



**UiO** : **Department of Mathematics**  
University of Oslo

MEK4540

# Komposittmaterialer og –konstruksjoner

## Composite materials and structures

**Introduction to Assignment no. 1**



## MEK 4540 Autumn 2011: Compulsory Assignment no. 1

To be submitted by 11:15 on 4 October 2011 as described below.

A glass fibre laminate consists of the following materials:

Glass fibres:	E-modulus:	$E = 72.4 \text{ GPa}$
	Poisson's ratio:	$\nu = 0.2$
	Density:	$\rho = 2.54 \text{ g/cm}^3$
Vinylester:	E-modulus:	$E = 3.4 \text{ GPa}$
	Poisson's ratio:	$\nu = 0.35$
	Density:	$\rho = 1.1 \text{ g/cm}^3$

The fibre fabric in each unidirectional ply weighs  $150 \text{ g/m}^2$ . After a burn-off test on  $7.5 \text{ g}$  of laminate,  $5.6 \text{ g}$  of glass is left.

- Calculate the density of the composite, the fibre volume fraction and the thickness of a single ply containing  $150 \text{ g/m}^2$  of reinforcement.
- Estimate  $E_L$ ,  $E_T$ ,  $G_{LT}$  and  $\nu_{LT}$ . Use the simple combination laws and, where appropriate, also the Halpin-Tsai formulae. Explain the differences.
- With the elastic properties derived in (b) above (taking the Halpin-Tsai values for  $E_T$  and  $G_{LT}$ ), set up the stiffness matrix ( $Q$ ) in the  $L$ - $T$  axes. Use the transformation matrix  $T$  as defined in the lectures (or the matrices  $T_1$  and  $T_2$  as defined in the text-book) to calculate the stiffness matrices ( $\bar{Q}$ ) in directions  $0^\circ$ ,  $90^\circ$ ,  $45^\circ$  and  $-45^\circ$ . Explain briefly how these matrices express the relationship between stress and strain for a single ply, and indicate clearly whether you are using the tensor shear strain or the engineering shear strain. NB: This part should be solved first *without* using MATLAB, but the results can be checked with MATLAB in connection with Part (d).

## Assignment no. 1 (contd.)

d) Study the MATLAB script provided and modify it to calculate the A-, B- and D-matrices for laminates consisting of the above plies in the three lay-ups:

- (i)  $(0/45/-45/0)_s$
- (ii)  $(0/45/-45/90)_s$  and
- (iii)  $(0/45/-45/0)$ .

Comment on the differences, identifying any special properties displayed by the lay-ups. Note: The MATLAB script may also be used to check the individual ply stiffness matrices ( $\bar{Q}$ ) calculated in part (c).

e) Calculate the total thickness of each of the laminates (i) and (ii). Modify the same MATLAB script to calculate the strains for each of these two laminates when they are exposed to a tensile stress in the  $0^\circ$  direction of 1 MPa (which must first be converted to a force per unit length). Hence find the effective E-modulus  $E_x$  and Poisson's ratio  $\nu_{xy}$  for each of these two laminates, where the  $x$ -axis lies parallel to the  $0^\circ$  direction. By applying a tensile stress in the  $90^\circ$  direction, find the effective  $E_y$  and  $\nu_{yx}$  and verify that  $\nu_{yx}/\nu_{xy} = E_y/E_x$ . Similarly, by applying a shear stress find  $G_{xy}$ .

Comment on the differences between the results for the two laminates.

## Assignment no. 1 (contd.)

- f) Laminate (i) is subjected simultaneously to stresses  $\sigma_x = 300$  MPa,  $\sigma_y = 140$  MPa and  $\tau_{xy} = 65$  MPa. By extending the MATLAB script, calculate the direct and shear stress components parallel and perpendicular to the fibre direction in each ply. Hence calculate also the Tsai-Wu factor  $F_{TW}$  and the inverse of the Tsai-Wu strength ratio index ( $1/R$ ) in each ply, and deduce whether the Tsai-Wu failure criterion is violated, given the following ply strength properties:

Tensile strength in fibre direction, $\hat{\sigma}_{1t}$ :	490 MPa
Compressive strength in fibre direction, $\hat{\sigma}_{1c}$ :	463 MPa,
Tensile strength in transverse direction, $\hat{\sigma}_{2t}$ :	170MPa
Compressive strength in transverse direction, $\hat{\sigma}_{2c}$ :	175 MPa
In-plane shear strength, $\hat{\tau}_{12}$ :	189 MPa
Normalised interaction coefficient, $F_{12}^*$ :	- 0.5

Note: Since there are no bending moments and deformations, in part (f) it is only necessary to consider one ply in each of the directions  $0^\circ$  and  $45^\circ$ . Thus it is sufficient to consider only the first two plies.

## Assignment no. 1 (contd.)

### Important: Submission of solutions

Solutions are to be submitted by 11:15 on 04.10.2011 by one of the following means:

- directly by email to brianha at math.uio.no
- placed in Brian's mail pigeon-hole on the 7th Floor
- delivered by hand at lecture on 4 October

Solutions may be either handwritten or typed using suitable word/text processing software. Solutions sent by email must be in either MS Word or (preferably) PDF format.

