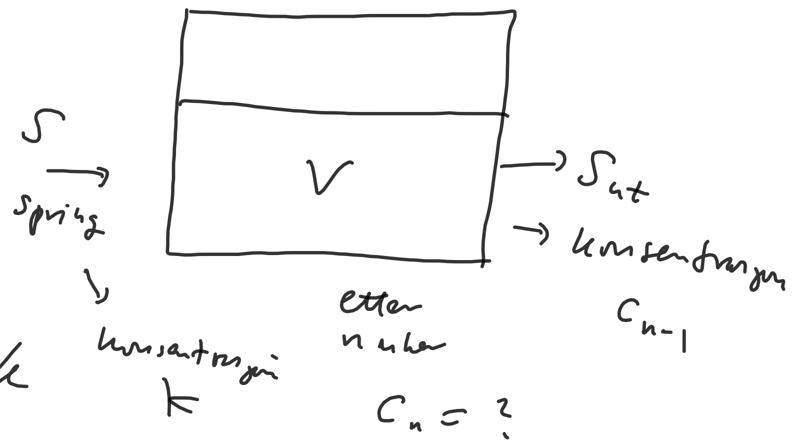
A 11 : 8, 9, 10

B2 : 12

a)

Akvarium



Konsentrasjon av salter $c_0 = 1.0$ g/l

(for hardt)

I uke n

Har $V-S$ liter av konsentrasjon c_{n-1} (g/l)

S —"— K (g/l)

Har $c_{n-1}(V-S) + KS$ g salt etter n uker.

Konsentrasjon $c_n = \dfrac{c_{n-1}(V-S)+KS}{V} = \left(1-\dfrac{S}{V}\right)c_{n-1} + \dfrac{S}{V}K$

b) $K = 0.1\, g/l$

$V = 100\, l$ Oppgitt

$S = 10\, l$ $c_0 = 1.0\, g/l$

Finn c_n

$$c_n - \left(1 - \frac{S}{V}\right) c_{n-1} = \frac{S}{V} K$$

$$c_n - (0.9)\, c_{n-1} = 0.01$$

$$c_n^s = A, \quad A - 0.9A = 0.01$$

$$0.1A = 0.01$$

$$A = 0.1, \quad c_n^h = B(0.9)^n$$

$$c_n = B(0.9)^n + 0.1$$

$$c_0 = B + 0.1 = 1, \quad B = 0.9$$

$$c_n = 0.9\,(0.9)^n + 0.1$$

Etter hvor mange har vannet konsentrasjon $\leq 0.5\, g/l$

Skal ha

$$0.9\,(0.9)^n + 0.1 \leq 0.5$$

$$(0.9)^n \leq \frac{0.4}{0.9} = \frac{4}{9}$$

$$n \ln 0.9 \leq \ln \frac{4}{9}$$

$$n \geqslant \frac{\ln 4.9}{\ln 0.9} \approx 7.7$$

dvs. etter 8 uker er

konsentrasjonen mindre

enn 0.5

A 11

8)

a) $\int (x^5 - 3x)\,dx =$

$= \frac{1}{6}x^6 - 3\frac{1}{2}x^2 + C$

$= \underline{\frac{x^6}{6} - \frac{3}{2}x^2 + C}$

$\int x^r dx =$

$= \frac{1}{r+1}x^{r+1} + C$

$r \neq -1$

b) $\int (x^{5/6} - 1)\,dx = \frac{x^{5/6+1}}{\frac{5}{6}+1} - x + C = \frac{x^{11/6}}{\frac{11}{6}} - x + C$

$= \underline{\underline{\frac{6}{11}x^{11/6} - x + C}}$

c) $\int 3x e^{x^2} dx = 3\int \frac{1}{2} e^u du =$

$u = x^2$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

$= \frac{3}{2}e^u + C =$

$= \underline{\frac{3}{2}e^{x^2} + C}$

$$\boxed{\begin{aligned} &\int g'(x) f(g(x))\,dx = \\ &u = g(x) \\ &du = g'(x)\,dx \\ &= \int f(u)\,du \end{aligned}}$$

d) $\int x^3 \sin(x^4)\,dx = \int \frac{1}{4}\sin(u)\,du =$

$u = x^4$, $du = 4x^3dx$

$x^3dx = \frac{1}{4}du$

$= -\frac{1}{4}\cos u + C =$

$= \underline{\underline{-\frac{1}{4}\cos(x^4) + C}}$

e) $\int \frac{3}{x(x-2)}\,dx =$, $\frac{3}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$

