3.2 After a burn-off test we have the following data:

(1) Mass of crucible: 47.6504 g

(2) Mass of crucible + composite: 50.1817 g

(3) Mass of crucible + glass: 49.4476 g

Densities:

Glass $\rho_f = 2.5 \text{ g/cm}^3$

Epoxy $\rho_m = 1.2 \text{ g/cm}^3$

(4) Mass of glass = (3) – (1) : $w_f = 1.7972$ g

(5) Mass of composite = (2) – (1): $w_c = 2.5313$ g

Mass of epoxy = (5) – (4) : $w_m = 0.7341$ g

Weight (mass) fractions:

$$W_f = \frac{w_f}{w_c} = \underline{\underline{0.710}} \qquad W_m = \frac{w_m}{w_c} = \underline{\underline{0.290}}$$

Volume of glass: $v_f = \dfrac{w_f}{\rho_f} = 0.7189 \text{ cm}^3$

Volume of epoxy: $v_m = \dfrac{w_m}{\rho_m} = 0.6118 \text{ cm}^3$

Volume fractions:

$$V_f = \frac{v_f}{v_f + v_m} = \underline{\underline{0.540}} \qquad V_m = \frac{v_m}{v_f + v_m} = \underline{\underline{0.460}}$$

3.4 Calculate the density of the composite in problem 3.2.

$$\rho_{ct} = \frac{w_c}{v_c} = \frac{w_c}{v_f + v_m} = \underline{\underline{1.90 \text{ g/cm}^3}}$$

Find the void content if $\rho_{ce} = 1.86 \text{ g/cm}^3$.

$$V_v = \frac{\rho_{ct} - \rho_{ce}}{\rho_{ct}} = 0.021, \quad \text{i.e. } \underline{\underline{2.1\%}}$$

3.5 Calculate σ_f/σ_m and σ_f/σ_c for UD composites with $V_f = 10, 25, 50$ and 75% if $E_f = 400$ GPa and $E_m = 3.2$ GPa.

We have $\epsilon = \frac{\sigma_f}{E_f} = \frac{\sigma_m}{E_m} = \frac{\sigma_c}{E_c} \Rightarrow \frac{\sigma_f}{\sigma_m} = \frac{E_f}{E_m} = \underline{\underline{125}}$

We have also $E_c = E_f V_f + E_m V_m$

$= E_f V_f + E_m (1 - V_f)$

so that $\frac{\sigma_f}{\sigma_c} = \frac{E_f}{E_c} = \frac{E_f}{E_f V_f + E_m(1-V_f)}$

This gives:

V_f (%) :	10	25	50	75
E_c (GPa):	42.9	102.4	201.6	300.8
$\frac{\sigma_f}{\sigma_c}$:	9.33	3.91	1.98	1.33

3.6 Calculate E_L, E_T, G_{LT} and ν_{LT} for glass-epoxy, graphite-epoxy, Kevlar-epoxy and boron-aluminium composites with $V_f = 25$, 50 and 75%.

$$E_L = E_f V_f + E_m (1 - V_f)$$

$$E_T = \frac{1}{V_f/E_f + (1-V_f)/E_m} = \frac{E_f E_m}{E_m V_f + E_f(1-V_f)}$$

$$G_{LT} = \frac{G_f G_m}{G_m V_f + G_f (1-V_f)} \quad \text{where } G = \frac{E}{2(1+\nu)} \text{ for each material.}$$

$$\nu_{LT} = \nu_f V_f + \nu_m (1 - V_f)$$

It is also interesting to calculate E_T and G_{LT} using Halpin-Tsai:

$$E_T = \frac{1 + \xi \eta V_f}{1 - \eta V_f} E_m, \quad \eta = \frac{(E_f/E_m) - 1}{(E_f/E_m) + \xi}, \quad \xi = 2$$

$$G_{LT} = \frac{1 + \xi \eta V_f}{1 - \eta V_f} G_m, \quad \eta = \frac{(G_f/G_m) - 1}{(G_f/G_m) + \xi}, \quad \xi = 1$$

See tables on next page.

