A10

10.1 Finn definisjonsmengden D_f til

a) $f(x) = x^2+1$, b) $f(x) = \dfrac{x^2+1}{2x-1}$,

c) $f(x) = x^{\frac{3}{2}} - 2x+1$, d) $f(x) = \ln|x| + 2\sin(x)$

---

a) x^2+1 er et polynom, definert overalt.

$\underline{D_f = \mathbb{R}.}$

b) $f(x) = \dfrac{x^2+1}{2x-1}$, definert når nevner $\neq 0$.

Løser: $2x-1=0 \Rightarrow \underline{x = \frac{1}{2}}$.

Definisjonsmengden: $D_f = \mathbb{R} \setminus \{\frac{1}{2}\}$.

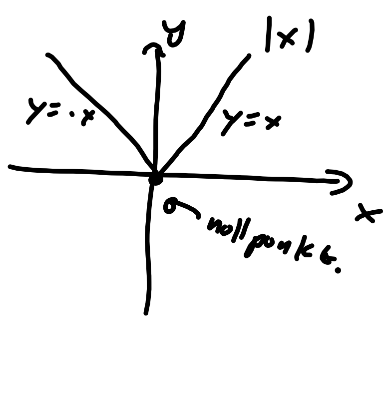
c) $\underline{f(x) = x^{\frac{3}{2}} - 2x+1}$. $x^{\frac{3}{2}} = \sqrt{x^3}$. Vi må ha $\boxed{x \geq 0}$

$\left(x^{\frac{n}{m}} \begin{array}{l} n \text{ — odde} \\ m \text{ — partall} \Rightarrow x \geq 0 \end{array} \right)$

$\underline{D_f = [0, \infty)}.$

d) $f(x) = \underbrace{\ln|x|}_{|x| \geq 0.} + \underbrace{2\sin(x)}_{\text{definert overalt.}}$

$\cdot$) $\ln(t)$ er definert når $t > 0$. $(0, \infty)$.



Eneste måte $\ln|x|$ ikke er definert, er at $|x| = 0$. Skjer kun når $x = 0$.

$\Rightarrow \underline{D_f = \mathbb{R} \setminus \{0\}}.$

10.2 Finn verdimengden V_f til:

a) $f(x) = \underline{x} + \underline{x^{\frac{3}{7}}} + \underline{2x^{\frac{3}{4}}}$.

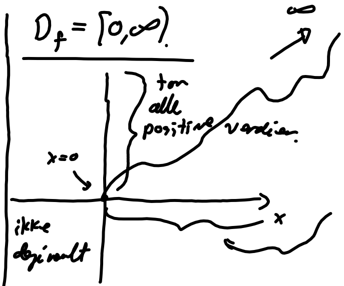
+, ikke −.

$x^{\frac{3}{7}}$ — odde, alltid definert.

$\boxed{\sqrt[3]{-1} = -1}$

$2x^{\frac{3}{4}}$ — partall, kan defineres når $x \geq 0$.

$D_f = [0, \infty)$.



$f(0) = 0 + 0^{\frac{3}{7}} + 2 \cdot 0^{\frac{3}{4}} = 0$.

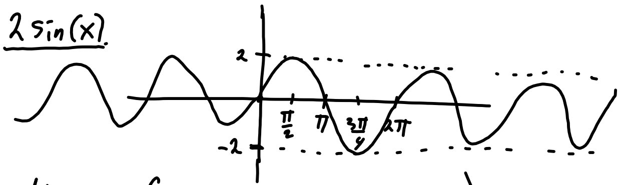
·) Hvordan oppfører $f(x)$ seg når $x \to \infty$?
$x, x^{\frac{3}{7}}, 2x^{\frac{3}{4}}$ går mot ∞. $\lim_{x \to \infty} f(x) = \infty$.

·) $f(x) \geq 0$ når $x \geq 0$ (fordi $x, x^{\frac{3}{7}}, 2x^{\frac{3}{4}} \geq 0$.)

$\Rightarrow V_f = [0, \infty)$.

b) $f(x) = \underline{2\sin(x)}$.

$D_f = \mathbb{R}$



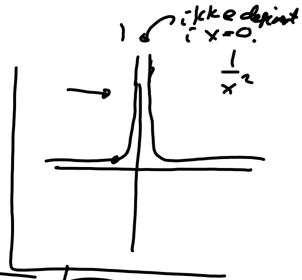
Maksverdi = 2 (f.eks $2\underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1} = 2$)

Min verdi = −2 ($2\underbrace{\sin\left(\frac{3\pi}{2}\right)}_{=-1} = -2$).

