24. november

Plan:
- Rep. andre ordens diffligning + eksempler (x3)
- Rep. andre ordens differensligning
- 2014: 2
- 2014: 3
- (2014: 5)
- Eksamenstidstips

Eksempler

12.6 a) $y'' + 2y' - 5y = 0$

$$\boxed{ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$r^2 + 2r - 5 = 0$$

$$r = \frac{-2 \pm \sqrt{4+20}}{2} = \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 \pm \sqrt{4 \cdot 6}}{2} = \frac{-2 \pm 2\sqrt{6}}{2} = -1 \pm \sqrt{6}$$

To reelle røtter $r_1 = -1 + \sqrt{6}$, $r_2 = -1 - \sqrt{6}$

$$\Rightarrow \quad y(x) = Ce^{r_1 x} + De^{r_2 x}$$

$$= \underline{\underline{Ce^{(-1+\sqrt{6})x} + De^{(-1-\sqrt{6})x}}}, \quad C, D \in \mathbb{R}$$

12.6 c) $y'' + 4y' + 4y = 0$

$r^2 + 4r + 4 = 0$

$$r = \frac{-4 \pm \sqrt{16-16}}{2} = \frac{-4}{2} = -2$$

Én reell løsning: $r_1 = -2$

$$\Rightarrow \quad y(x) = Ce^{r_1 x} + Dxe^{r_1 x}$$

$$= \underline{\underline{Ce^{-2x} + Dxe^{-2x}}}, \quad C, D \in \mathbb{R}$$

12.7 d) $y'' - 4y' + 5y = 0$

$r^2 - 4r + 5 = 0$

$$r = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm \sqrt{-4}}{2}$$

$$= \frac{4 \pm 2\sqrt{-1}}{2} = 2 \pm \sqrt{-1}$$

$$= 2 \pm i$$

To komplekse løsninger: $r_1 = 2 + i$, $\overline{r_1} = 2 - i$

$$\boxed{\begin{array}{c} z = a + ib \\ \hline a = 2 \\ b = 1 \end{array}}$$

$$\Rightarrow \quad y(x) = e^{ax}\left(C \cdot \cos(bx) + D \cdot \sin(bx)\right)$$

$$= \underline{\underline{e^{2x}\left(C \cdot \cos(x) + D \cdot \sin(x)\right)}}, \quad C, D \in \mathbb{R}$$

Rep: Andre ordens lineære differenslikninger

$$x_{n+2} + b x_{n+1} + c x_n = f(n)$$

↑ inhomogen

Metode:

1) Finn løsningene til den homogene:

$$x^h_{n+2} + b x^h_{n+1} + c x^h_n = 0$$

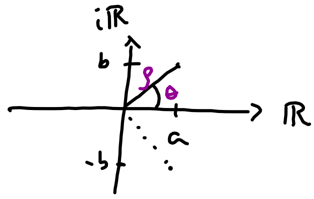
Løs: $r^2 + br + c = 0$

To reelle: $x^h_n = C r_1^n + D r_2^n , \quad C, D \in \mathbb{R}$

Én reell: $x^h_n = C r_1^n + D n r_1^n , \quad C, D \in \mathbb{R}$

To komplekse: $r_1 = a + ib = \rho \cdot e^{i\theta}$

$\bar{r}_1 = a - ib$



$$x^h_n = C \rho^n \cos(n\theta) + D \rho^n \sin(n\theta) , \quad C, D \in \mathbb{R}$$

2) Gjett på spesiell løsning:

$$x^s_n = A_k n^k + \cdots + A_1 n + A_0$$

↳ et polynom av samme grad (k) som $f(n)$

3) Legg sammen:

$$x_n = x^h_n + x^s_n$$

Evt. 4) Bruk initialverdier til å finne C, D.

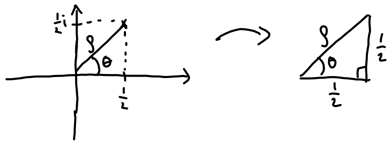
2014: oppg. 2:

$$2x_{n+2} - 2x_{n+1} + x_n = 1$$

1)
$$2x^h_{n+2} - 2x^h_{n+1} + x^h_n = 0$$
$$2r^2 - 2r + 1 = 0$$

$$r = \frac{2 \pm \sqrt{4-8}}{4} = \frac{2 \pm \sqrt{-4}}{4}$$
$$= \frac{2 \pm 2i}{4} = \frac{1 \pm i}{2}$$

$$r_1 = \tfrac{1}{2} + \tfrac{1}{2}i \,, \quad \bar{r}_1 = \tfrac{1}{2} - \tfrac{1}{2}i$$



$$\theta = \frac{\pi}{4}$$

$$\rho^2 = \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2$$
$$\rho = \sqrt{\tfrac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$x^h_n = \rho^n C \cos(n\theta) + \rho^n D \sin(n\theta)$$
$$= \left(\frac{1}{\sqrt{2}}\right)^n \cdot C \cdot \cos\left(n\frac{\pi}{4}\right) + \left(\frac{1}{\sqrt{2}}\right)^n D \sin\left(n\frac{\pi}{4}\right)$$

2) Gjett på løsning: siden $f(n) = 1$, gjetter vi

$$x^s_n = A \quad \text{(en konstant)}$$
$$x^s_{n+1} = A$$
$$x^s_{n+2} = A$$

$$\Rightarrow \quad 2A - 2A + A = 1$$
$$A = 1 \,, \quad \text{så} \quad x^s_n = 1$$

3) Legger sammen:

$$x_n = x^h_n + x^s_n$$
$$= \left(\frac{1}{\sqrt{2}}\right)^n \left(C\cos\left(n\frac{\pi}{4}\right) + D\sin\left(n\frac{\pi}{4}\right)\right) + 1$$

Hva blir $\lim_{n\to\infty} x_n$?

- $\left(\frac{1}{\sqrt{2}}\right)^n = \frac{1}{(\sqrt{2})^n}$ går mot null når $n \to \infty$ fordi $\lim_{n\to\infty} \sqrt{2}^n = \infty$
- $\cos\left(n\frac{\pi}{4}\right)$ og $\sin\left(n\frac{\pi}{4}\right)$ blir aldri større enn 1 eller mindre enn -1.
- 1 endres ikke når $n \to \infty$

Så $\lim_{n\to\infty} x_n = 1$

2014: 3

En funksjon f tilfredstiller $f'(x) = e^x \sin(x)$

$f(0) = 1$

Finn $f(x)$.

Altså: finn f som blir $e^x \sin(x)$ når vi deriverer.

Vi antideriverer!

$$f(x) = \int f'(x)\,dx = \int e^x \sin(x)\,dx$$

Ikke substitusjon! Delvis: $\int u' \cdot v\,dx = uv - \int u v'\,dx$

$$\int e^x \sin x\,dx$$

$u' = e^x$	$u = e^x$
$v = \sin x$	$v' = \cos x$

Ny delvis!

$u' = e^x$	$u = e^x$
$v = \cos x$	$v' = -\sin x$

$$= e^x \sin x - \int e^x \cos x\,dx$$

$$= e^x \sin x - \left(e^x \cos x + \int e^x \sin x\,dx\right)$$

$$= e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = e^x(\sin x - \cos x) + C$$

$$f(x) = \int e^x \sin x\,dx = \underline{\frac{1}{2} e^x (\sin x - \cos x) + C_1}$$