$= \frac{(A+B)x - 2A}{x(x-2)}$, $A+B=0$, $-2A = 3$

$A = -\frac{3}{2}$, $B = \frac{3}{2}$

$= \int \left(-\frac{3}{2}\frac{1}{x} + \frac{3}{2}\frac{1}{x-2}\right)dx$

$= -\frac{3}{2}\int\frac{dx}{x} + \frac{3}{2}\int\frac{dx}{x-2} = -\frac{3}{2}\ln|x| + \frac{3}{2}\ln|x-2| + C$

$= \frac{3}{2}\left(\ln|x-2| - \ln|x|\right) + C = \underline{\underline{\frac{3}{2}\ln\left|\frac{x-2}{x}\right| + C}}$

f) $\int \frac{1}{(x-1)(x+2)}\,dx =$ $\quad \frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$

$$= \frac{(A+B)x + 2A - B}{(x-1)(x+2)}$$

$$= \int\left(\frac{1}{3}\frac{dx}{x-1} - \int\frac{1}{3}\frac{dx}{x+2}\right)$$

$A + B = 0, \; B = -A$

$2A - B = 1, \; 3A = 1$

$A = \frac{1}{3}$

$B = -\frac{1}{3}$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C$$

$$= \underline{\underline{\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C}}$$

11.9

$$\int u v' dx = uv - \int u' v\, dx$$

a) $\int e^x \sin x\, dx = \sin x\, e^x - \int \cos x\, e^x dx =$

$u = \sin x,\ u' = \cos x$
$v' = e^x,\ v = e^x$

$u = \cos x,\ u' = -\sin x$
$v' = e^x,\ v = e^x$

$= e^x \sin x - \left[\cos x\, e^x - \int (-\sin x) e^x dx\right] =$

$= e^x(\sin x - \cos x) - \int e^x \sin x\, dx$

$2\int e^x \sin x\, dx = e^x(\sin x - \cos x) + C$

$$\int e^x \sin x\, dx = \frac{e^x(\sin x - \cos x)}{2} + C$$

b) $\int \frac{\ln x}{x} dx = \int u\, du = \frac{1}{2} u^2 + C$

$= \frac{1}{2}(\ln x)^2 + C$

$u = \ln x$
$du = \frac{1}{x} dx$

c) $I = \int \frac{1}{x^2-4}\,dx$

$$\frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

$$= \frac{(A+B)x + (2A-2B)}{(x-2)(x+2)}$$

$A + B = 0$, $B = -A$

$2A - 2B = 1$ $\quad 4A = 1$

$A = \frac{1}{4}$, $B = -\frac{1}{4}$

$$I = \int\left(\frac{1}{4}\frac{1}{x-2} - \frac{1}{4}\frac{1}{x+2}\right)dx = \frac{1}{4}\int\frac{dx}{x-2} - \frac{1}{4}\int\frac{dx}{x+2} =$$

$$= \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| + C$$

$$= \underline{\underline{\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C}}$$

10) Delvis

a) $\int 3\ln x\,dx = 3\int 1\cdot\ln x\,dx = 3\left(x\ln x - \int x\frac{1}{x}dx\right) =$

