

Question- and answer sheet for midterm exam in MAT1030

CANDIDATE NUMBER _____

DATE:

OCTOBER 16TH, 2003.

ENCLOSURES:

NONE.

THIS IS A PENCIL AND PAPER EXAM. THERE ARE TEN QUESTIONS.

YOU ARE NOT ALLOWED TO USE BOOKS, NOTES, CALCULATORS, ETC.

Mark one answer only for each question. Multiple answers will score 0.

1) To which of the following statements is $\sim p \wedge \sim (p \vee q)$ equivalent ?

a) $\sim p \vee q$

b) $\sim (p \vee q)$

c) $\sim p$

d) $\sim p \vee \sim q$

2) Which one of the following statements is equivalent to $((p \rightarrow q) \rightarrow q) \rightarrow p$?

a) $q \rightarrow p$

b) T (tautology)

c) p

d) $\sim p \rightarrow q$

3) Which of the following rules of inference is not valid ?

a)
$$\begin{array}{l} p \rightarrow r \\ q \rightarrow r \\ \therefore p \vee q \rightarrow r \end{array}$$

b)
$$\begin{array}{l} \sim p \rightarrow \sim q \\ q \\ \therefore p \end{array}$$

c)
$$\begin{array}{l} p \rightarrow q \\ \sim q \rightarrow \sim r \\ \therefore p \rightarrow r \end{array}$$

d)
$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ p \\ \therefore r \end{array}$$

4) Which one of the following set-theoretical formulas is valid for arbitrary sets A, B, C ?

a) $(A \cap C) - (B \cap C) = (A \cap C) - B$

b) $(A \cup B) - C = A \cup (B - C)$

c) $A - (B - C) \subseteq (A - B) - C$

d) $A - B \subseteq (A \cup C) - (B \cup C)$

5) What is the binary representation of 643 ?

a) 1100000101

b) 1011000101

c) 1010000011

d) 1010010010

6) If $n \in \mathbf{Z}^+$, then let $n! = 1 \cdot 2 \cdot \dots \cdot n$. Which one of the following statements is not valid ?

a) $\frac{11!}{10!} = 11$

b) $n \mid n!$ for all integers $n > 1$

c) $10! \mid (10! + 1)$

d) $13! = 13 \cdot 12 \cdot 11 \cdot (10!)$

7) Which one of the following subsets of \mathbf{Z} has a least element ?

a) $\{n \in \mathbf{Z} : n < n^2 - 1\}$

b) $\{n \in \mathbf{Z} : (n > 1) \wedge (n \mid (n^2 + 1))\}$

c) $\{n \in \mathbf{Z} : n < n^3 - 1\}$

d) $\{n \in \mathbf{Z} : 8 \mid (n^2 - 1)\}$

8) If $n \in \mathbf{Z}^+$, then what is the value of

$$\sum_{k=1}^n (3k - 1) = 2 + 5 + 8 + \dots + (3n - 1) ?$$

a) $\frac{n(3n+1)}{2}$

b) $n^2 + 2n - 1$

c) $\frac{(n+1)(4n-1)}{3}$

d) $\frac{5n^2+1}{3}$

9) How many integers from 1 through 1000 are divisible by 5 or 7 ?

a) 342

b) 286

c) 314

d) 325

10) How many 5-tuples of nonnegative integers $(x_1, x_2, x_3, x_4, x_5)$ satisfy the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 8 ?$$

a) $\binom{12}{4}$

b) $\binom{8}{5}$

c) $\binom{12}{5}$

d) $\binom{13}{8}$