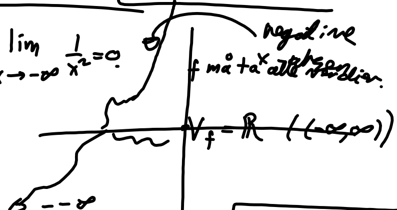
$\Rightarrow V_f = [-2, 2]$.

c) $f(x) = \underline{\frac{1}{x^2}} + x^3$.

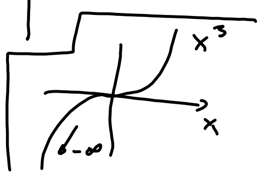
kan defineres når $x \neq 0$. | $D_f = \mathbb{R} \setminus \{0\}$.



·) $x \to -\infty$: $\frac{1}{x^2} \to 0$. $\lim_{x \to -\infty} \frac{1}{x^2} = 0$
$x^3 \to -\infty$.



$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{1}{x^2} + x^3 = 0 + (-\infty) = -\infty$



·) $x \to 0^-$: $\frac{1}{x^2} \to \infty$. $x^3 \to 0$.

$\lim_{x \to 0^-} f(x) = \infty + 0 = \infty$.

$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$
x^2 alltid positivt

10.3 Finn grenseverdiene:

a) $\lim_{x\to\infty} \frac{7x^2 + \underline{4x^4}}{\underline{3x^3} - 2x^2}$.

dominerende

dominerende

Finne de dominerende leddene: de med høyest eksponent

$= \lim_{x\to\infty} \frac{4\cancel{x^4}\,x}{3\cancel{x^3}}$.

(Kan alltid fjerne ikke-dominerende ledd)

[$x^4 = x^3 \cdot x$ (deler på x^3 oppe og nede)]

$= \lim_{x\to\infty} \frac{4}{3}x = \infty$.

Metoden gjelder når $x \to \infty$. Når $x \to 0$, da er de dominerende leddene de med lavest eksponent.

b) $\lim_{x\to\infty} \frac{8x^2 + 2x + 7}{\sqrt{x} - 4x^2} = \lim_{x\to\infty} \frac{\underline{8x^2} + 2x + 7}{x^{\frac{1}{2}} - \underline{4x^2}}$.

$= \lim_{x\to\infty} \frac{8\cancel{x^2}}{-4\cancel{x^2}} = \lim_{x\to\infty} \frac{8}{-4} = \underline{\underline{-2}}$

10.4 a) $\lim_{x \to \infty} \frac{2^x(3^x+8)}{3^x(2^x-5)}$ går mot ∞ når $x \to \infty$.

(Kan prøve å dele på $2^x \cdot 3^x$.)

$$= \lim_{x\to\infty} \frac{\frac{\cancel{2^x}(3^x+8)}{\cancel{2^x}\cdot 3^x}}{\frac{\cancel{3^x}(2^x-5)}{2^x\cdot\cancel{3^x}}}$$

$$= \lim_{x\to\infty} \frac{\frac{3^x+8}{3^x}}{\frac{2^x-5}{2^x}} = \lim_{x\to\infty} \frac{\frac{\cancel{3^x}}{\cancel{3^x}} + \frac{8}{3^x}}{\frac{\cancel{2^x}}{\cancel{2^x}} - \frac{5}{2^x}}$$

$$= \lim_{x\to\infty} \frac{1 + \frac{8}{3^x}}{1 - \frac{5}{2^x}}$$

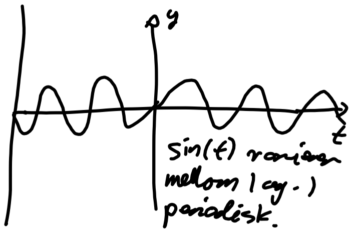
$\frac{8}{3^x} \to 0$, $3^x \to \infty$, $\frac{5}{2^x} \to 0$, $2^x \to \infty$.

$$= \frac{1+0}{1-0} = \underline{\underline{1}}$$

10.5/ Eksisterer grensene?

a) $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$.

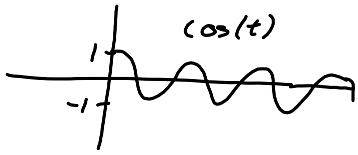
$\boxed{\lim_{x\to 0^+} \frac{1}{x} = \infty.}$



Gjør et variabelskifte: $t = \frac{1}{x}$. Når $x \to 0$, så går $t \to \infty$.

