MAT4300-Fall '08-Exercises

Week 37 - September 10

From Chapter 3 of the textbook:
Problems A, B, F, G, I, J, K and L.

Exercise 1

Let μ be a measure on the algebra $\mathbf{B_{[0,1]}}$ of Borel subsets of $[0, 1]$ such that $\mu(I) = \text{length}(I)$ for all intervals I. (We will prove the existence of μ later.) For $x \in [0, 1]$, let $f(x) = 0$ in case there is a decimal expansion of x consisting only of 0's and 9's. In the other cases, we let $f(x)$ be the least number n such that decimal number n of x differs from both 0 and 9.
Show that f is continuous μ-a.e., but that f has an uncountable set of discontinuity points.

Exercise 2

A $0-1$-*measure* is a measure μ on a σ-algebra $\mathbf{X}$ on a set X such that $\mu(X) = 1$ and such that $\mu(A) \in \{0, 1\}$ for all $E \in \mathbf{X}$.
Show that if μ is a $0-1$-measure on the Borel algebra, then there is a real such that $\mu(\{a\}) = 1$, i.e. that the measure is concentrated in a.