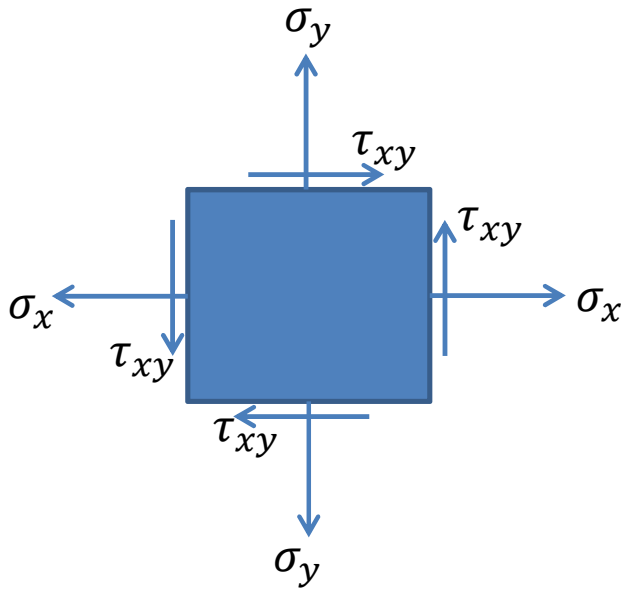
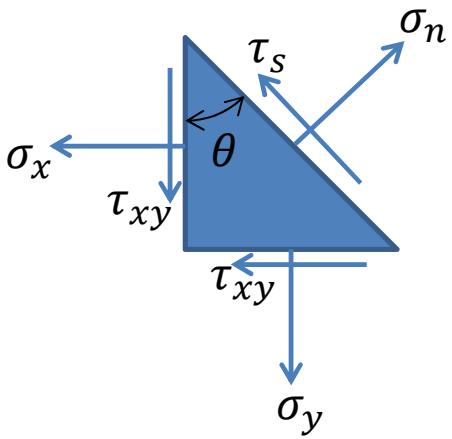


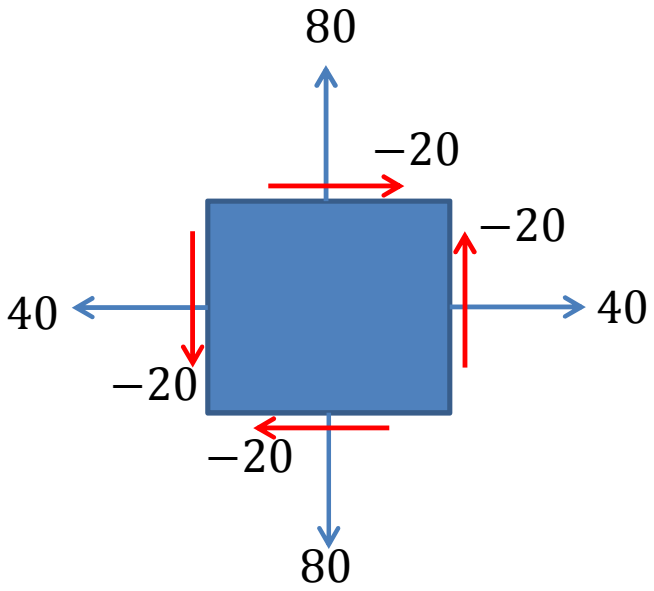


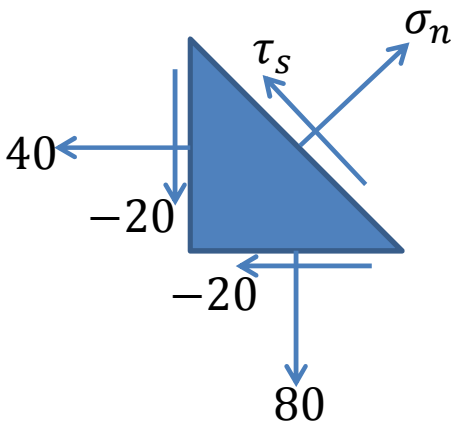
$$\sigma_n = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)cos2\theta + \tau_{xy}sin2\theta$$

$$\tau_s = -\frac{1}{2}(\sigma_x - \sigma_y)sin2\theta + \tau_{xy}cos2\theta$$

Eksempel 11.2 (NB! Feil i figur)

$$\sigma = \begin{bmatrix} 40 & -20 \\ -20 & 80 \end{bmatrix} \quad \theta = 45°$$

