UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in:	MEK2500 — Solid mechanics
Day of examination:	Trial Exam 2015
Examination hours:	00.00 – 04.00
This examination set consists of 3 pages.	
Appendices:	None
Permitted aids:	Rottmann's and Calculator of approved type

Make sure that your copy of the examination set is complete before you start solving the problems.

Problem 1.

This problem consists of 5 independent questions. Each question has a max score of 2 points.

a) (2 points) Express Cauchy's infinitesimal strain tensor ε for a deformation f of a domain Ω with coordinates x.

b) (2 points) What is the SI unit for strain?

c) (2 points) What is the definition of stress and what is its SI unit?

d) (2 points) Define the tensile strength of an elastic material.

e) (2 points) What are the natural bounds for the Poisson ratio ν? If a body (say a sphere) has Poisson ratio larger than the natural upper bound, how will the body deform when a pure pressure is applied to its boundaries?

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Problem 2.

Assume that an elastic body B occupies a domain $\Omega \subset \mathbb{R}^3$ with coordinates $x = (x_1, x_2, x_3)$. Assume that the body is deformed with a deformation f given by:

$$f(x) = ((\kappa_1 + 1)x_1, (\kappa_2 + 1)x_2, x_3) \tag{1}$$

for real, positive constants κ_1, κ_2. Let the Frobenius norm of an $n \times n$ matrix A be denoted $\|A\|$ with

$$\|A\|^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2 \tag{2}$$

a) (5 points) Compute the inverse deformation $g = f^{-1}$ that maps a coordinate $y \in f(\Omega)$ to $x \in \Omega$.

b) (5 points) Compute the displacement $u = u(x)$ of the body B and Cauchy's infinitesimal strain tensor ε associated with u.

c) (5 points) Compute the principal strains and principal directions of strain associated with ε. Give a condition on κ_1, κ_2 such that the largest principal strain is less then 1%.

d) (5 points) Compute the Cauchy stress tensor of the body, assuming that the body is isotropic, homogeneous and linearly elastic with Lame parameters $\mu = 2.0$ MPa, $\lambda = 100.0$ MPa.

Problem 3.

Assume a linear regime with small strains and no distinction between Eulerian and Lagrangian coordinates.

Consider a two-dimensional rectangular body of length a (m) and height b (m) with coordinates $(x_1, x_2) \in [0, a] \times [0, b]$.

- Assume that the body is isotropic and homogeneous with Lamé parameters μ and λ and density ρ.
- We shall consider the case where the body is clamped at the ends where $x_0 = 0$ or $x_0 = a$ (and $x_1 \in [0, b]$), hence $u = (0, 0)$ there.

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- Assume that a constant body force f acts downwards in the body, that is: $f = (0, -g)$ for some constant g (N/m^d), $d = 2$.

Consider this setting for the questions below.

a) (5 points) State the dynamic elasticity equations describing the displacement $u = u(x,t) = (u_1(x,t), u_2(x,t))$ of a homogeneous, isotropic, linearly elastic body with coordinates $x \in \mathbb{R}^2$, $t \in (0,T)$, Lamé parameters μ and λ and density ρ and a given body force f.

b) (5 points) Assume that the body is in elastic equilibrium and that there are no normal stresses. There may be tangential stresses on all boundaries. Compute the resulting stress tensor σ.

c) (5 points) From the result in b), compute the corresponding strain tensor in terms of the applied body force f and the material parameters μ and λ.

d) (5 points) From the result in c), compute the corresponding displacement in terms of the applied body force f and the material parameters μ and λ.

Hint: Recall the inverse (isotropic, homogeneous) Hooke's law

$$\varepsilon = \frac{1}{2\mu}\left(\sigma - \frac{\lambda}{3\lambda + 2\mu}\operatorname{tr}\sigma I\right) \tag{3}$$

END