_{n=0}^{\infty}$ konvergerer mot 1000

g) $\left\{ \frac{(-1)^n}{2n} \right\}_{n=1}^{\infty}$ konvergerer mot 0
siden $2n \to \infty$

h) $\{(-1)^n\}_{n=0}^{\infty}$ hopper mellom 1 og −1
og konvergerer ikke

i) $\{(0.5)^n\}_{n=0}^{\infty}$ Går mot 0

Hvis $a \in (-1, 1)$ så vil alltid $a^n \to 0$

5.5 Fibonacci følgen. (s. 113)

Starter ved tiden $t = 0$ med et par (nyfødte) kaniner. X_n antall kaniner ved tiden n. $X_0 = 1$, $X_1 = 1$

$$X_2 = X_1 + X_0 ,$$

$$X_3 = \underset{\text{en måned gamle}}{(X_2 - X_1)} + \underset{\text{nyfødt}}{X_1} + \underset{\text{eldre}}{X_1} = X_2 + X_1$$

$$X_4 = (X_3 - X_2) + X_2 + X_2 = X_3 + X_2$$

$$X_{n+2} = X_{n+1} + X_n , \quad n \geq 0$$

$X_{17} = ?$

$X_0 = 1$ $\quad$ $X_3 = 3$

$X_1 = 1$ $\quad$ $X_5 = 5$ $\quad$ Regner slik og får

$X_2 = 2$ $\quad$ $X_6 = 8$ $\quad$ $X_{17} = 2584$

Bruker teori om differenslikninger kommer en frem til at

$$X_n = \frac{\sqrt{5}}{5}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$$

Sett $n = 4$

$$X_n = \frac{\sqrt{5}}{5}\,\frac{1}{2^5}\left((1+\sqrt{5})^5 - (1-\sqrt{5})^5\right)$$

$$(1 \pm \sqrt{5})^5 = (1 \pm \sqrt{5})^4 (1 \pm \sqrt{5})$$

$$(1 \pm \sqrt{5})^4 = ((1 \pm \sqrt{5})^2)^2$$

$$= (1 \pm 2\sqrt{5} + 5)^2 =$$

$$= (6 \pm 2\sqrt{5})^2 = 36 \pm 24\sqrt{5} + 20$$

$$= 56 \pm 24\sqrt{5}$$

$$(56 \pm 24\sqrt{5})(1 \pm \sqrt{5})$$

$$= 56 \pm 24\sqrt{5} \pm 56\sqrt{5} + 120$$

$$= 176 \pm 80\sqrt{5}$$

$$x_n = \frac{\sqrt{5}}{32 \cdot 5}((\cancel{176} + 80\sqrt{5}) - (\cancel{176} - 80\sqrt{5}))$$

$$= \frac{\sqrt{5}}{1\cancel{6}0} 1\cancel{6}0\sqrt{5} = \underline{\underline{5 = x_4}}$$