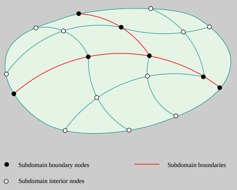
Chapter: 7

MEK4560
The Finite Element Method in Solid Mechanics II

(March 5, 2008)

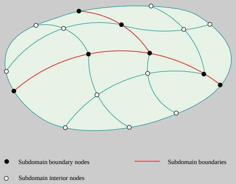
TORGEIR RUSTEN

(E-post:torgeiru@math.uio.no)



Contents

Department of Mathematics
University of Oslo



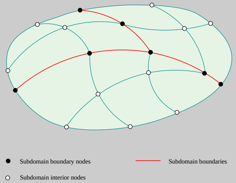
7. Shell models

The topic of the present chapter is circular arches and shell models. See also[Cook et al., 2002][1] sections 16.1, 16.2, 16.4 and 16.5.

Shell constructions, and consequently shell analysis, are used frequently.

[1] R. D. Cook, D. S. Malkus, M. E. Plesha, and R. J. Witt. *Concepts and Applications of Finite Element Analysis*. Number ISBN: 0-471-35605-0. John Wiley & Sons, Inc., 4th edition, October 2002.

Department of Mathematics University of Oslo

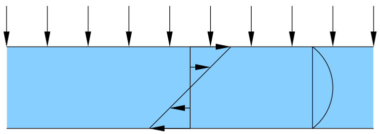


7.1. Shell behavior

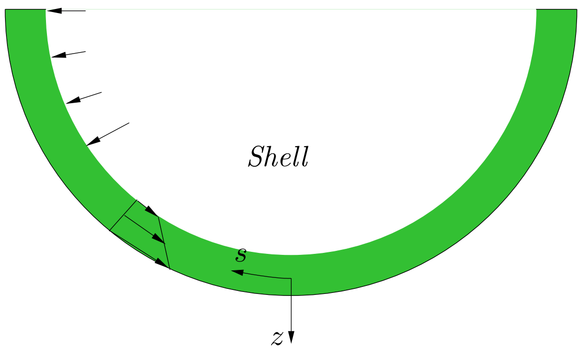
Shell constructions are found frequently both in nature and in man made constructions. The primary reason is the way shells behave.

A shell has curved inner and outer surfaces separated by a distance t, called the thickness. The models considered here hare using the mid surface, i.e. the surface with distance $t/2$ from both inner and outer surfaces, to describe the shell.

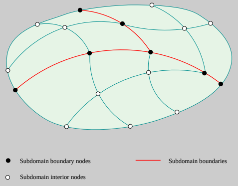
A plate carry loads through bending and high stresses, while shell constructions use relatively moderate membrane stresses.



Plate



Some examples of constructions carrying loads using membrane stresses:



Contents
Page 4 of 26
Go Back
Close
Quit

1. Containers,
2. cylindrical roofs,
3. circular arches (Condeep platform, Colosseum movie theater),

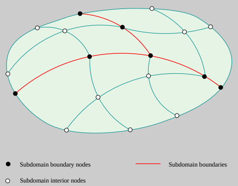
The state of stress in the local x and y coordinates, tangential plane, can be represented as *membrane* and *bending* stresses. In a thin-walled shell composed of a linearly elastic and homogeneous material the membrane stresses are independent of z and the bending stresses vary linearly with z. Bending stresses result primarily from:

1. Concentrated loads.
2. Boundaries.
3. Changes in the radius of curvature.

The bending effects are often localized near loads or disturbances that cause them, a boundary layer.

Shell theory can be viewed as a modification of plate theory where membrane and bending effects are coupled.

At each point of the shell mid-surface circles tangent to the surface exist. The circle with the smallest and largest radii are the two principal radii of curvature at the point. In a cylindrical shell one radii is constant and one is infinite, in a conical shell on is varying and one is infinite.



If one principal radius is finite the shell is *singly curved*; when both are finite the shell is *doubly curved*. Different shells are:

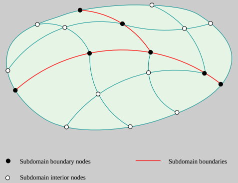
1. Singly curved.
2. Doubly curved.
3. Prismatic.
4. Rotational symmetric.

Classic shell theory result in complicated *differential equations* which are difficult to solve, even after some simplifications:

1. *Love*
2. *Donnell*
3. *Flügge*
4. *Vlasov*

⋮

Department of Mathematics
University of Oslo



Mek 4560
Torgeir Rusten

Classic theory is usually for thin shells:

$$\frac{t}{R} < \frac{1}{20}$$

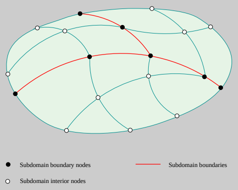
are frequently based on *Kirchhoff* hypothesis.

Finite element analysis are usually based on one of the following three methods:

1. The shell surface is approximated using a set of plane element using both membrane and bending plate models for each element.
2. Curved elements based on classic shell theory.
3. Mindlin-type (C^0) elements. They can be modeled as a special type of three dimensional elements with special properties to account for the small dimension in one direction.

We mainly discuss item 1, for formulations based on item 2 and item 3 the main ideas are outlined.

Department of Mathematics University of Oslo



Mek 4560
Torgeir Rusten

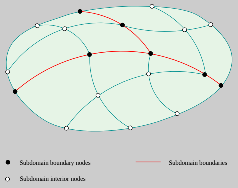
7.2. Circular arches and arch elements

In this brief introduction some of the challenges in shell modeling is outlined, for further details see [Cook et al., 2002][1] chapter 16.2.

