<u>A</u>2

4)

$$\underbrace{\begin{bmatrix} 4 & 0 & 3 \\ 1 & 0 & 2 \\ 5 & 1 & 4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 4 \\ 2 & 0 & 5 \end{bmatrix}}_{B} = \begin{bmatrix} 4+0+6 & 12 & 16+15 \\ 1+0+4 & 3 & 4+10 \\ 5+1+8 & 15+2 & 20+4+20 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 12 & 31 \\ 5 & 3 & 14 \\ 14 & 17 & 44 \end{bmatrix} = AB$$

$$\underbrace{\begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 4 \\ 2 & 0 & 5 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} 4 & 0 & 3 \\ 1 & 0 & 2 \\ 5 & 1 & 4 \end{bmatrix}}_{A} = \begin{bmatrix} 27 & 4 & 25 \\ 26 & 4 & 23 \\ 33 & 5 & 26 \end{bmatrix} = BA$$

Ser at $AB \neq BA$

7)

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 0 & 4 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 2 & 0 & 0 & 3 \\ 4 & 1 & 2 & 1 \\ 0 & 0 & 1 & 3 \\ -1 & 0 & 0 & 1 \end{bmatrix}}_{B} =$$

$$= \begin{bmatrix} -5 & -1 & -2 & 5 \\ 4 & 0 & 4 & 18 \\ 7 & 2 & 3 & 0 \\ 1 & 0 & 1 & 7 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 2 & 0 & 0 & 3 \\ 4 & 1 & 2 & 1 \\ 0 & 0 & 1 & 3 \\ -1 & 0 & 0 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 0 & 4 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}}_{A} = \begin{bmatrix} 5 & -2 & 3 & 9 \\ 7 & 0 & 3 & 15 \\ 3 & 2 & 2 & 4 \\ 0 & 1 & 1 & -2 \end{bmatrix}$$

$AB \neq BA$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (identitets 4x4 matrisen)

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 0 & 4 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}}_{=A} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{=I} = \underbrace{\begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 0 & 4 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}}_{=A} \qquad AI = A$$

$BI = B$ Generelt $AI = A$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 0 & 4 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 0 & 4 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \qquad IA = A$$

På samme måte $IB = B$. Dette holder generelt

8)

a) $\underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}}_{= A} \underbrace{\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}}_{= B} = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}_{= C} = \begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix}$

Ganger en en $n \times n$ diagonal matrise A med en $n \times q$ matrise B, får en $n \times q$-matrise AB slik at

første rad i AB = første rad i B ganget med 1. diagonal element i A

2. rad i AB = 2. rad i B —"— 2. diagonalelement i A

osv.

$\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} xa & xb \\ yc & yd \end{bmatrix}$

b)
$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 6 & 4 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} xa & yb \\ xc & yd \end{bmatrix}$$

A $n \times n$ diagonalmatrise

B $m \times n$ matrise

1. kolonne i BA = 1. kolonne i B multiplisert med 1. diagonal element i A

2. —"— = 2. —"— 2. diagonalelement i A

osv.

B.1

a) a_1, a_2, a_3 tre plantearter

b_1, b_2, b_3 tre arter plantetende dyr

c_1, c_2 to rovdyrarter.

$$\begin{array}{c|ccc} & b_1 & b_2 & b_3 \\ \hline a_1 & & & \\ a_2 & & \begin{pmatrix} 7 & 4 & 0 \\ 8 & 4 & 3 \\ 3 & 0 & 8 \end{pmatrix} & \\ a_3 & & & \end{array} \qquad \begin{array}{c|cc} & c_1 & c_2 \\ \hline b_1 & & \\ b_2 & \begin{pmatrix} 4 & 0 \\ 3 & 4 \\ 6 & 5 \end{pmatrix} & \\ b_3 & & \end{array}$$

Hvor mye spiser c_1 av a_1 (indirekte)

$$7 \cdot 4 + 4 \cdot 3 + 0 \cdot 6 = 40$$

c_1 spiser av a_2;

$$8 \cdot 4 + 4 \cdot 3 + 3 \cdot 6 =$$

$$\begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} c_1 \quad c_2 \\ \begin{pmatrix} \quad & \quad \\ & \\ & \end{pmatrix} \end{array} = \begin{pmatrix} 7 & 4 & 0 \\ 8 & 4 & 3 \\ 3 & 0 & 8 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 3 & 4 \\ 6 & 5 \end{pmatrix} =$$

$$= \underline{\begin{pmatrix} 40 & 16 \\ 62 & 31 \\ 60 & 40 \end{pmatrix}}$$

b) Sett opp likningsystem som gir oss hvor mye c_1 må spise av b_1, b_2, b_3 for å fortære 14, 46, 86 enheter av a_1, a_2, a_3

Må ha $\begin{pmatrix} 7 & 4 & 0 \\ 8 & 4 & 3 \\ 3 & 0 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 14 \\ 46 \\ 86 \end{pmatrix}$

$L_1 \quad 7x_1 + 4x_2 = 14$

$L_2 \quad 8x_1 + 4x_2 + 3x_3 = 46$

$L_3 \quad 3x_1 + 8x_3 = 86$

$L_2 - L_1 \quad x_1 + 3x_3 = 32 \quad L_2'$

$L_3: \quad 3x_1 + 8x_3 = 86$

$L_3 - 3L_2' \quad -x_3 = 86 - 96 = -10, \; x_3 = 10$

$x_1 = 32 - 3x_3 = 32 - 30 = 2$

$L_1; \quad x_2 = \frac{14 - 7x_1}{4} = 0 \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 \\ 0 \\ 10 \end{pmatrix}}}$

1.22

$$(1-t)x + 2y = 5 \quad L_1$$

$$3x + (2-t)y = -5 \quad L_2$$

For hvilke t har dette éntydig løsning.

$$L_1: \quad y = \frac{5+(t-1)x}{2}$$

setter inn i L_2;

$$3x + (2-t)\left(\frac{5}{2} + \frac{(t-1)}{2}x\right) = -5$$

$$3x + (2-t)\frac{(t-1)}{2}x = -5 + \frac{5}{2}(t-2)$$

$$6x + (2-t)(t-1)x = -10+5(t-2)$$

$$6x + (-2+3t-t^2)x = -10+5t-10$$

$$(t^2-3t-4)x = 20-5t$$

Har når $t^2-3t-4 \neq 0$

$$x = \frac{20-5t}{t^2-3t-4}, \qquad t^2-3t-4=0$$

$$t = \frac{3\pm\sqrt{9+16}}{2} = \frac{3\pm 5}{2} = \begin{cases}4\\-1\end{cases}$$

Entydig løsning når $t \neq 4$ og $t \neq -1$