FASIT

Oppgave 1

a) Karakteristisk ligning:

$$r^2 - 4r + 13 = 0$$

$$r = \frac{4 \pm \sqrt{16-52}}{2} = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

Generell løsning

$$\underline{\underline{y = e^{2x}(C\cos 3x + D\sin 3x)}}$$

b) $y(0) = e^0(C\cos 0 + D\sin 0) = C = 1$

$y'(x) = 2e^{2x}(C\cos 3x + D\sin 3x) + e^{2x}(C\cdot(-\sin 3x)\cdot 3 + D(\cos 3x)\cdot 3)$

$y'(0) = 2C + 3D = -1$

$$3D = -1 - 2C = -3 \Rightarrow D = -1$$

$$\underline{\underline{y = e^{2x}(\cos 3x - \sin 3x).}}$$

Oppgave 2

a) $\int x \cos x \, dx = x \sin x - \int 1 \cdot \sin x \, dx =$

$$\boxed{\begin{array}{ll} u = x & u' = 1 \\ v' = \cos x & v = \sin x \end{array}}$$

$x \sin x - (-\cos x) + \cancel{C} = \underline{\underline{x \sin x + \cos x + C}}$

b) $y' + \frac{1}{x} y = \cos x$ (Første ordens lineær diffligning).

Integrerende faktor $m(x) = e^{\int \frac{1}{x} dx} = e^{\ln|x|} = |x|$.

Vi kan velge $m(x) = x$.

Ved å multiplisere med x blir ligningen

$$(xy)' = x \cos x$$

$$xy = \int x \cos x \, dx = x \sin x + \cos x + C$$

Generell løsning:

$$y = \sin x + \frac{\cos x + C}{x}$$

$$y\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} + \frac{\cos \frac{\pi}{2} + C}{\frac{\pi}{2}} = 1 + \frac{2C}{\pi} = 3$$

$$\frac{2C}{\pi} = 2, \quad C = \pi.$$

$$\underline{\underline{y = \sin x + \frac{\cos x + \pi}{x}}}$$

Oppgave 3

a) Egenverdien:

$$|M-\lambda I| = \begin{vmatrix} -0.5-\lambda & 3 \\ -0.5 & 2-\lambda \end{vmatrix} = (-0.5-\lambda)(2-\lambda)-(-0.5)\cdot 3$$

$$= -1+0.5\lambda-2\lambda+\lambda^2+1.5 = \lambda^2-1.5\lambda+0.5$$

$$\lambda = \frac{1.5 \pm \sqrt{2.25-2}}{2} = \frac{1.5\pm\sqrt{0.25}}{2} = \frac{1.5\pm 0.5}{2} = \begin{cases} 1 \\ 0.5 \end{cases}$$

Egenverdiene er 1 og 0.5.

Egenvektoren for $\lambda = 1$:

$$-1.5x+3y=0$$
$$-1.5x=-3y$$
$$x=2y$$

y velges fritt $y=s$.

Egenvektoren $\begin{bmatrix} x \\ y \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2s \\ s \end{bmatrix}}}$

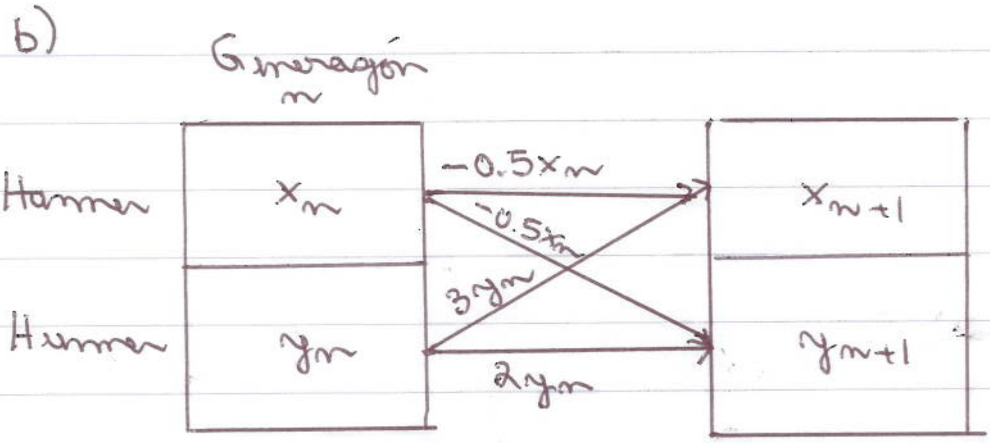
Egenvektoren for $\lambda = 0.5$:

$$-x+3y=0$$
$$x=3y$$

y velges fritt $y=t$.

Egenvektoren $\begin{bmatrix} x \\ y \end{bmatrix} = \underline{\underline{\begin{bmatrix} 3t \\ t \end{bmatrix}}}$

b)



$$x_{n+1} = -0.5x_n + 3y_n$$

$$y_{n+1} = -0.5x_n + 2y_n$$

så M er overgangsmatrisen.

c) Skriver $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 50 \\ 50 \end{bmatrix}$ som sum av egenvektorer:

$$\begin{bmatrix} 50 \\ 50 \end{bmatrix} = \begin{bmatrix} 2s \\ s \end{bmatrix} + \begin{bmatrix} 3t \\ t \end{bmatrix} = \begin{bmatrix} 2s+3t \\ s+t \end{bmatrix}$$

$$\begin{aligned} 2s+3t &= 50 \\ s+t &= 50 \quad \cdot(-2) \end{aligned} \qquad \begin{aligned} t &= 50-2\cdot 50 = -50 \\ s &= 50-t = 100 \end{aligned}$$

$$\begin{bmatrix} 50 \\ 50 \end{bmatrix} = \underbrace{\begin{bmatrix} 200 \\ 100 \end{bmatrix}}_{\text{Eg.v. } 1} - \underbrace{\begin{bmatrix} 150 \\ 50 \end{bmatrix}}_{\text{Eg.v. } \frac{1}{2}} \Rightarrow \begin{bmatrix} x_n \\ y_n \end{bmatrix} = M^n \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = 1^n \cdot \begin{bmatrix} 200 \\ 100 \end{bmatrix} - \left(\frac{1}{2}\right)^n \begin{bmatrix} 150 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 200 - \frac{150}{2^n} \\ 100 - \frac{50}{2^n} \end{bmatrix}$$

Siden $\frac{1}{2^n} \to 0$ når $n \to \infty$

vil

$$\lim_{n\to\infty} \begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} 200 \\ 100 \end{bmatrix}$$

Oppgave 4

a) $\int 2x \cos x^2 \, dx = \int \cos u \, du = \sin u + C = \underline{\underline{\sin x^2 + C}}$

$u = x^2$

$du = 2x\,dx$

b) $\frac{dy}{dx} = e^{-y} 2x \cos x^2.$

Separerer variable:

$e^{y} dy = 2x \cos x^2 \, dx$

Integrerer hver side

V.S. $\int e^{y} dy = e^{y}$

H.S $\int 2x \cos x^2 \, dx = \sin x^2 + C$

$e^{y} = \sin x^2 + C$

$y = \ln(\sin x^2 + C)$ (Generell løsning)

$y(0) = \ln C = 0 \Rightarrow C = 1$

$\underline{\underline{y = \ln(\sin x^2 + 1)}}$