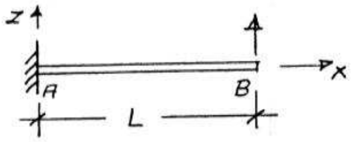
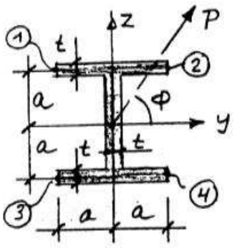
ME 150 OPPGAVESETT B

OPPGAVE 1





Utkragerbjelken med I-tverrsnitt er påkjent av en skrålast P ved B.

For $\varphi = 45°$,
beregn normalspenningene (σ_x) i tverrsnitts-punktene 1, 2, 3 og 4 for innspenningstverrsnittet og skissér (i diagramform) spenningsfordelingen over tverrsnittet.

NB! Det kan antas at $a \gg t$ (→ gir f.eks. $I_z = 4a^3t/3$.

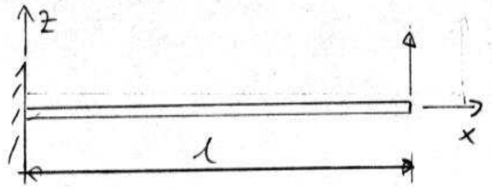
OPPGAVE 2

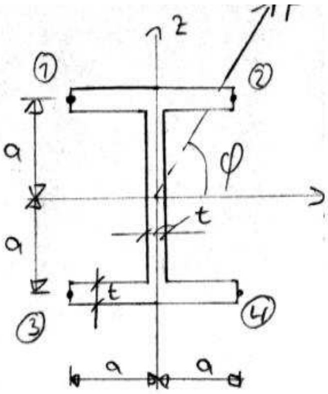
Beregn skjærspenningsfordelingen over I-tverrsnittet til bjelken i Oppg. 1 for $\varphi = 90°$. Angi for hvilken x-koord. du beregner spenn.

OPPGAVER FRA LÆREBOKA:

6.21 , 6.22 , 6.23

Opps: 4





$\varphi = 45^\circ$

$$I_z = \frac{1}{6} t (2a)^3 + \frac{1}{12} (2a - t) t^3$$

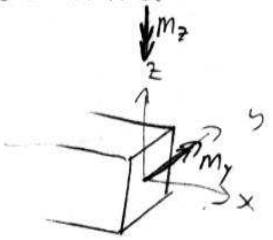
$$\simeq \frac{4}{3} t a^3 \qquad \text{for } a \gg t$$

$$I_y = 2\left(\frac{1}{12} \cdot 2a \cdot t^3 + 2a \cdot t \left(a - \frac{t}{2}\right)^2\right) + \frac{1}{12} t (2a - t)^3$$

$$\simeq \frac{1}{3} a t^3 + 4 t a^3 + \frac{2}{3} t a^3$$

$$\approx \frac{14}{3} t a^3 \qquad \Big\} \; a \gg t$$

Bøyemomenter i innspenningstverrsnitt:



$$M_y = -P \sin\varphi \cdot l = -\frac{\sqrt{2}}{2} P l$$

$$M_z = -P \cos\varphi \cdot l = -\frac{\sqrt{2}}{2} P l$$

Dobbeltsymmetrisk tverrsnitt

$\Rightarrow$ y- og z-akser er hovedakser

$$\Rightarrow \sigma_x = \frac{M_z \cdot y}{I_z} + \frac{M_y z}{I_y}$$

$$① : \sigma_x = \frac{-\frac{\sqrt{2}}{2} P l (-a)}{\frac{4}{3} t a^3} - \frac{\frac{\sqrt{2}}{2} P l a}{\frac{14}{3} t a^3}$$