V_f (%):	E_L [GPa]			E_T [GPa]			G_{LT} [GPa]			ν_{LT}		
	25	50	75	25	50	75	25	50	75	25	50	75
Glass-epoxy	20.13	36.75	53.38	4.59	6.67	12.17	1.70	2.48	4.58	0.313	0.275	0.238
Graphite-epoxy	65.13	126.75	188.38	4.64	6.90	13.44	1.72	2.56	5.00	0.313	0.275	0.238
Kevlar-epoxy	37.63	71.75	105.88	4.63	6.83	13.02	1.72	2.54	4.86	0.313	0.275	0.238
Boron-aluminium	140.0	210.0	280.0	87.5	116.7	175.0	33.1	44.6	68.3	0.298	0.265	0.233

Using Halpin-Tsai

V_f (%):	E_T [GPa]			G_{LT} [GPa]		
	25	50	75	25	50	75
Glass-epoxy	6.39	11.48	22.81	2.07	3.48	6.96
Graphite-epoxy	6.81	13.18	30.41	2.13	3.76	8.36
Kevlar-epoxy	6.67	12.60	27.59	2.11	3.67	7.88
Boron-aluminium	105.0	154.0	227.5	37.4	54.3	83.5

3

3.7 A rod consists of a matrix and two types of fibre as shown in the table in the textbook.

(a) What is the maximum load the rod can carry without rupturing, given that the area of cross-section is $10\ cm^2$?

we need first ρ_c: $\frac{1}{\rho_c} = \frac{W_m}{\rho_m} + \frac{W_{fA}}{\rho_{fA}} + \frac{W_{fB}}{\rho_{fB}}$

$$\Rightarrow \quad \rho_c = 1.7415\ g/cm^3$$

Volume fractions:

$$V_m = \frac{\rho_c}{\rho_m} W_m , \quad V_{fA} = \frac{\rho_c}{\rho_{fA}} W_{fA} , \quad V_{fB} = \frac{\rho_c}{\rho_{fB}} W_{fB}$$

$$\Rightarrow \quad V_m = 0.469 , \quad V_{fA} = 0.313 , \quad V_{fB} = 0.218$$

Calculate maximum strains:

$$\epsilon_{ui} = \frac{\sigma_{ui}}{E_i} \quad \Rightarrow \quad \epsilon_{um} = 0.0171$$

$$\epsilon_{ufA} = 0.0200$$

$$\epsilon_{ufB} = 0.0750$$

The largest strain the rod can withstand without rupture is thus

$$\epsilon_{um} = 0.0171$$

We have $E_c = E_m V_m + E_{fA} V_{fA} + E_{fB} V_{fB} = 24.90\ GPa$

$$\sigma_{cmax} = \epsilon_{um} E_c = 0.4267\ GPa = 426.7\ MPa \quad (= N/mm^2)$$

$$\Rightarrow \quad F_{max} = 426.7 \times 10^3\ N \simeq \underline{\underline{427\ kN}}$$

(b) What is the greatest load the rod can carry?

Here we can make two different assumptions:

(i) The matrix cracks up completely when $\epsilon > \epsilon_{um}$ and carries no load.

(ii) The matrix continues to carry that part of the load it carried when F reached 427 kN, but only the fibres contribute to further increase.

We investigate both cases:

(i) At fracture (cracking) of the matrix we have $\epsilon = \epsilon_{um} = 0.0171$ and $F = 427$ kN. For $\epsilon > \epsilon_{um}$ we assume that the matrix cracks completely giving $\sigma_m = 0$ and $E_m = 0$.

Then $E_c' = E_{fA} V_{fA} + E_{fB} V_{fB} = 23.21$ GPa.

At rupture of fibres A we have $\epsilon = \epsilon_{ufA}$ and $\sigma_c = E_c' \epsilon_{ufA} = 464$ MPa

so that $F = 464$ kN.

For $\epsilon > \epsilon_{ufA}$ only fibres B are intact.

Now $E_c'' = E_{fB} V_{fB} = 1.303$ GPa.

At rupture of fibres B, $\epsilon = \epsilon_{ufB}$ and $\sigma_c = E_c'' \epsilon_{ufB} = 98.1$ MPa.

so that $F = 98.1$ kN

We conclude that the greatest load the rod can carry is

$$F_{max} = \underline{\underline{464 \text{ kN}}}$$

and that this is reached when fibres A rupture.

