A7 $a+bi$, a realdel, b imaginærdel.

3) a) $i\cdot(i-1) = ii - i = -1-i$

b) $(2+i)(2+i) = 4+2i+2i-1 = 3+4i$

c) $(4-3i)(4+5i) = 16+20i-12i+15$
$= 31+8i$

d) $i^3 = (i\cdot i)\cdot i = -i$

e) $i^4 = i^3\cdot i = (-i)\cdot i = -(-1) = 1$

f) $(3-2i)(3+2i) = 9-6i+6i+4 = 13$

Obs! $(a+bi)(a-bi) = a^2+ab\cancel{i}-ab\cancel{i}+b^2$
$= \underline{a^2+b^2}$

7.4

a) $\frac{4-8i}{1-i} = \frac{(4-8i)(1+i)}{(1-i)(1+i)} = \frac{4-8i+4i+8}{2}$

$((1-i)(1+i) = 1-i+i+1 = 2)$ $= \frac{12-4i}{2} = 6-2i$

b) $\frac{1}{2+4i} = \frac{2-4i}{(2+4i)(2-4i)} =$

$= \frac{2-4i}{4+16} = \frac{2-4i}{20} = \frac{1}{10} - \frac{1}{5}i$

c) $\frac{3+16i}{2-6i} = \frac{(3+16i)(2+6i)}{(2-6i)(2+6i)} = \frac{6+18i+32i-96}{4+36} =$

$= \frac{-90+50i}{40} = -\frac{9}{4} + \frac{5}{4}i$

d) $\frac{1+5i}{5-i} = \frac{(1+5i)(5+i)}{(5-i)(5+i)} = \frac{5+25i+i-5}{26} = \frac{26i}{26} = i$

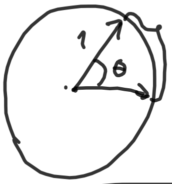
e) $\frac{i}{6+7i} = \frac{i(6-7i)}{(6+7i)(6-7i)} = \frac{6i+7}{36+49} = \frac{7+6i}{85}$

$= \frac{7}{85} + \frac{6}{85}i$

f) $\frac{i^2}{i+1} = \frac{(-1)(-i+1)}{(i+1)(-i+1)} = \frac{-1+i}{2}$

$= -\frac{1}{2} + \frac{1}{2}i$

10)

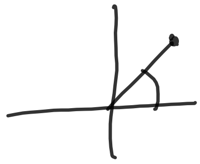


lengden av buen er θ målt i radianer.

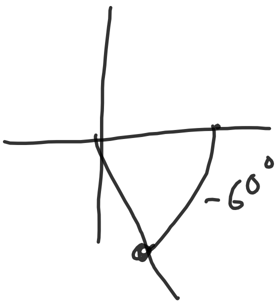
$$\boxed{360^\circ = 2\pi}$$

a) $\frac{\pi}{4} = \left(\frac{\frac{\pi}{4}}{2\pi} \cdot 360\right)^\circ = \left(\frac{\pi}{8\pi} \cdot 360\right)^\circ$

$= \left(\frac{360}{8}\right)^\circ = 45^\circ$; 1 kvadrant.



b) $-\frac{\pi}{3} = \left(\frac{-\frac{\pi}{3}}{2\pi} \cdot 360\right)^\circ = \left(-\frac{360}{6}\right)^\circ = -60^\circ$

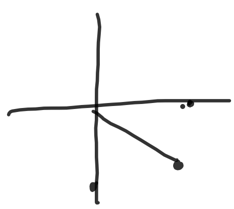


er i 4. kvadrant.
(Svarer $360^\circ - 60^\circ \simeq 300^\circ$)

c) $5 = \left(\frac{5}{2\pi} \cdot 360\right)^\circ = \left(\frac{1800}{2\pi}\right)^\circ = \left(\frac{900}{\pi}\right)^\circ$