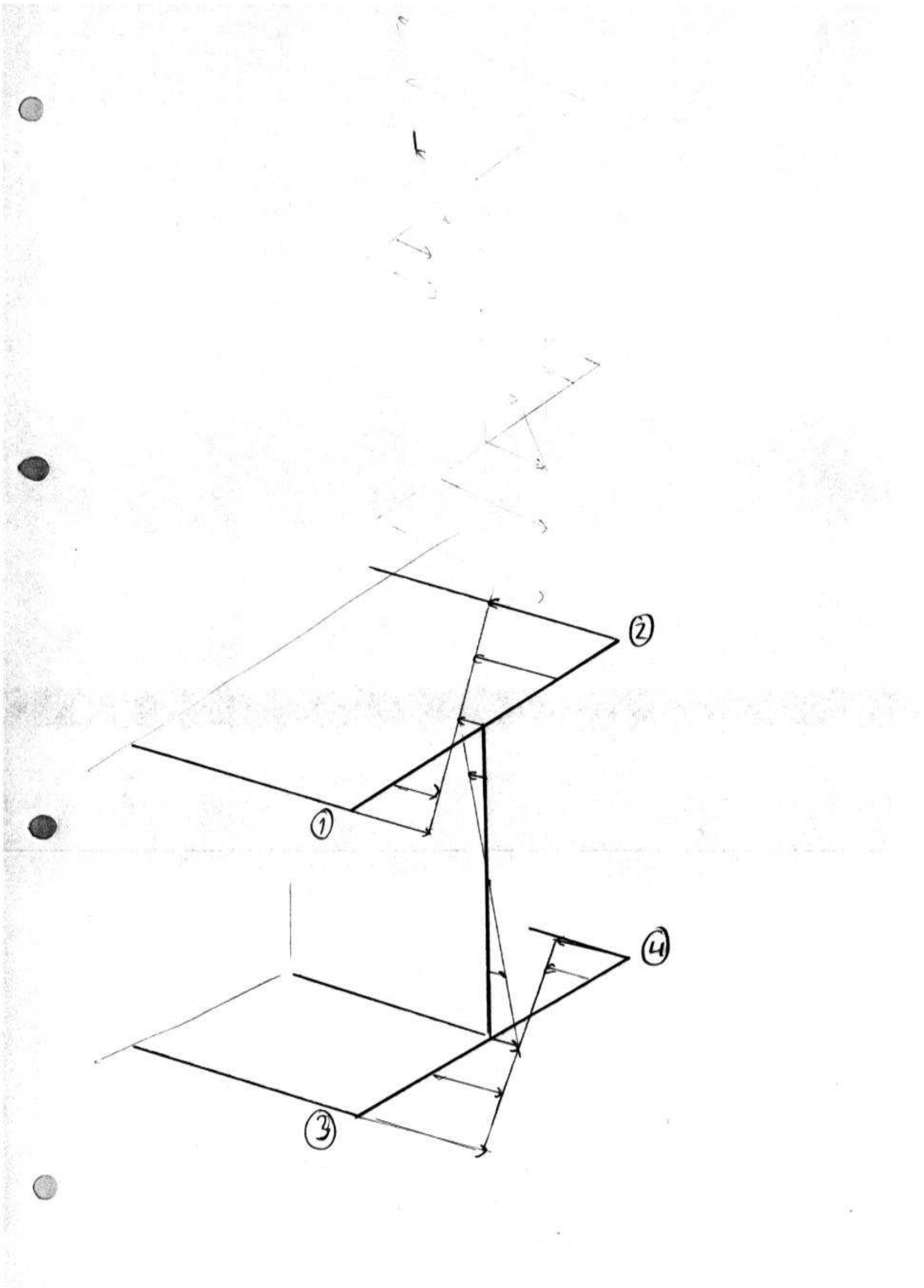
$$= \frac{3\sqrt{2}\, P l}{8 t a^2} - \frac{3\sqrt{2}\, P l}{28 t a^2} = \underline{\frac{15\sqrt{2}\, P l}{56 t a^2}}$$

$$② : \sigma_x = \frac{-\frac{\sqrt{2}}{2} P l \cdot a}{\frac{4}{3} t a^3} + \frac{-\frac{\sqrt{2}}{2} P l a}{\frac{14}{3} t a^2}$$

$$= -\frac{3\sqrt{2}\, P l}{8 t a^2} - \frac{3\sqrt{2}\, P l}{28 t a^2} = \underline{\frac{-27\sqrt{2}\, P l}{56 t a^2}}$$

$$③ : \sigma_x = \frac{-\frac{\sqrt{2}}{2} P l (-a)}{\frac{4}{3} t a^3} + \frac{-\frac{\sqrt{2}}{2} P l (-a)}{\frac{14}{3} t a^3} = \frac{27\sqrt{2}\, P l}{56 t a^2}$$

$$④ : \sigma_x = \frac{-\frac{\sqrt{2}}{2} P l a}{\frac{4}{3} t a^3} + \frac{-\frac{\sqrt{2}}{2} P l (-a)}{\frac{14}{3} t a^3} = -\frac{15\sqrt{2}\, P l}{56 t a^2}$$

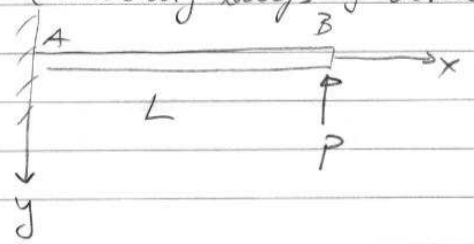


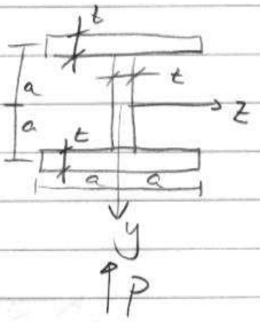
①
②
③
④

Oppgavesett 8

Oppg 2.

Skjærspenningsfordeling over I tverrsnitt. ($\varphi = 90°$)
(Fordeling langs y-aksen)

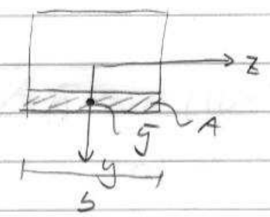




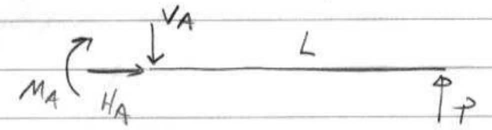
$$I_z = \int_A y^2 dA = \frac{14}{3} t a^3$$

Skjærspenningsfordeling

$$\tau_{xy} = \frac{Q A \bar{y}}{b I}$$

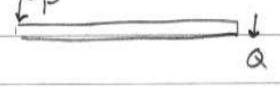


Reaksjonskrefter:



$V_A = P$, $H_A = 0$ $M_A = PL$

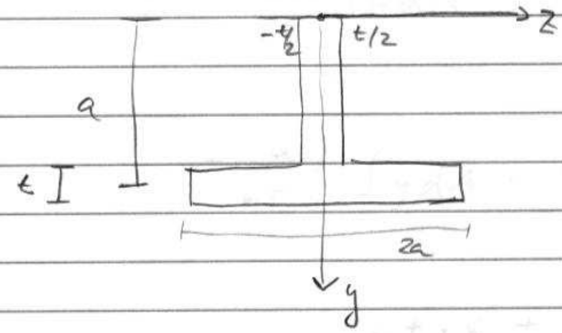
Skjær



$\sum F_y = 0 \Rightarrow \quad Q + P = 0$

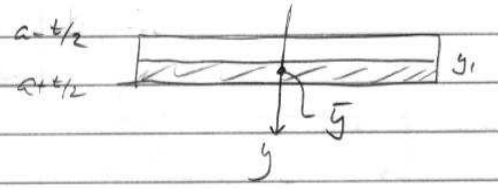
$\underline{Q = -P}$

Ser på nedre del av I tverrsnitt (symmetri)



1) $y_1 \in [a - t/2,\ a + t/2]$

$$\tau_{xy} = \frac{Q}{I}\frac{A\bar{y}}{b}$$



$$b = 2a$$

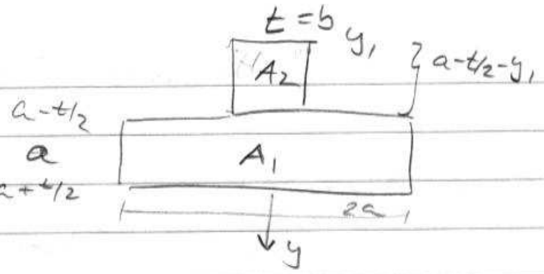
$$A = 2a\left(a + t/2 - y_1\right)$$

$$\bar{y} = \frac{1}{2}\left(a + \frac{t}{2} + y_1\right)$$

$$A\bar{y} = a\left(\left(a + t/2\right)^2 - y_1^2\right)$$

$$\tau_{xy} = \frac{Q}{I}\frac{A\bar{y}}{b} = \frac{Q}{I}\,\frac{\left(a + t/2\right)^2 - y_1^2}{2}$$

2) $y_1 \in [0, a - t/2)$



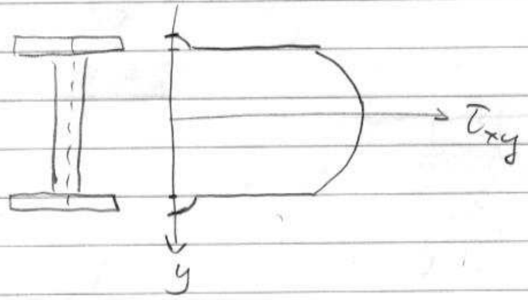
$$\tau_{xy} = \frac{Q}{bI}\int_A y\,dA$$

$$= \frac{Q}{bI}\left\{\int_{A_1} y\,dA + \int_{A_2} y\,dA\right\}$$

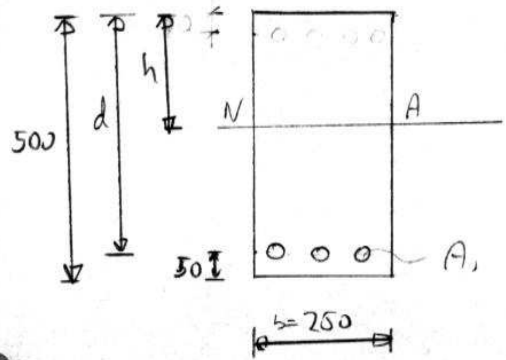
$$= \frac{Q}{tI}\left\{2at \cdot a + t\cdot\left(a - \tfrac{t}{2} - y_1\right)\cdot\tfrac{1}{2}\left(a - \tfrac{t}{2} + y_1\right)\right\}$$

$$= \frac{Q}{I}\left\{2a^2 + \tfrac{1}{2}\left[(a - t/2)^2 - y_1^2\right]\right\}$$

Skisse av τ_{xy}



6.21



$$\frac{E_s}{E_b} = m = 15 \qquad A_s = 1100\ mm^2$$

$$\sigma_{b\,till} = 4{,}2\ MN/m^2$$

Finner nøytralaksens plassering: (ligning 6.21):

$$h = \left[\left(\frac{mA_s}{b} \right)^2 + \frac{2mA_s d}{b} \right]^{1/2} - \frac{mA_s}{b}$$

$$= \left[\left(\frac{15 \cdot 1100}{250} \right)^2 + \frac{2 \cdot 15 \cdot 1100 \cdot 450}{250} \right]^{1/2} - \frac{15 \cdot 1100}{250}$$

$$= \underline{186{,}5\ mm}$$

Ligning 6.22:

$$\sigma_b = \frac{M \cdot y_b}{\frac{1}{3} b h^3 + m A_s (d-h)^2}$$

$$M_{max} = \frac{\sigma_{btill} \left(\frac{1}{3} b h^3 + m A_s (d-h)^2 \right)}{h}$$

$$= \frac{4{,}2 \left(\frac{1}{3} \cdot 250 \cdot 186{,}5^3 + 15 \cdot 1100 (450 - 186{,}5)^2 \right)}{186.5}$$

$$= \underline{38 \text{ kNm}}$$

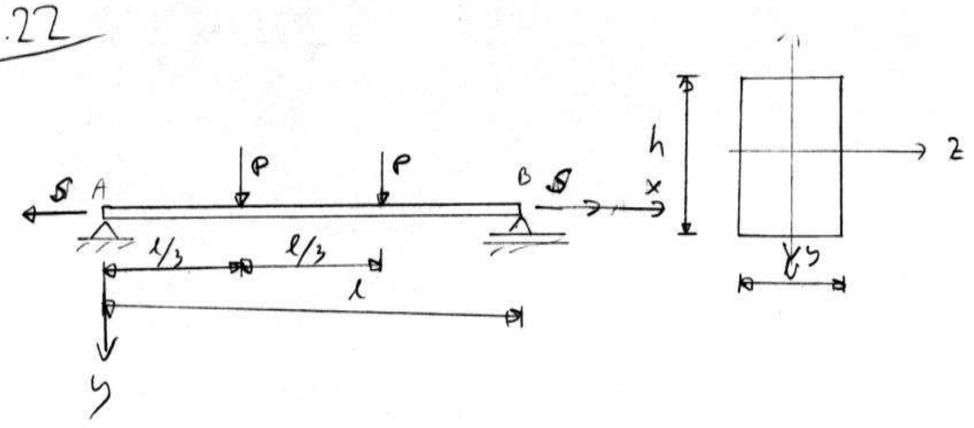
Spenning i armering: (likn. 6.22)

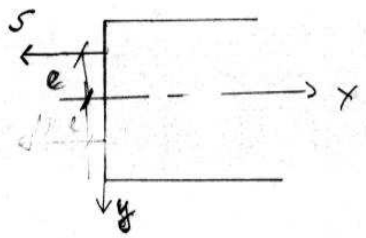
$$\sigma_s = \frac{M_{max} (d-h) m}{\frac{1}{3} b h^3 + m A_s (d-h)^2}$$

$$= \frac{38 \cdot 10^6 (450 - 186{,}5) \cdot 15}{\frac{1}{3} \cdot 250 \cdot 186.5^3 + 15 \cdot 1100 (450 - 186{,}5)^2}$$

$$= \underline{89{,}1 \text{ MN/m}^2}$$

6.22

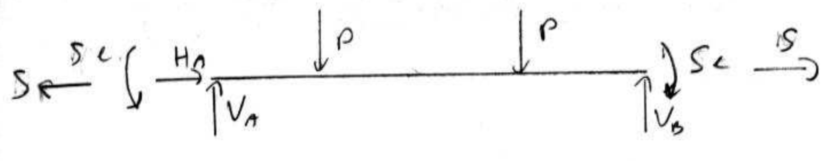




$S = 40\ kN$ $\quad l = 1500\,mm$ $\quad b = 50\,mm$

$P = 5\,kN$ $\quad h = 100\,mm$ $\quad e = 25\,mm$

Opplagerkrefter



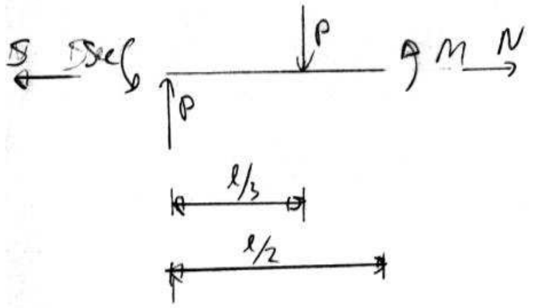
$$\Sigma F_x = 0 \Rightarrow H_A + S - S = 0 \Rightarrow H_A = 0$$

$$\Sigma M_A = 0 \Rightarrow V_B \cdot l - P \cdot l/3 - P \cdot \frac{2l}{3} + S \cdot e - S \cdot e = 0$$

$\underline{V_B = P}$

$$\Sigma F_y = 0 \Rightarrow V_A - P - P + V_B = 0 \Rightarrow \underline{V_A = P}$$

Moment og normalkraft



$$\Sigma M_{l/2} = 0 \Rightarrow M + P\cdot\frac{l}{6} - P\cdot\frac{l}{2} + S\cdot e = 0$$

$$\Rightarrow \underline{M = \frac{1}{3}Pl - S\cdot e}$$

$$\Sigma F_x = 0 \Rightarrow S - N = 0 \Rightarrow N = S$$

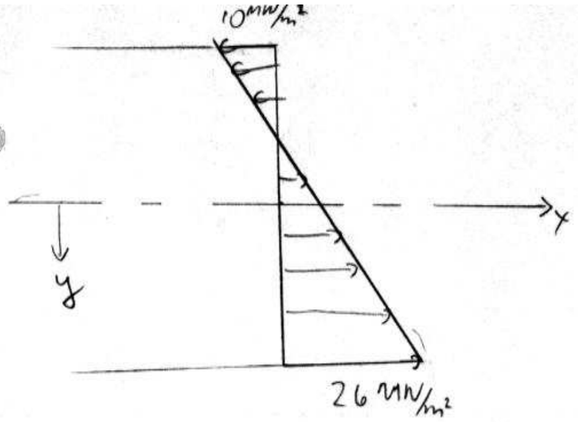
Spenning: $\sigma = \frac{N}{A} + \frac{M\cdot y}{I}$