The MATLAB script that is developed to solve parts (d), (e) and (f) is to be handed in together with the solution. Even if the main solution is handed in on paper, the script must be sent in digital format (MATLAB input file) by email to brianha at math.uio.no.

Brian Hayman, 30.08.2011

# The MATLAB script provided – 1

```
% Script for Assignment 1, MEK4540 Autumn 2011
% Brian Hayman, UiO, 2011
%
% The material properties and laminate lay-ups must be replaced with the correct data.
%

clear all; % sets all variables to zero

% Material data (to be replaced by correct values)

EL=40000;
ET=7700;
VLT=0.25;
GLT=2800;
t=0.5; % Thickness of one UD ply

% Ply orientations (to be replaced by correct lay-ups, which have more plies than this)

R=[45 0 -45]*3.14159/180; % ply angles in radians; alternative is R=[45 0 -45]*rad;
h=[-1.5*t -0.5*t 0.5*t 1.5*t]; % z-coordinates for top and bottom surfaces of plies as defined in AB&C Fig. 6-5

% Compliance (S-matrix)

S(1,1)=1/EL;
S(2,2)=1/ET;
S(1,2)=-VLT/EL;
S(3,3)=1/GLT;
S(2,1)=S(1,2);

% Stiffness (Q-matrix)

Q=inv(S);

% The procedure shown here uses tensor shear strains.
% You may keep to engineering shear strains if you prefer, but then you need the T1 and T2 matrices as defined in AB&C.

Q(3,3)=2*Q(3,3); % convert from engineering to tensor shear strains
```

# The MATLAB script provided – 2

```
A(1:3,1:3)=0;           % Set matrices to zero
B(1:3,1:3)=0;
D(1:3,1:3)=0;

for i=1:length(R)       % calculate T and Q for each ply 'k'. Assemble A, B and D

    % Transformation matrix (T-matrix)
    T(1,1)=cos(R(i))^2;
    T(2,2)=cos(R(i))^2;
    T(1,2)=sin(R(i))^2;
    T(2,1)=sin(R(i))^2;
    T(3,1)=-sin(R(i))*cos(R(i));
    T(3,2)=sin(R(i))*cos(R(i));
    T(1,3)=2*sin(R(i))*cos(R(i));
    T(2,3)=-2*sin(R(i))*cos(R(i));
    T(3,3)=cos(R(i))^2-sin(R(i))^2;

    Qk(:,:,i)=inv(T)*Q*T;           % Stiffness matrix for tensor strains, following rotation
    Qk(:,3,i)=Qk(:,3,i)/2;         % Convert back to engineering shear strains
    A=A+Qk(:,:,i)*(h(i+1)-h(i));    % Calculate A-matrix
    B=B+.5*Qk(:,:,i)*(h(i+1)^2-h(i)^2); % Calculate B-matrix
    D=D+1/3*Qk(:,:,i)*(h(i+1)^3-h(i)^3); % Calculate D-matrix
end
% Print ABD-matrices
A
B
D

% Assemble and print total stiffness matrix for the laminate
% This gives the relation between stresses and strains with engineering shear strains
Qtot=[[A B];[B D]] % Places contributions A,B,D with [B D] in rows below [A B]
```

# How to run MATLAB at UiO

- If you are logged on to a Linux machine in the Science Faculty at UiO, just write the command `matlab` or `matlab &`.
- If you are using a Windows PC somewhere else (e.g. your own laptop):
  - Make sure you have an internet connection, then use [Accessories – Communications – Remote desktop connection](#)
  - A login box will appear. Type `kontor.uio.no` and press [Connect](#). You will then be faced with a normal UiO login box. Log in to UiO with your UiO user and password.
  - Click on the Linux (penguin) icon to open a Linux login box – log in on this with your UiO user and password, then you will get a Linux window.
  - Proceed as described above.
- You can also purchase a student MATLAB CD and licence at the bookshop.



# MATLAB – some basic principles

- Use the help function to find out how each type of command works.
- MATLAB also contains some tutorials for beginners.
- MATLAB is specially designed for matrix operations.
- It uses scalars, vectors and matrices.
- Matrix operations are simple to write – e.g.
  - Invert matrix A to give matrix B:  $B = \text{inv}(A)$
  - Transpose matrix A to give matrix B:  $B = \text{transpose}(A)$  or  $B = A'$
  - Pre-multiply vector V by matrix A to give vector W:  $W = A*V$
- Basic input file is [filename].m
- This can call up subroutine files but this facility is not needed for this assignment.
- Comments (ignored by program) are preceded by “%” sign – can be separate line or end of line.
- The result of each line will be written to the command window unless it ends with “;” sign.
- Values of variables are retained unless set to zero by “clear” command.

# MATLAB – some basic principles

- Open MATLAB. You will see an empty command window.
- Use **File – Open** to open the m-file
- You can edit this file as you wish.
- Save it, then press the green arrow button to run it.
- Results will appear in the command window.
- You can copy any part of the command window and paste into a text file.

## Further notes:

- MATLAB script files are simply text files. They can be edited either within MATLAB (as above) or by use of a text editor such as Notepad or Wordpad in Windows.
- Much information is available from the web page of MAT1110, including a user manual (in Norwegian) but also some links to internet websites.