MAT 1010 – Våren 2008.

Oppgave 1.

(Tallfølgen blir 1, 1, 7, 13, 55, ...)

$$x_{n+2} - x_{n+1} - 6x_n = 0$$

Karakteristisk ligning: $r^2 - r - 6 = 0 \Rightarrow r = \frac{1 \pm \sqrt{1+24}}{2}$

$$= \frac{1 \pm 5}{2} = \begin{cases} 3 \\ -2 \end{cases}$$

Generell løsning $x_n = A \cdot 3^n + B \cdot (-2)^n$

$$\begin{aligned} x_0 &= A + B = 1 \\ x_1 &= 3A - 2B = 1 \end{aligned} \Rightarrow A = \frac{3}{5}, \quad B = \frac{2}{5}$$

$$\underline{\underline{x_n = 0.6 \cdot 3^n + 0.4 \cdot (-2)^n}}$$

Oppgave 2

$$\frac{dx}{dt} = y \qquad M = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}$$

$$\frac{dy}{dt} = -4x$$

Egenverdier $|M - \lambda I| = \begin{vmatrix} -\lambda & 1 \\ -4 & -\lambda \end{vmatrix} = \lambda^2 + 4 = 0 \Rightarrow \lambda = \pm 2i$

$(a = 0, \ \omega = 2)$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \frac{1}{\omega}(M - aI)\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$x = x_0 \cos 2t + x_1 \sin 2t = \cos 2t + 2 \sin 2t$$

$$y = y_0 \cos 2t + y_1 \sin 2t = \underline{\underline{4 \cos 2t - 2 \sin 2t}}.$$

Oppgave 3

$$(1+2i)^2 = 1 + 4i + 4i^2 = \underline{\underline{-3+4i}}.$$

$$3z^2 + (2+i)z + \frac{1}{2} = 0$$

$$z = \frac{-(2+i) \pm \sqrt{4+4i+i^2-6}}{6} = \frac{-(2+i) \pm \sqrt{-3+4i}}{6} =$$

$$\frac{1}{6}\left(-2-i \pm (1+2i)\right) = \begin{cases} \frac{1}{6}(-1+i) \\ \frac{1}{6}(-3-3i) \end{cases} = \begin{cases} \frac{1}{6}(-1+i) \\ \frac{1}{2}(-1-i) \end{cases}.$$

Oppgave 4.

$f = (x-1)y + x^2 y^3$

a) $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] = \left[y + 2xy^3, x - 1 + 3x^2y^2\right]$

$\nabla f(2,1) = \left[1 + 2\cdot 2\cdot 1^3, 2 - 1 + 3\cdot 2^2 \cdot 1^2\right] = [3, 13]$

$f'_{\vec{v}}(2,1) = \frac{\nabla f(2,1)\cdot \vec{v}}{|\vec{v}|} = \frac{[3,13]\cdot[-4,3]}{\sqrt{16+9}} = \frac{-12+39}{5} = \underline{\underline{\frac{27}{5}}}$

b) I $f_x = y + 2xy^3 = y(1 + 2xy^2) = 0 \Rightarrow y = 0$ eller $2xy^2 = -1$

II $f_y = x - 1 + 3x^2y^2 = 0$

$y = 0$: II gir $x - 1 = 0$, dvs $x = 1$

$xy^2 = -\frac{1}{2}$: II gir $x - 1 + 3x\cdot\left(-\frac{1}{2}\right) = 0$

$-1 - \frac{1}{2}x = 0 \Rightarrow x = -2$

$y^2 = \frac{1}{4}, \quad y = \pm\frac{1}{2}$

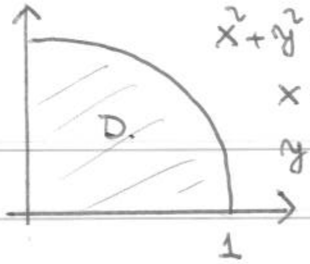
Altså $(1,0), \left(-2, \frac{1}{2}\right), \left(-2, -\frac{1}{2}\right)$

$f_{xx} = 2y^3 \qquad f_{xy} = 1 + 6xy^2 \qquad f_{yy} = 6x^2y$

$\Delta = 12x^2y^4 - (1 + 6xy^2)^2, \quad r = 2y^3$

	Δ	r	
$(1,0)$	-1		Sadelpunkt.
$(-2, \pm\frac{1}{2})$	-1		Sadelpunkt.

c)



$x^2+y^2=1$

$x=\sqrt{1-y^2}$

$y=\sqrt{1-x^2}$

$$\iint_D (x-1)y + x^2y^3 \, dx\,dy = \int_0^1 \left(\int_0^{\sqrt{1-x^2}} (x-1)y + x^2y^3 \, dy \right) dx$$

$$= \int_0^1 \frac{1}{2}(x-1)y^2 + \frac{1}{4}x^2y^4 \Big|_0^{\sqrt{1-x^2}} dx = \int_0^1 \frac{1}{2}(x-1)(1-x^2) + \frac{1}{4}x^2(1-x^2)^2 \, dx$$

$$= \int_0^1 \frac{1}{2}x - \frac{1}{2}x^3 - \frac{1}{2} + \frac{1}{2}x^2 + \frac{1}{4}x^2 - \frac{1}{2}x^4 + \frac{1}{4}x^6 \, dx$$

$$= \int_0^1 -\frac{1}{2} + \frac{1}{2}x + \frac{3}{4}x^2 - \frac{1}{2}x^3 - \frac{1}{2}x^4 + \frac{1}{4}x^6 \, dx$$

$$= -\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} + \frac{3}{4}\cdot\frac{1}{3} - \frac{1}{2}\cdot\frac{1}{4} - \frac{1}{2}\cdot\frac{1}{5} + \frac{1}{4}\cdot\frac{1}{7} = -\frac{31}{140}$$

MAT 1010 – Våren 2008

Oppgave 5.

a) Ledig kapasitet er $10000 - y$

Relativ vekst er $\frac{1}{y}\frac{dy}{dt}$

At relativ vekst er proporsjonal med ledig kapasitet betyr at

$$\frac{1}{y}\frac{dy}{dt} = a(10000-y) \quad \text{for en konstant } a$$

dvs.

$$\underline{\underline{\frac{dy}{dt} = ay(10000-y)}}$$

I følge formel er generell løsning

$$y = \frac{10000}{1+ke^{-10000at}}$$

$$y(0) = \frac{10000}{1+k} = 5000 \Rightarrow k = 1, \text{ altså } \underline{\underline{y = \frac{10000}{1+e^{-10000at}}}}$$

b) Det er gitt at for $t = 0$ er $\frac{1}{y}\frac{dy}{dt} = 0.05$ og $y = 5000$

Dette gir ved tiden $t = 0$

$$0.05 = \frac{1}{y}\cdot\frac{dy}{dt} = a(10000-y) = a(10000-5000) = 5000a$$

$$\Rightarrow a = 10^{-5} \text{ og } y(t) = \frac{10000}{1+e^{-0.1t}}, \text{ så}$$

$$y(20) = \frac{10000}{1+e^{-2}} = \underline{\underline{8808}} \text{ kaniner}$$