(ii) For $\epsilon > \epsilon_{um}$ the increase of loading occurs only in the fibres:

$$\frac{\Delta \sigma_c}{\Delta \epsilon} = E_{fAB} = E_{fA} V_{fA} + E_{fB} V_{fB} = 23.21 \text{ GPa}$$

Up to rupture of fibres A, $\Delta \epsilon = \epsilon_{ufA} - \epsilon_{um} = 0.0029$

$\Rightarrow \Delta \sigma_c = 0.0673$ GPa $= 67.3$ MPa

$\Rightarrow \Delta F = 67.3$ kN and $F = 427 + 67 = 494$ kN.

For $\epsilon > \epsilon_{ufA}$ we must assume that all A-fibres are fractured so there are only a constant contribution from the matrix $\sigma_m = \sigma_{um} = 0.06$ GPa and a contribution from fibres B of $\sigma_{fB} = E_{fB} \epsilon$.

At rupture of fibres B, $\sigma_{fB} = E_{fB} \epsilon_{ufB} = \sigma_{ufB} = 0.45$ GPa.

Then the greatest stress the rod can take after rupture of fibres A is given by $\sigma_c = \sigma_{um} V_m + \sigma_{ufB} V_{fB} = 126$ MPa

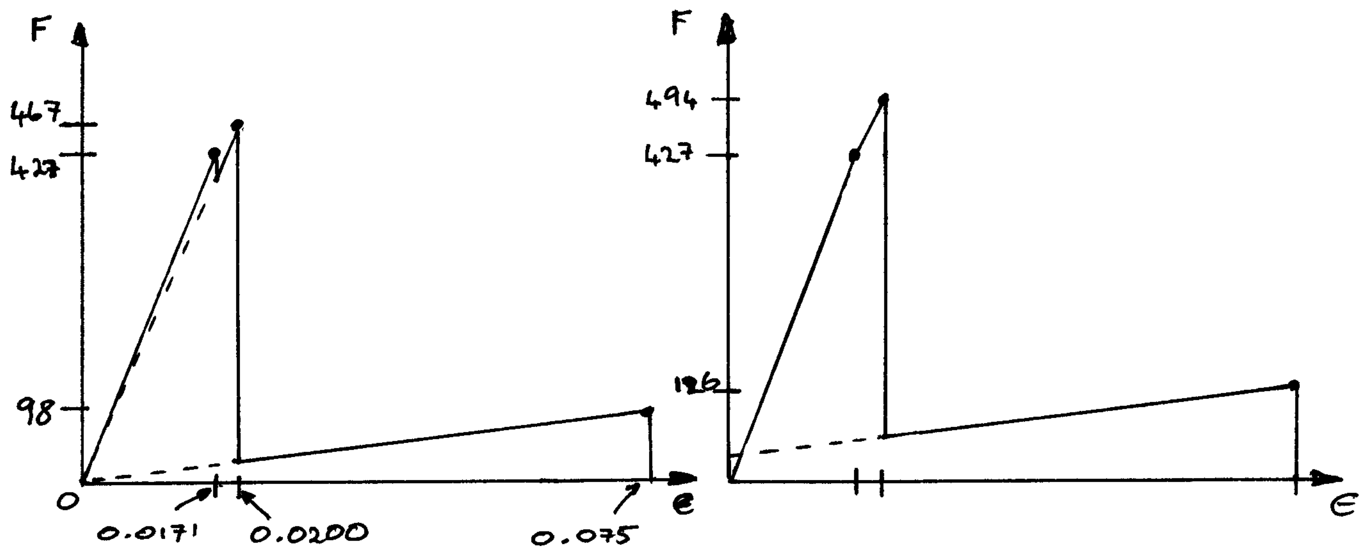
with $F = 126$ kN.

This is less than when fibres A rupture so $F_{max} = \underline{\underline{494 \text{ kN}}}$

(c) The matrix ruptures first, then fibres A and finally fibres B.

(d) The load-elongation curve depends on whether we adopt assumption (i) or (ii). Assumption (i) means that the matrix has a ductile behaviour with $\sigma_m = \sigma_{mu}$ for $\epsilon > \epsilon_{mu}$, while (ii) means that the matrix is brittle so that $\sigma_m = 0$ for $\epsilon > \epsilon_{mu}$.

With displacement control:



With load control the drops in loading cannot occur in a stable manner. The behaviour is thus: