6.1 Assume that $E_s = 210$ GPa, $\nu_s = 0.3$

$E_a = 70$ GPa, $\nu_a = 0.33$ (can vary dependent on alloy)

Thickness = 5mm for each layer

stiffnesses: $Q_{11} = Q_{22} = \frac{E}{1-\nu^2}$, $Q_{12} = \frac{\nu E}{1-\nu^2}$

$Q_{66} = G = \frac{E}{2(1+\nu)}$, $Q_{16} = Q_{26} = 0$ } isotropic material

These give

| | Steel | Aluminium | |
|---|---|---|---|
| $Q_{11} = Q_{22}$ | 230.8 | 78.6 | (GPa) |
| $Q_{12}$ | 69.2 | 25.9 | (GPa) |
| $Q_{66}$ | 80.8 | 26.3 | (GPa) |

ABD-matrix:

(in N, mm units)

$$\left[\begin{array}{ccc|ccc} 1547 & 476 & 0 & -1903 & -541 & 0 \\ 476 & 1547 & 0 & -541 & -1903 & 0 \\ 0 & 0 & 535 & 0 & 0 & -681 \\ \hline -1903 & -541 & 0 & 6444 & 1982 & 0 \\ -541 & -1903 & 0 & 1982 & 6444 & 0 \\ 0 & 0 & -681 & 0 & 0 & 2231 \end{array}\right] \times 10^3$$

This comes from equations (6.16) and (6.17) in AB&C. Remember that $\bar{Q}_{ij} = Q_{ij}$ for an isotropic material and take care with +/- signs in $\int_{h_{k-1}}^{h_k} dz$, $\int_{h_{k-1}}^{h_k} z\,dz$ and $\int_{h_{k-1}}^{h_k} z^2\,dz$.

The B-matrix is not zero so there is coupling between in-plane deformation and bending.

6.4–6.6 See AB&C Section 6.6.3. For a quasi-isotropic material (6.25) must be satisfied, i.e.

$$\left.\begin{aligned} A_{11} &= A_{22} \\ A_{11} - A_{12} &= 2A_{66} \\ A_{16} &= A_{26} = 0 \end{aligned}\right\}$$

For each ply we can use $\bar{Q}_{ij} = T^{-1} Q_{ij} T$, where $Q_{ij}$ is the same for each ply but the angles in the T-matrix are different.

When we multiply the matrices we obtain the equations (5.95) in AB&C. We substitute in the respective $\theta$ values, e.g. in problem 6.5 we have

$\theta = 0°$ : $c = 1$, $s = 0$

$\theta = 60°$ : $c = 1/2$, $s = \sqrt{3}/2$

$\theta = -60°$ : $c = 1/2$, $s = -\sqrt{3}/2$

The A-matrix is built up by summing $\bar{Q}_{ij}$ multiplied by the ply thickness for each ply. Since all plies are identical, all the thicknesses are equal, so we can just as well add together all the $\bar{Q}_{ij}$ matrices.

There is a good deal of algebra but it works!

We find that all the conditions given above are satisfied for the laminates in problems 6.5 and 6.6 but $A_{11} - A_{12} \neq 2A_{66}$ for the laminate in 6.4.