

1. desember

Plan: 2 2013

3 2013

Notatarkut

Eksamenstips

② 2013

$$\frac{dp}{dx} = -kp$$

$$p' = -kp$$

separabel:

$$\frac{1}{p} \cdot p' = -k$$



$$\frac{1}{p} p' = -k$$

$$\int \frac{1}{p} dp = \int -k dx$$

$$\ln(p) = -kx + C$$

$$e^{\ln(p)} = e^{-kx+C}$$

$$p = e^{-kx} \cdot e^C \text{ konstant}$$

$$p(x) = C_1 e^{-kx}$$

$$= C_1 e^{-1,26 \cdot 10^{-4} x}$$

Har:  $p(x) = C_1 e^{-1,26 \cdot 10^{-4} x}$   
 $p(0) = p_0$   
 $p(0) = C_1 e^0 = C_1 = p_0$

$$p(x) = \underline{\underline{p_0 e^{-1,26 \cdot 10^{-4} x}}}$$

(sjekk:  $\frac{p(2469)}{p(0)} = \dots$ )

③ 2013

a)  $\sqrt{3} \sin\left(\frac{1}{3}t\right) + \cos\left(\frac{1}{3}t\right) = \underline{A} \cos(\underline{\omega}t - \underline{\varphi})$

( $\cos(u-v) = \cos(u)\cos(v) + \sin(u)\sin(v)$ )

s. 242 (?):

$$C \cdot \cos(bx) + D \cdot \sin(bx) = A \cos(bx - \varphi)$$

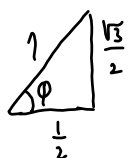
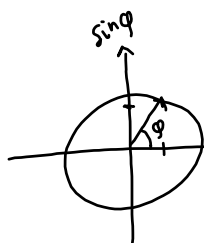
$$\text{der } A = \sqrt{C^2 + D^2}$$

$$\cos \varphi = \frac{C}{A} \quad \text{og} \quad \sin \varphi = \frac{D}{A}$$

Vi har:  $A = \sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4} = 2$

$$\cos \varphi = \frac{1}{2}$$

$$\sin \varphi = \frac{\sqrt{3}}{2}$$



$$\varphi = \frac{\pi}{3}$$

$\omega = \frac{1}{3}$ , så vi får

$$\cos\left(\frac{1}{3}t\right) + \sqrt{3} \sin\left(\frac{1}{3}t\right) = \underline{\underline{2 \cos\left(\frac{1}{3}t - \frac{\pi}{3}\right)}}$$

$$b) \quad 9y'' + y = 0$$

Karakteristisk polynom:

$$9r^2 + 1 = 0$$

$$9r^2 = -1$$

$$r^2 = -\frac{1}{9}$$

$$r = \pm \sqrt{\frac{-1}{9}} = \pm \frac{\sqrt{-1}}{3} = \pm \frac{i}{3}$$

To komplekse:  $r_1 = \frac{1}{3}i$ ,  $\bar{r}_1 = -\frac{1}{3}i$

$$\text{Så: } y(t) = C \cdot \cos\left(\frac{1}{3}t\right) + D \cdot \sin\left(\frac{1}{3}t\right)$$

der  $C, D \in \mathbb{R}$

$$\text{Vi vet: } y(\pi) = 2, \quad y'(\pi) = 0$$

Skal finne  $C$  og  $D$

$$y(t) = C \cdot \cos\left(\frac{1}{3}t\right) + D \sin\left(\frac{1}{3}t\right)$$

$$y'(t) = -\frac{1}{3}C \sin\left(\frac{1}{3}t\right) + \frac{1}{3}D \cos\left(\frac{1}{3}t\right)$$

$$\text{I } y(\pi) = C \cdot \cos\left(\frac{\pi}{3}\right) + D \sin\left(\frac{\pi}{3}\right) = 2$$

$$\text{II } y'(\pi) = -\frac{1}{3}C \cdot \sin\left(\frac{\pi}{3}\right) + \frac{1}{3}D \cos\left(\frac{\pi}{3}\right) = 0$$

$$\text{I } \frac{1}{2}C + \frac{\sqrt{3}}{2}D = 2$$

$$\text{II } -\frac{1}{3} \cdot \frac{\sqrt{3}}{2}C + \frac{1}{3} \cdot \frac{1}{2} \cdot D = 0$$

$$-\frac{\sqrt{3}}{6}C + \frac{1}{6}D = 0$$

$$\frac{1}{6}D = \frac{\sqrt{3}}{6}C$$

$$D = \sqrt{3}C$$

setter inn i  
lign. I

$$\hookrightarrow \text{I: } \frac{1}{2}C + \frac{\sqrt{3}}{2} \sqrt{3}C = 2$$

$$C \left( \frac{1}{2} + \frac{3}{2} \right) = 2$$

$$2C = 2$$

$$C = 1$$

$$D = \sqrt{3} \cdot C = \sqrt{3}$$

$$y(t) = \underline{\underline{\cos\left(\frac{1}{3}t\right) + \sqrt{3} \sin\left(\frac{1}{3}t\right)}}$$

På notatarket

Se snublenotater fra 8. oktober

Annnet: Trigonometri: enhets sirkel  
eksakte verdier

- Antiderivasjon:  $\left. \begin{array}{l} \text{substitusjon} \\ \text{delvis integrasjon} \\ \text{delbrokopp spalting} \end{array} \right\} \text{eksempler}$

- Differensiallikninger: løsningsmetoder,  
eksempler (separabel)

- Trigonometriske funksjoner: akrofase, amplitude, ...<sup>def. 10.13</sup>  
+ teorem 10.18

- Differensiallikninger: inhomogene (løsningsmetode)

Tips til eksamen

- Mat, søvn  
↳ ha med nok (sunn + premie)
- Les gjennom hele eksamenen først
- Bruk masse plass
- Bruk hjelpsetninger ("siden (...), så (...)"  
"vi bruker", ...)