MAT4300-Fall '08-Exercises

Week 36 - September 3

From Chapter 2 of Bartle

Problems M, N, O, P, Q and R.

Exercise 1

Let X be an uncountable set and let $\mathbf{X}$ be the σ-algebra on X consisting of the countable subsets of X and of those with countable complements.

a) Let $f : X \to \mathbf{R}$ be $\mathbf{X}$-measurable.
   Show that f is constant except on a countable subset of X.

b) Show that the least σ-algebra on $\mathbf{R}$ generated from the class of countable subsets is a proper subalgebra of the Borel algebra.

Exercise 2

We have seen that the intersection of an arbitrary nonempty family of σ-algebras on a set X is a σ-algebra on X (Where?).
Show by an example that the union of two σ-algebras on the same set X does not need to be a σ-algebra on X.