



UiO : **Matematisk institutt**

Det matematisk-naturvitenskapelige fakultet

MEK2500

Faststoffmekanikk 7. forelesning



Generalisert Hookes lov

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$

Uttrykket $\varepsilon = C^{-1}\sigma$
 kan inverteres for å
 finne spenninger
 uttrykt ved tøyninger

Generalisert Hookes lov

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

Plan spenning og plan tøyning

- Ofte kan den generelle 3-dim. spennings-/tøyningstilstanden forenkles (tilnærmes) til en 2-dim. Tilstand
- To tilfeller er av spesiell interesse
 - Plan spenning
 - Plan tøyning

Plan spenning

- (Normal)spenningen i én retning er neglisjerbar, for eksempel i z-retn.

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} & 0 & 0 & 0 \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$

Kan stryke
 "spenningsløse"
 ledd, og som før kan
 uttrykket $\epsilon = C^{-1}\sigma$
 inverteres for å finne
 spenninger uttrykt
 ved tøyninger

Plan spenning

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$\begin{aligned} \sigma_z &= 0 \\ \epsilon_z &= -\frac{(\sigma_x + \sigma_y)\nu}{E} \end{aligned}$$

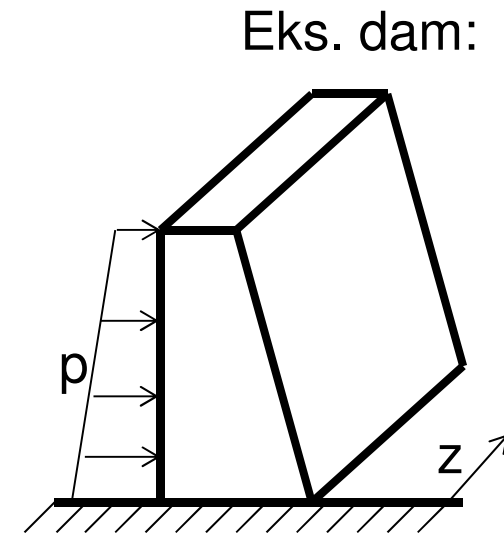
Tilnærmelsen er god for tynne skiver.

Plan tøyning

- (Normal)tøyningen i én retning er neglisjerbar, for eksempel i z-retn.

$$\epsilon_z = 0$$
$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

- Tilnærmelsen kan brukes når utstrekningen i z-retn. er mye større enn i x og y.