$\boxed{\lim_{t\to\infty} \sin(t)}$ Periodisk funksjon. Har ingen grenseverdi når $t \to \infty$.

b) $\lim_{x\to 0} \cos\left(\frac{1}{x}\right)$.



Variabelskifte: $t = \frac{1}{x}$

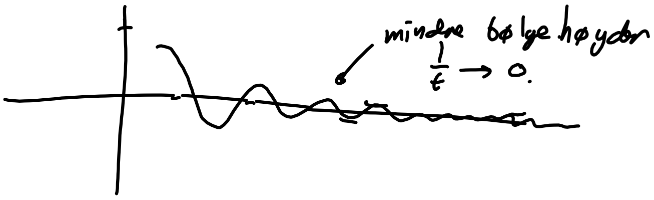
$\lim_{t\to\infty} \cos(t)$. Som over, $\cos(t)$ varierer (oscillerer) mellom 1 og -1.

Har ingen grense.

c) $\lim_{x\to 0} x \cdot \cos\left(\frac{1}{x}\right)$

$t = \frac{1}{x}$, studerer : $x = \frac{1}{t}$

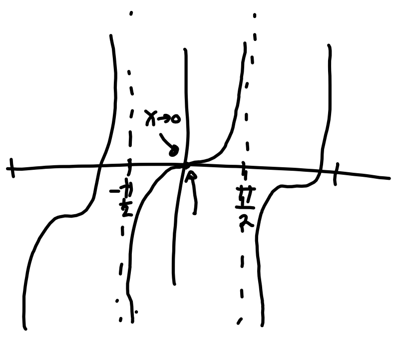
$\lim_{t\to\infty} \frac{1}{t} \cdot \cos(t)$



Siden $\cos(t)$ ligger mellom 1 og -1, mens $\frac{1}{t} \to 0$.

$\Rightarrow \lim_{t\to\infty} \underbrace{\frac{1}{t}}_{\to 0} \underbrace{\cos(t)}_{\text{begrenset}} = 0.$

d) $\lim_{x\to 0} \tan(x)$

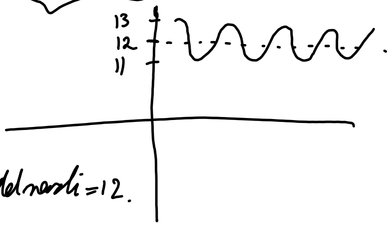


$\tan(x)$ er definert på $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (mm.) og er kontinuerlig.

$\Rightarrow \lim_{x\to 0} \tan(x) = \tan(0) = 0.$

10.6 Finn middelverdi, maks og minverdi til:

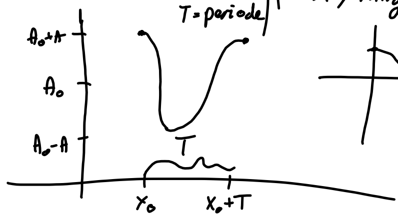
$$f(t) = \underline{12} - \underbrace{\cos\left(\frac{\pi}{6}(t+6)\right)}$$

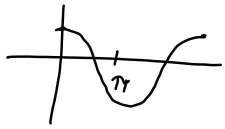


$\Rightarrow$ Maks $= 13$ | Middelverdi $= 12$.
Min $= 11$

$$f(x) = \underbrace{A_0}_{\text{middelverdien}} + \underbrace{A}_{\text{amplitude}} \cos\left(\underbrace{\frac{2\pi}{T}}_{w=\frac{2\pi}{T}}(x - x_0)\right), \quad A \geq 0, T \geq 0.$$

$w = \frac{2\pi}{T}$ frekvens, T = periode | akrofase forskyvning





+middelverdi + amplitude

10.8 Finn frekvens w, periode T og akrofase til:

a) $f(t) = \underline{-4} + 1 \cdot \cos\left(2\pi\,(t-2)\right)$ t_0.

$$f(x) = A_0 + A\cos\left(\frac{2\pi}{T}(x - x_0)\right), \quad A \geq 0, T \geq 0.$$

$A_0 = -4$ (middelverdi)

$A = 1$ (amplitude)

$w = 2\pi$ (frekvens)

$\frac{2\pi}{T} = w$: $\frac{2\pi}{T} = 2\pi \Rightarrow T = 1$ (periode)

$t_0 = 2$ (forskyvning/akrofase)

9a) $f(t) = \underline{-2} + \underline{4}\cos(\pi t - \pi)$.