$\simeq (286{,}5)^\circ$ $\quad 270 < 286{,}5 < 360$

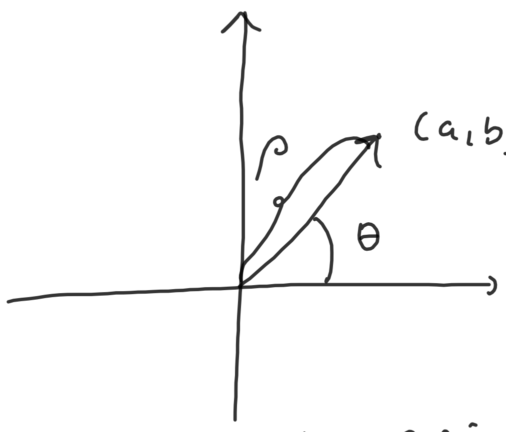
er i 4. kvadrant.



d) $\frac{2\pi}{3} = \left(\frac{\frac{2\pi}{3}}{2\pi} \cdot 360\right)^\circ = \left(\frac{1}{3} \cdot 360\right)^\circ = \underline{120^\circ}$

$90 < 120 < 180$

2. kvadrant.



$a + bi = z$

$\rho = \sqrt{a^2 + b^2}$

$\theta \in [0, 2\pi)$

$$a = \rho\cos\theta, \quad b = \rho\sin\theta$$

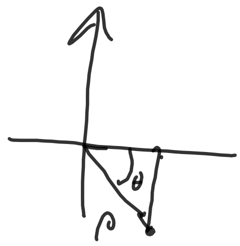
$$z = a + bi = \rho\cos\theta + \rho\sin\theta\, i$$ z på polar form.

7.11 Skriv på polar form

a) $z = 2 - 2i, \quad \rho = \sqrt{4+4} = 2\sqrt{2}$

$$\cos\theta = \frac{a}{\rho} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\sin\theta = \frac{b}{\rho} = \frac{-2}{2\sqrt{2}} = -\frac{1}{2}\sqrt{2}$$



$$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$z = 2\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$$

b) $2 + 2i, \quad \rho = \sqrt{4+4} = 2\sqrt{2}$

$$\cos\theta = \frac{2}{2\sqrt{2}} = \frac{1}{2}\sqrt{2}, \quad \sin\theta = \frac{2}{2\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\theta = \frac{\pi}{4}, \quad z = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

c) $z = 1 + i, \quad \rho = \sqrt{1+1} = \sqrt{2},$

$$\cos\theta = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}, \quad \sin\theta = \frac{1}{2}\sqrt{2} \qquad z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

d) $z = \sqrt{3} + i$

$\rho = \sqrt{3+1} = 2$

$\cos\theta = \frac{\sqrt{3}}{2}$, $\sin\theta = \frac{1}{2}$, $\theta = \frac{\pi}{6}$

$z = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$

e) $z = i$, $\rho = \sqrt{0+1} = 1$, $\cos\theta = \frac{0}{1} = 0$

$\sin\theta = \frac{1}{1} = 1$, $\theta = \frac{\pi}{2}$, $i = 1\cdot(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\, i)$

12) $z = \rho e^{i\theta} = \overline{\rho}(\cos\theta + i\sin\theta)$

a) $z = 2 - 2i = 2\sqrt{2}\, e^{i\frac{7\pi}{4}}$

b) $2 + 2i = 2\sqrt{2}\, e^{i\frac{\pi}{4}}$

c) $1 + i = \sqrt{2}\, e^{i\frac{\pi}{4}}$

d) $\sqrt{3} + i = 2e^{i\frac{\pi}{6}}$

e) $i = e^{i\frac{\pi}{2}}$

Eksamensoppgaver:

2.6 $\quad z = 1 + i, \; w = 2 - i$

Skriv $\frac{z}{w}$ og $\bar{z} - \frac{w}{z}$ på formen

$a + bi$

$$\frac{z}{w} = \frac{1+i}{2-i} = \frac{(1+i)}{(2-i)}\frac{(2+i)}{(2+i)} = \frac{2+i+2i-1}{4+1}$$

$$= \frac{1+3i}{5} = \frac{1}{5} + \frac{3}{5}i$$

$$\bar{z} = 1 - i, \quad \frac{w}{z} = \frac{2-i}{1+i} = \frac{(2-i)(1-i)}{(1+i)(1-i)} =$$

$$= \frac{2-i-2i-1}{1+1} = \frac{1-3i}{2} = \frac{1}{2} - \frac{3}{2}i$$

$$\bar{z} - \frac{z}{w} = (1-i) - \left(\frac{1}{2} - \frac{3}{2}i\right) = \underline{\underline{\frac{1}{2} + \frac{1}{2}i}}$$