$u = \ln x$, $u' = \frac{1}{x}$

$v' = 1$, $v = x$

$$= 3\left(x\ln x - \int dx\right) = 3(x\ln x - x) + C = \underline{\underline{3x\ln x - 3x + C}}$$

$$\boxed{\int \ln x\,dx = x\ln x - x + C}$$

(følger av regningen over)

b) $\int \cos^2 x\,dx = \cos x\sin x - \int \sin x(-\sin x)dx$

$u = \cos x$, $u' = -\sin x$

$v' = \cos x$, $v = \sin x$

$= \cos x\sin x + \int \underbrace{\sin x\sin x}_{\sin^2 x}dx$

$= \cos x\sin x + \int(1 - \cos^2 x)dx$

$= \cos x\sin x + \int dx - \int \cos^2 x\,dx$

$$2\int\cos^2 x\,dx = \cos x\sin x + \int dx$$

$$\int \cos^2 x\,dx = \frac{\cos x\sin x}{2} + \frac{1}{2}\int dx = \underline{\frac{\cos x\sin x + x}{2} + C}$$

Alternativ

Vi husker

$\cos 2x = \cos^2 x - \sin^2 x =$

$\cos^2 x - (1 - \cos^2 x)$

$= 2\cos^2 x - 1, \quad \cos^2 x = \dfrac{\cos 2x + 1}{2}$

$$\int \cos^2 x \, dx = \int \frac{\cos 2x + 1}{2} \, dx$$

$$= \frac{1}{4} \sin 2x + \frac{x}{2} + C =$$

$$= \frac{1}{4} 2 \sin x \cos x + \frac{x}{2} + C$$

$$= \frac{\cos x \sin x + x}{2} + C$$

c) $\int x(\ln x^2)\,dx = \int \frac{1}{2} \ln u \, du$

$u = x^2$

Hadde nsted

$du = 2x\,dx$ $\quad = \frac{1}{2} \int \ln u \, du = \frac{1}{2}(u \ln u - u) + C$

$= \underline{\underline{\frac{1}{2}(x^2 \ln x^2 - x^2) + C}} =$

d) $\int x \sin\sqrt{x}\, dx = \int t^2 \sin t \, 2t\, dt =$

$t = \sqrt{x}, \quad x = t^2$

$dt = \frac{1}{2\sqrt{x}} dx \qquad = 2\int t^3 \sin t\, dt =$

$dx = 2\sqrt{x}\, dt \qquad v' = \sin t, \quad u = t^3$

$= 2t\, dt \qquad v = -\cos t \quad u' = 3t^2$

$= 2\left[-t^3\cos t - \int 3t^2(-\cos t)\, dt\right] =$

$= 2\left[-t^3\cos t + 3\int t^2\cos t\, dt\right] = -2t^3\cos t + 6\int t^2 \cos t\, dt$

$v' = \cos t \quad u = t^2$

$v = \sin t \quad u' = 2t$

$= -2t^3\cos t + 6\left(t^2\sin t - \int 2t\sin t\right) dt$

$= -2t^3\cos t + 6t^2\sin t - 12\int t\sin t\, dt =$

$v' = \sin t \quad u = t$

$v = -\cos t \quad u' = 1$

$= -2t^3\cos t + 6t^2\sin t - 12\left(-t\cos t - \int 1\cdot(-\cos t)\, dt\right)$

$= -2t^3\cos t + 6t^2\sin t + 12t\cos t - 12\int \cos t\, dt$

$= -2t^3\cos t + 6t^2\sin t + 12t\cos t - 12\sin t + C, \quad t = \sqrt{x}$

$= -2x^{3/2}\cos\sqrt{x} + 6x\sin\sqrt{x} + 12\sqrt{x}\cos\sqrt{x} - 12\sin\sqrt{x} + C$

e) $I = \int (x^2 + x) \sin x \, dx$

$I_1 = \int x^2 \sin x \, dx = x^2(-\cos x) - \int 2x(-\cos x) dx$

$v' = \sin x, u = x^2$

$= -x^2 \cos x + 2 \int x \cos x \, dx =$

$v' = \cos x, u = x$

$= -x^2 \cos x + 2 \left[x \sin x - \int \sin x \, dx \right]$

$= -x^2 \cos x + 2x \sin x - 2(-\cos x) + C$

$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$

$I_2 = \int x \sin x \, dx = (x(-\cos x)) - \int 1 \cdot (-\cos x) dx$

$v' = \sin x, u = x$

$v = -\cos x \quad u' = 1$

$= -x \cos x + \int \cos x \, dx = -x \cos x + \sin x$

$I = I_1 + I_2$