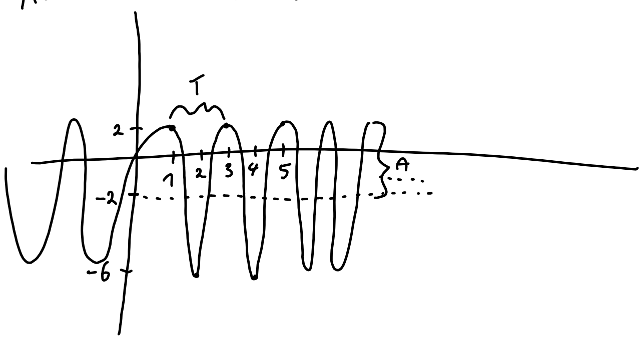
$A_0 = -2$
$A = 4$
$w = \pi$
$t_0 = 1$

$\pi t - \pi = \pi(t-1)$ — frekvens, akrofase.

$\frac{2\pi}{T} = w$

$\frac{2\pi}{T} = \pi$. $\underline{\underline{T = 2}}$

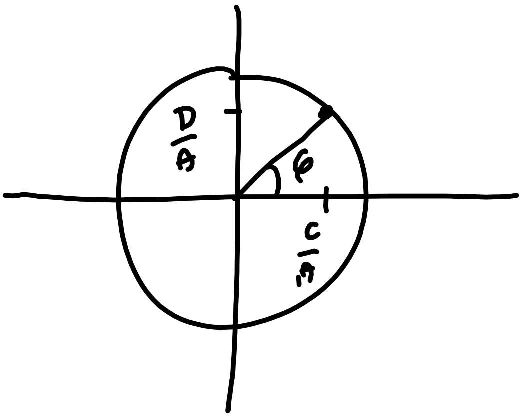
$f(t) = -2 + 4\cos(\pi(t-1))$.



10.10: Skriv om: $f(x) = \cos(x) - \sin(x)$ på fase form.

Generelt: $\underbrace{C\cos(bx) + D\sin(bx)} = \underbrace{A\cos(bx - \varphi)}_{\text{faseformen}}$

$A = \sqrt{C^2 + D^2}$, $\cos(\varphi) = \dfrac{C}{A}$, $\sin(\varphi) = \dfrac{D}{A}$.



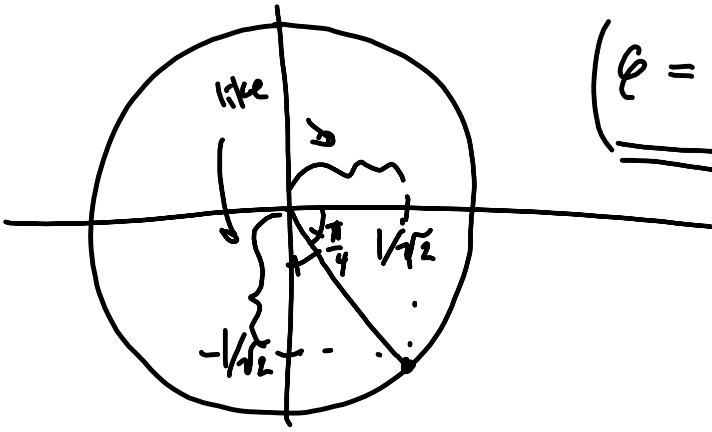
$f(x) = \cos(x) - \sin(x)$

$= 1 \cdot \cos(1 \cdot x) - 1 \cdot \sin(1 \cdot x)$

$C = 1$, $D = -1$, $b = 1$.

$A = \sqrt{C^2 + D^2} = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$.

$\cos(\varphi) = \dfrac{1}{\sqrt{2}} \left(= \dfrac{\sqrt{2}}{2}\right)$, $\sin(\varphi) = \dfrac{-1}{\sqrt{2}} \left(= -\dfrac{\sqrt{2}}{2}\right)$.



$\left(\underline{\underline{\varphi = -\dfrac{\pi}{4}}}\right)$. $\left(\varphi = \dfrac{7\pi}{4}\right)$.

$\rightsquigarrow$ faseformen er: $A\cos(bx - \varphi)$

$= \underline{\underline{\sqrt{2}\cos\left(x + \dfrac{\pi}{4}\right)}}$ $\left[\sqrt{2}\cos\left(x - \dfrac{7\pi}{4}\right)\right]$