$$A = bh \quad , \quad I = \frac{1}{12}bh^3$$

$$\sigma = \frac{S}{bh} + \frac{\frac{1}{3}Pl - S\cdot e}{\frac{1}{12}bh^3}\cdot y$$

$$= \frac{S}{bh} + \frac{4Pl - 12Se}{bh^3}\,y$$

$$= \underline{8 + 0{,}36\,y} \quad [N/mm^2]$$



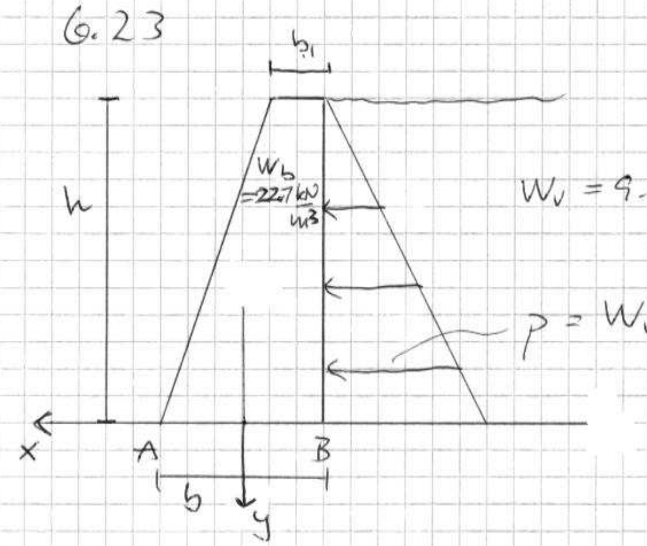
Ingen trykkspenning ved øvre rand:

$$\frac{S}{bh} + \frac{4Pl - 12Se}{bh^3} \cdot \left(-\frac{h}{2}\right) = 0$$

$$-2Pl + 6Se + Sh = 0$$

$$e = \frac{2Pl - Sh}{6S} = \underline{45{,}8\,mm}$$

6.23

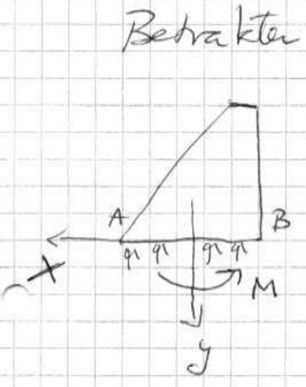


$W_v = 9.81\ kN/m^3$ ↓g

$p = W_v \cdot 1 \cdot (h+y)$

$b_1 = 10m$, $h = 60m$

Betrakter enhetstykkelse av dammen:



$$\Sigma M_{AB} = 0 \Rightarrow M + p(0)\frac{h}{2}\cdot\frac{h}{3} = 0$$

$$\Rightarrow M = -\frac{1}{6} W_v h^3$$

Spenning pga demningens tyngde:

$$\frac{-W_b V}{A}$$

Normalsp. ved B:

$$\sigma_B = \frac{-W_b V}{A_{AB}} + \frac{M_{AB}}{I_{AB}}\left(-\frac{b}{2}\right)$$

$$V = \frac{(b+b_1)h}{2}$$

$$I_{AB} = \frac{1}{12}\cdot 1 \cdot b^3 = \frac{1}{12} b^3$$

Får da: Ønsker $\sigma_B = 0$

$$\frac{-W_b \frac{(b+b_1)h}{2}}{b \cdot 1} + \frac{-\frac{1}{6} W_v h^3}{\frac{1}{12} b^3}\left(-\frac{b}{2}\right) \overset{!}{=} 0$$

$\Rightarrow$ 2. gradsligning for b

$$\Rightarrow \quad b^2 + b_1 b - 2\frac{W_v}{W_b} h^2 = 0$$

$$\Rightarrow \quad b = 5\,1\,\text{m} \qquad (b = -6\,1\,\text{m})$$

Spenning ved A:

$$\sigma_A = \frac{-W_b V}{A_{AB}} + \frac{M_{AB}}{I_{AB}}\left(\frac{b}{2}\right)$$

$$= -1{,}63\ \text{MN/m}^2 \qquad (\text{Kompresjon})$$