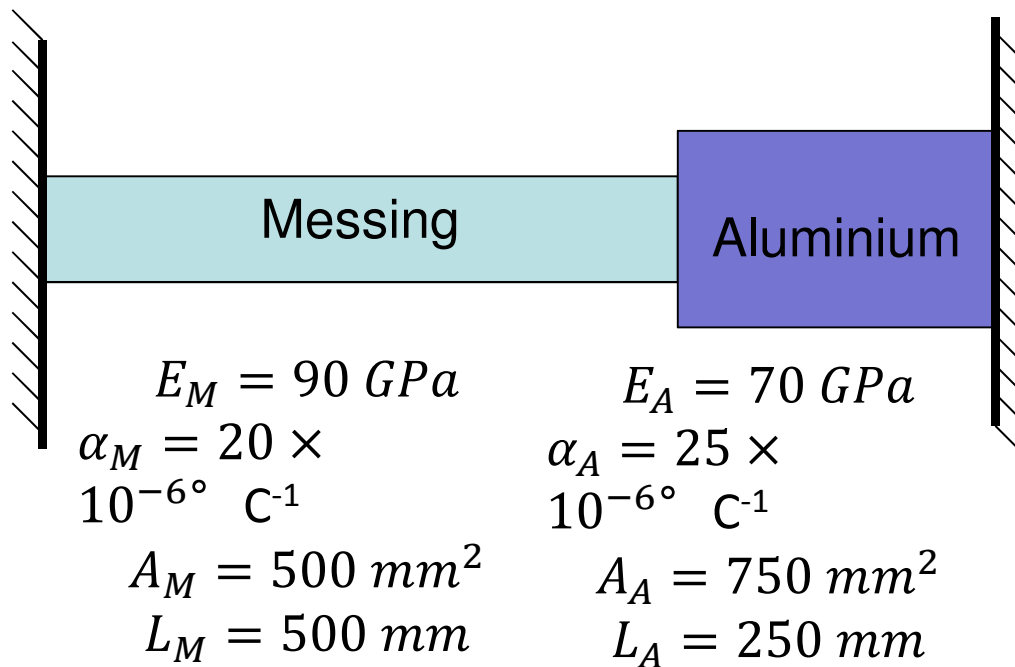
Termiske spenninger/tøyninger

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu(\sigma_y + \sigma_z)) + \alpha(T - T_0)$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu(\sigma_x + \sigma_z)) + \alpha(T - T_0)$$

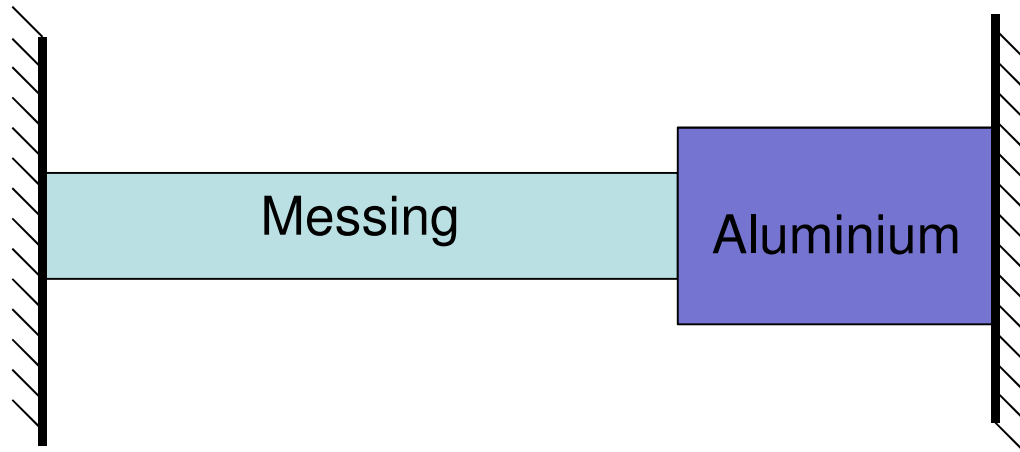
$$\epsilon_z = \frac{1}{E} (\sigma_z - \nu(\sigma_x + \sigma_y)) + \alpha(T - T_0)$$

Termiske spenninger/tøyninger



Systemet er spenningsfritt, men så faller temperaturen med $20 \text{ } ^\circ\text{C}$. Bestem spenningene i de to delene.

Termiske spenninger/tøyninger



Kraftlikevekt:

$$F_M = F_A$$

$$\sigma_M A_M = \sigma_A A_A$$

$$\sigma_A = \frac{2}{3} \sigma_M$$

Randbetingelser:

$$\epsilon_M L_M + \epsilon_A L_A = 0$$

$$\epsilon_A = -2\epsilon_M$$

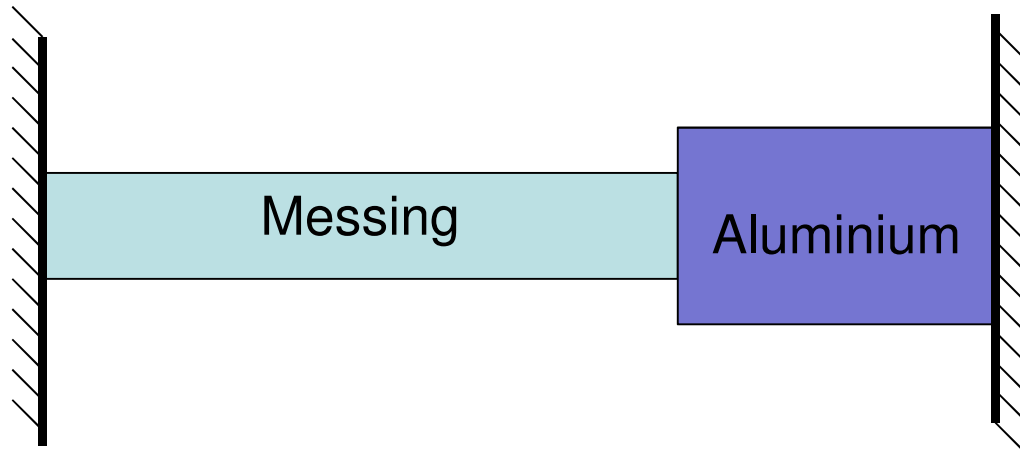
$$\sigma_A = \frac{2}{3} \sigma_M$$

$$(\epsilon_A - \alpha_A \Delta T) E_A = \frac{2}{3} (\epsilon_M - \alpha_M \Delta T) E_M$$

$$(-2\epsilon_M - \alpha_A \Delta T) E_A = \frac{2}{3} (\epsilon_M - \alpha_M \Delta T) E_M$$

$$\epsilon_M = \frac{\Delta T (-3\alpha_A E_A + 2\alpha_M E_M)}{2(3E_A + E_M)}$$

Termiske spenninger/tøyninger



$$\begin{aligned}\epsilon_M &= \frac{\Delta T(-3\alpha_A E_A + 2\alpha_M E_M)}{2(3E_A + E_M)} \\ &= \frac{-20(-3 \times 25 \times 70 + 2 \times 20 \times 90)}{2(3 \times 70 + 90)} \times 10^{-6} = \frac{11}{200000}\end{aligned}$$

$$\sigma_M = (\epsilon_M - \alpha_M \Delta T) E_M = \left(\frac{11}{200000} + 20 \times 10^{-6} \times 20 \right) 90 \times 10^9 = 40.95 \times 10^6 \text{ Pa}$$

$$\sigma_M = 41 \text{ MPa} \rightarrow \sigma_A = 27.3 \text{ MPa}$$

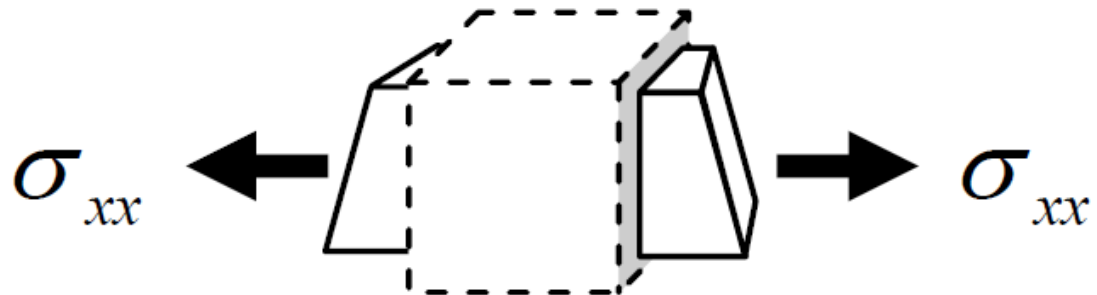
Anisotrope materialer

- En del materialer kan betraktes som lineært elastiske, men har ulik stivhet i ulike retninger
- Eksempler på slike materialer er
 - Tre
 - Kompositter
- For slike materialer er det ikke tilstrekkelig med to elastiske konstanter

Anisotrope materialer

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{45} \\ & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

NB: kobling mellom normalspenning og skjærtøyning og mellom skjærspenning og normaltøyning



Ortotrope materialer

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix}$$

3 symmetriplan
i materialet

Skjærtøyninger $2\varepsilon = \gamma$

Transvers isotrope materialer

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{E_t} & -\frac{\nu_t}{E_t} & -\frac{\nu_f}{E_f} & 0 & 0 & 0 \\ -\frac{\nu_t}{E_t} & \frac{1}{E_t} & -\frac{\nu_f}{E_f} & 0 & 0 & 0 \\ -\frac{\nu_f}{E_f} & -\frac{\nu_f}{E_f} & \frac{1}{E_f} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_f} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_f} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_t} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix}$$

$$\frac{1}{2G_t} = \frac{1 + \nu_t}{E_t}$$

Symmetriakse /
 et isotropt plan

