1.4: Likningssystemer.

- ·) Addisjonsmetoden
- ·) Substitusjonsmetoden

~ Matriser

a) $L_1: 7x - 5x = 3$

$(7-5)x = 3$

$+2x = 3$

$x = +\frac{3}{2}$

Løsningsmengden: $\{\frac{3}{2}\}$.

b) $L_1: \boxed{7x - 5y = 3}$

to variable
en likning
ofte vendelig mange løsninger.

Isolerer x:

$7x = 3 + 5y$

$x = \frac{3}{7} + \frac{5}{7}y.$

Innfører en parameter $t = y$.

$x = \frac{3}{7} + \frac{5}{7}t.$

parametriserer løsningene.

Løsningsmengden: $\{(\frac{3}{7} + \frac{5}{7}t, t) : t \in \mathbb{R}\} \ni (x, y)$.

En-dimensjonal løsningsmengde.

c) $\begin{cases} L_1: 7x - 5y = 3 \\ L_2: x + 5y = 1 \end{cases}$

Addisjonsmetoden: $L_1 + L_2 = 3 + 1 = 4.$

$L_1 + L_2:$ $\begin{array}{r} 7x - 5y \\ +\ x + 5y \\ \hline = 8x \end{array}$

$8x = 4.$ Eliminert y.

$x = \frac{4}{8} = \frac{1}{2}$

$L_2: x + 5y = 1.$ $x = \frac{1}{2}.$

$\frac{1}{2} + 5y = 1$

$5y = 1 - \frac{1}{2}$

$5y = \frac{1}{2}$

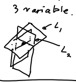
$\frac{5y}{5} = \frac{\frac{1}{2}}{5} = \frac{1}{2\cdot 5} = \frac{1}{10}.$

$y = \frac{1}{10}.$

Løsningsmengden: $\{(\frac{1}{2}, \frac{1}{10})\}$.

d) $\begin{cases} L_1: x + 2y - z = 5 \\ L_2: 2x - y = 2 \\ L_3: x + y + z = 3 \end{cases}$

3 likninger
3 variable.



$L_1 + L_3 = 5 + 3 = 8.$

$L_1 + L_3:$ $\begin{array}{r} x + 2y - z \\ +\ x + y + z \\ \hline = 2x + 3y \end{array}$

$2x + 3y = 8$ $(L_1 + L_3)$.

$(L_1 + L_3) - L_2 = 8 - 2 = 6.$

$\begin{array}{r} 2x + 3y \\ -\ 2x + y \\ \hline = 4y \end{array}$

$4y = 6.$

$y = \frac{6}{4} = \frac{3}{2}.$

$L_1 + L_3: 2x + 3\cdot\frac{3}{2} = 8.$

$2x + \frac{3\cdot 3}{2} = 8$

$2x + \frac{9}{2} = 8$

$2x = 8 - \frac{9}{2}$

$2x = \frac{8\cdot 2 - 9}{2}$

$2x = \frac{16 - 9}{2}$

$2x = \frac{7}{2}.$

$x = \frac{7}{2\cdot 2} = \frac{7}{4}.$

$L_3: x + y + z = 3.$

$\frac{7}{4} + \frac{3}{2} + z = 3.$ $|\cdot 4$

$7 + 4\cdot\frac{3}{2} + 4z = 4\cdot 3$

$7 + 6 + 4z = 12.$

$13 + 4z = 12.$

$4z = 12 - 13 = -1.$

$4z = -1.$

$z = -\frac{1}{4}.$

Løsningsmengde: $\{(\frac{7}{4}, \frac{3}{2}, -\frac{1}{4})\} \ni (x, y, z)$

1.5 a) $\begin{cases} L_1: x + y + z = 0 \\ L_2: x - y + 3z = 0 \end{cases}$

Snittet.

Substitusjon: (løser for y)

$L_2: x - y + 3z = 0$

$x + 3z = y$

$y = x + 3z$ · Setter inn i L_1:

$L_1: x + (x + 3z) + z = 0$

$x + x + 3z + z = 0$

$2x + 4z = 0$

$2x = -4z.$

$x = \frac{-4z}{2} = -2z$

Parameter: $t = z$

$x = -2t$

$y = x + 3z$

$y = -2t + 3t$

$y = t$

Løsningsmengde: $\{(-2t, t, t) : t \in \mathbb{R}\}$.

b) $\begin{cases} L_1: x - y + 2z = 4 \\ L_2: 2x - 2y + 4z = 8 \end{cases}$

Addisjonsm. $2\cdot L_1 - L_2 = 2\cdot 4 - 8 = 0$

$\begin{array}{r} \cancel{2x} - \cancel{2y} + \cancel{4z} \\ -\ \cancel{2x} + \cancel{2y} - \cancel{4z} \\ \hline = 0 \end{array}$

$2L_1 - L_2: 0 = 0.$

($0 = 0$-uttrykk)

Overflødig info. $L_1: x - y + 2z = 4.$

Isolerer x:

$x - y + 2z = 4$

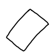
$x + 2z = 4 + y$

$x = 4 + y - 2z.$

Innfører parametre: $t = y$, $s = z$.

$x = 4 + t - 2s.$

LM: $\{(4 + t - 2s, t, s) : \underbrace{(t, s) \in \mathbb{R}^2}_{t \in \mathbb{R},\, s \in \mathbb{R}}\}$



c) $\begin{cases} L_1: x + y + z + w = 15 \\ L_2: x - 2y + 4z - w = 12 \end{cases}$

$L_1 + L_2 = 15 + 12 = 27$

$\begin{array}{r} x + y + z + w \\ +\ x - 2y + 4z - w \\ \hline = 2x - y + 5z \end{array}$

$L_1 + L_2: 2x - y + 5z = 27.$

$2x + 5z = 27 + y$

$2x + 5z - 27 = y$

$y = 2x + 5z - 27.$

$t = x$, $s = z$: $y = 2t + 5s - 27.$

$L_1: x + y + z + w = 15$

$t + (2t + 5s - 27) + s + w = 15.$

$t + 2t + 5s - 27 + s + w = 15.$

$3t + 6s - 27 + w = 15.$

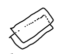
$w = 15 - 3t - 6s + 27$

$w = 42 - 3t - 6s.$

L.M.: $\{(t, 2t + 5s - 27, s, 42 - 3t - 6s) : (t, s) \in \mathbb{R}^2\}$.

d) $L_1: x - y + 2z = 4$

$L_2: 2x - 2y + 4z = 9$



Addisjonsm. $L_2 - 2\cdot L_1 = 9 - 2\cdot 4 = 9 - 8 = 1.$

$\begin{array}{rl} 2x - 2y + 4z & \leftarrow L_2 \\ -2x + 2y - 4z & \leftarrow -2L_1 \\ \hline = 0 & \end{array}$

$L_2 - 2L_1: 0 = 1.$

Inkonsistent.

Ingen løsninger