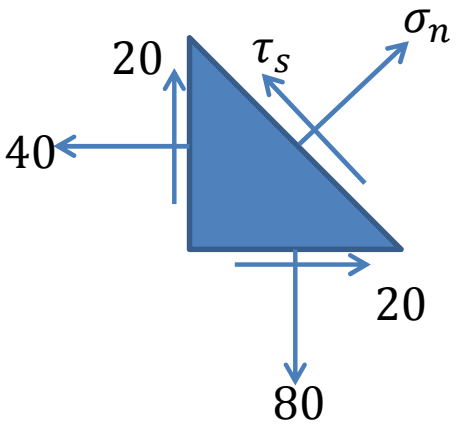


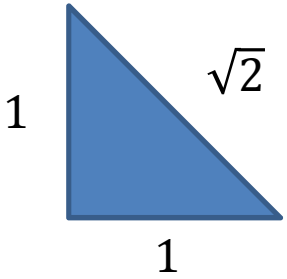


$$\sigma_n = \frac{1}{2}(40 + 80) + \frac{1}{2}(40 - 80)cos90° - 20sin90° = 60 - 20 = 40$$

$$\tau_s = -\frac{1}{2}(40 - 80)sin90° - 20cos90° = 20$$

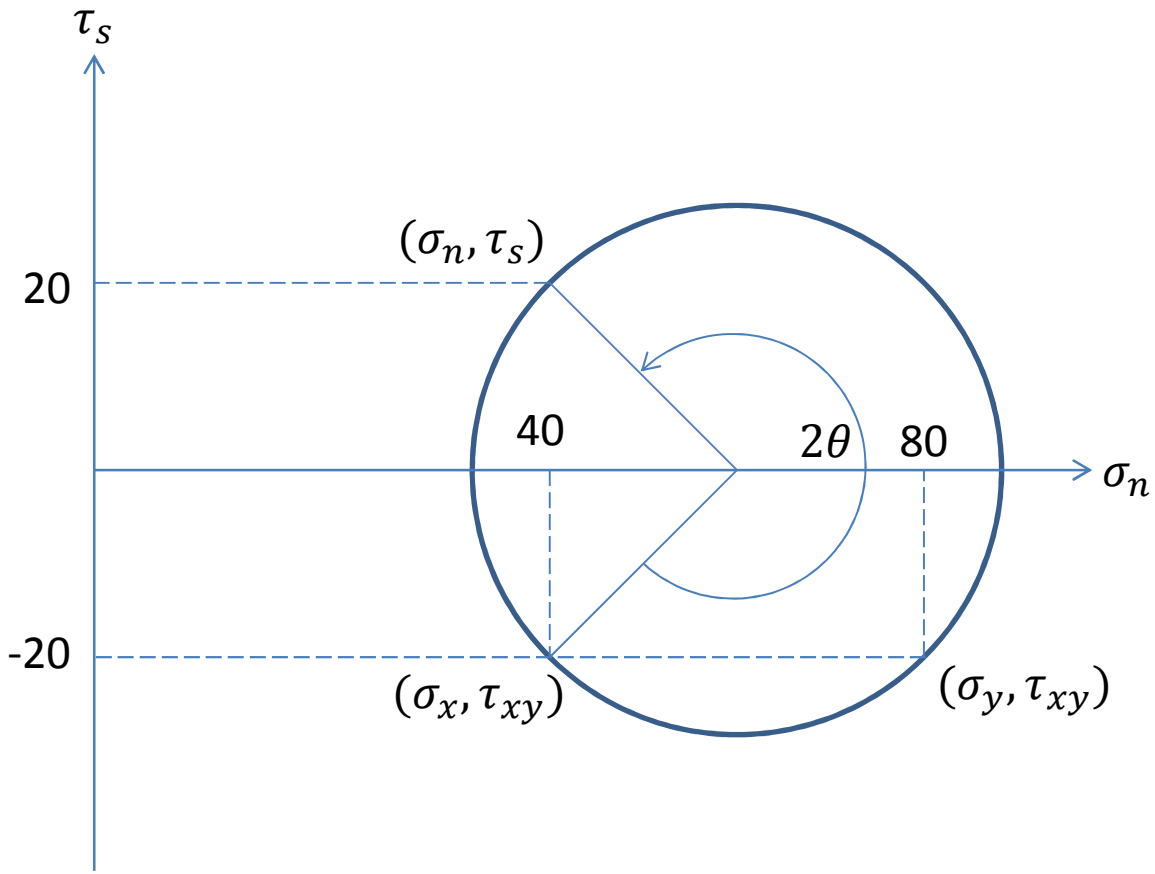
Likevektsbetraktning

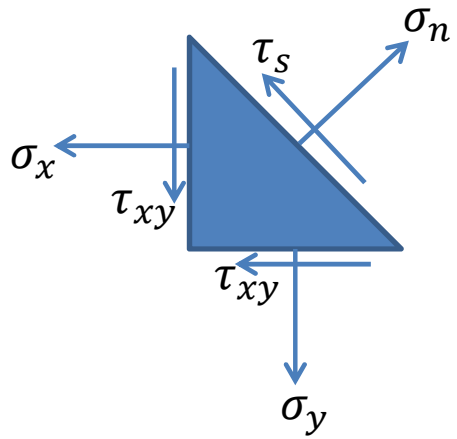


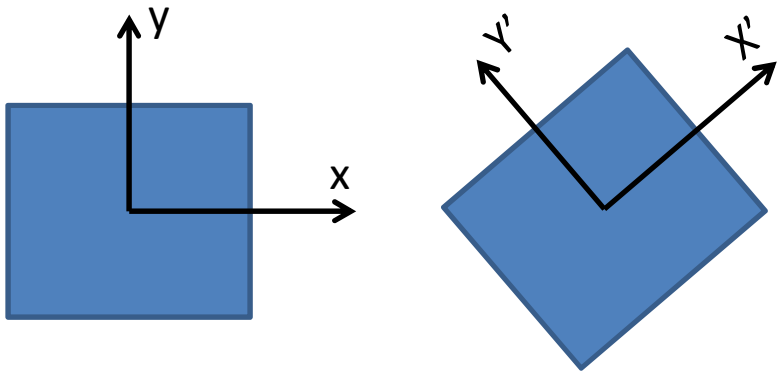


$$\sum F_n = 0$$
$$\downarrow$$
$$\sigma_n\sqrt{2} + \frac{20}{\sqrt{2}} + \frac{20}{\sqrt{2}} - \frac{40}{\sqrt{2}} - \frac{80}{\sqrt{2}} = 0$$
$$\downarrow$$
$$\sigma_n = \frac{40 + 80 - 20 - 20}{2} = 40$$

$$\sum F_s = 0$$
$$\downarrow$$
$$\tau_s\sqrt{2} + \frac{20}{\sqrt{2}} - \frac{20}{\sqrt{2}} + \frac{40}{\sqrt{2}} - \frac{80}{\sqrt{2}} = 0$$
$$\downarrow$$
$$\tau_s = \frac{80 - 40 + 20 - 20}{2} = 20$$


τ_s
20
-20
(σ_n, τ_s)
40
2θ
80
σ_n
(σ_x, τ_{xy})
(σ_y, τ_{xy})

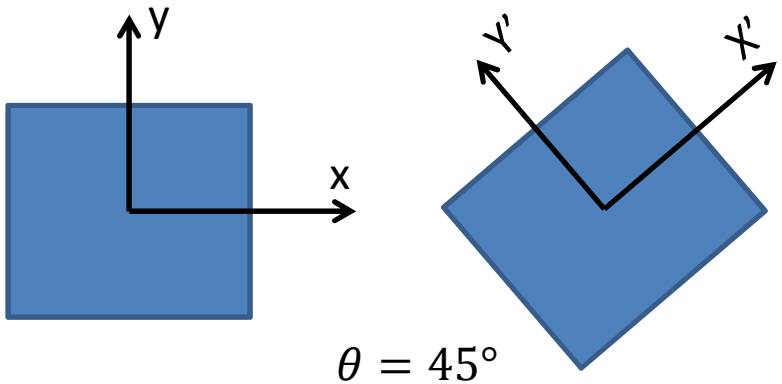

σ_n
τ_s
σ_x
τ_{xy}
τ_{xy}
σ_y



$$\boldsymbol{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\boldsymbol{\sigma'} = \boldsymbol{R}^T \boldsymbol{\sigma} \boldsymbol{R}$$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} sin\theta cos\theta$$
$$\sigma_{y'} = \sigma_x sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} sin\theta cos\theta$$
$$\tau_{x'y'} = (\sigma_y - \sigma_x) sin\theta cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$



$\theta = 45°$

$$\boldsymbol{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\boldsymbol{\sigma'} = \boldsymbol{R}^T \boldsymbol{\sigma} \boldsymbol{R}$$

$$\sigma_{x'} = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} + \tau_{xy} = 20 + 40 - 20 = 40 = \sigma_n$$

$$\sigma_{y'} = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - \tau_{xy} = 20 + 40 + 20 = 80$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2} = 20 = \tau_s$$