MAT4300 - Fall 09 - Old exams, set 3.

Notation : we let $\mathcal{B} = \mathcal{B}(\mathbb{R})$ denote the σ-algebra of Borel subsets of $\mathbb{R}$ and λ the Lebesgue measure on $\mathcal{B}$. If $E \in \mathcal{B}$, then $\mathcal{B}_E$ denotes all Borel subsets of E and λ_E denotes the restriction of λ to $\mathcal{B}_E$.

Exercise 1, MAT254, Spring 1998

Let $f \in \mathcal{L}^1(\lambda)$.

a) Show that

$$\lim_{n\to\infty} \int f(x)\, e^{-n\,x^2}\, d\lambda(x) = 0\,.$$

b) Show that

$$\lim_{n\to\infty} \int f(x)\, e^{-\frac{1}{n}\,x^2}\, d\lambda(x) = \int f\, d\lambda\,.$$

Exercise 1, MAT254, Spring 2000

Set $\mu = \lambda_{[0,\infty)}$. Compute

$$\lim_{n\to\infty} \int e^{-(\frac{x^2\,y^2}{n}+x+y)}\, d(\mu\times\mu)(x,y)\,.$$

Exercise 3, MAT254, Spring 2004

Let $(X, \mathcal{A}, \mu)$ be a probability space. Consider the following inequality :

$$(*) \qquad e^{\int f\, d\mu} \leq \int e^{f(x)}\, d\mu(x)\,.$$

a) Show that $(*)$ holds for all $f \in \mathcal{E}(\mathcal{A})$.

b) Show that $(*)$ holds for all f in $\mathcal{M}(A)$ which are bounded.