2.10

$$z = \frac{1}{\sqrt{2}}(1+i)e^{-i\frac{\pi}{4}}$$

Er dette lik

$\boxed{X}$ 1 , $\square$ $\frac{1}{\sqrt{2}}$, $\square$ $1-i$

$$z = \frac{1}{\sqrt{2}}(1+i)e^{-i\frac{\pi}{4}}$$

$$= \frac{1}{\sqrt{2}}(1+i)\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

$$= \frac{1}{\sqrt{2}}(1+i)\left(\frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt{2}i\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{2}\sqrt{2} + \cancel{\frac{1}{2}\sqrt{2}i} - \cancel{\frac{1}{2}\sqrt{2}i} + \frac{1}{2}\sqrt{2}\right) = \frac{1}{\sqrt{2}}\sqrt{2} = 1$$

2.11 (Samme spørsmål som i 2.10)

$$z = \frac{1}{2}(\sqrt{3} + i)\, e^{\frac{\pi}{3} i}$$

$\boxed{X}$ i $\square$ 1 $\square$ $\frac{1}{2}\sqrt{3} + 2i$

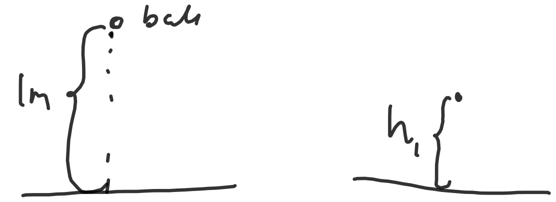
$$z = \frac{1}{2}(\sqrt{3}+i)\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= \frac{1}{2}(\sqrt{3}+i)\left(\frac{1}{2} + \frac{1}{2}\sqrt{3}\, i\right) =$$

$$= \frac{1}{2}\left(\cancel{\frac{1}{2}\sqrt{3}} + \frac{1}{2} i + \frac{3}{2} i - \cancel{\frac{1}{2}\sqrt{3}}\right) = \frac{1}{2}(2i) = i$$

Fra forige uke :

A.6 2)



$h_1 = 0.8$, $h_{n+1} = 0.8 h_n$ (h_n høyden etter n-sprett)

Likning av formen $x_{n+1} = r x_n$

har generell løsning $x_n = C r^n$

Får $h_n = C(0.8)^n$, $h_0 = C(0.8)^0 = C = 1$

$h_n = (0.8)^n$ etter 20 sprett har vi

$h_{20} = (0.8)^{20} \approx 0.012\,m$

6.3

Banken gir 6.2 % rente på et innskudd

Setter inn 10.000 ved tiden $t = 0$

$X_0 = 10.000$, etter $n+1$ år

$$\boxed{X_{n+1} = X_n + (0.062)X_n = (1.062)X_n}$$

$$X_n = C(1.062)^n, \quad X_0 = 10.000 = C$$

(Husk $r^0 = 1$) $\quad X_n = (1.062)^n 10\,000$

b) Når har innskuddet fordoblet seg?

$(1.062)^n \cdot 10\cancel{000} = 2\cancel{0000}, \quad n = ?$

$\ln\left((1.062)^n\right) = \ln 2$

$n \ln(1.062) = \ln 2, \quad n = \frac{\ln 2}{\ln(1.062)} \approx \underline{11.523}$

c) Hva om vi sette K kronen i banken ved tiden $t=0$

Får da $\quad X_n = K(1.062)^n$

$$\cancel{K}(1.062)^n = 2\cancel{K} \quad n = ?$$

$(1.062)^n = 2$ samme likning som i stad så

$n \approx 11.523$