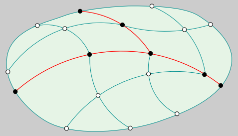
The model is base on a local coordinate system (s, z), where s is the tangential direction and z is the radial.

Henceforth we assume that the arch is sufficiently thin such that shear deformation can be neglected.

[1] R. D. Cook, D. S. Malkus, M. E. Plesha, and R. J. Witt. *Concepts and Applications of Finite Element Analysis*. Number ISBN: 0-471-35605-0. John Wiley & Sons, Inc., 4th edition, October 2002.

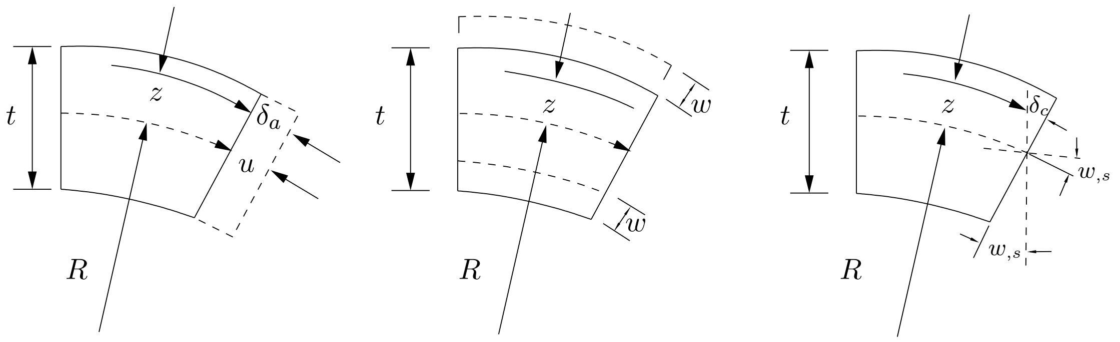


Department of Mathematics
University of Oslo



Mek 4560
Torgeir Rusten

Displacements and strains: Using the figure below



the following kinematic relation can be derived:

$$\frac{\delta_a}{R+z} = \frac{u}{R}, \qquad \varepsilon_s = \frac{w}{R+z} \approx \frac{w}{R} \qquad \text{and} \qquad \delta_c = -z\frac{\partial w}{\partial s}$$

here u is the displacements in the s-direction at the mid-plane, w is the displacement in the z-direction at the mid-plane. It is assumed that the thickness t is small compared to the radius of curvature.

The strains can be found from the displacements together with the strains given by the radial displacement:

$$\varepsilon_s = \frac{d}{ds}(\delta_a + \delta_c) + \frac{w}{R} = u_{,s} + \frac{w}{R} + z\left(\frac{u_{,s}}{R} - w_{,ss}\right)$$

This result in

$$\varepsilon_s = \varepsilon_m + z\,\kappa \qquad \text{where} \qquad \begin{aligned} \varepsilon_m &= u_{,s} + \frac{w}{R} \\ \kappa &= \frac{u_{,s}}{R} - w_{,ss} \end{aligned}$$

Membrane strain is in the mid line and is related to the membrane forces in the s-direction of the arch. The rate of change of curvature is associated with bending moments.

Strain energy: …is a result of contributions from the membrane strains and curvature change:

$$U = U_m + U_b = \frac{1}{2}\int_\ell \left(EA\varepsilon_m^2 + EI\kappa^2\right)\,ds$$

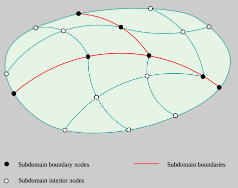
where E is the module of elasticity, A is the area of the cross section of the arch and I is the moment of inertia about the neutral axis of bending.

Bending: Most loadings of an slender arch result in bending, but the membrane strains are small. If $t \to 0$

$$\varepsilon_m = 0 \qquad \text{thus} \qquad u_{,s} + \frac{w}{R} = 0$$

This is known as the *the inextensibility condition.*

Department of Mathematics
University of Oslo



Mek 4560
Torgeir Rusten

Membrane locking: *Membrane locking* refers to excessive stiffness in bending. This a problem for some elements, and is caused by nodal displacements that should be resisted only by bending are resisted by membrane deformations as well. Since the membrane stiffness is much higher than the bending stiffness in a slender arch the desired bending mode tend to be excluded from element response.

> *Straight elements do not suffer from membrane locking.*

Membrane locking is mainly seen in curved elements with low order interpolation, e.g:

$$\begin{aligned} u &= a_1 + a_2 s \\ w &= a_3 + a_4 s + a_5 s^2 + a_6 s^3 \end{aligned}$$

where a_i are generalized coordinates. The strains are found from

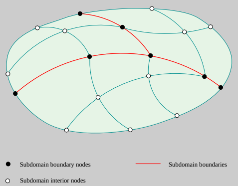
$$\varepsilon_m = \left(a_2 + \frac{a_3}{R}\right) + \frac{a_4}{R}s + \frac{a_5}{R}s^2 + \frac{a_6}{R}s^3 \qquad \kappa = \frac{a_2}{R} - 2a_5 - 6a_6 s$$

If the element are inextensible, $\varepsilon_m = 0$,

$$a_2 + \frac{a_3}{R} = a_4 = a_5 = a_6 = 0$$

The first condition $a_2 + \frac{a_3}{R} = 0$ result in $\varepsilon_m = 0$ for $s = 0$, the local midpoint of the element. This condition cause no problems. The remaining conditions, $a_4 = a_5 = a_6 = 0$, result in $w_{,s} = w_{,ss} = w_{,sss} = 0$. This is not true. This result in membrane locking.

Department of Mathematics
University of Oslo



Mek 4560
Torgeir Rusten

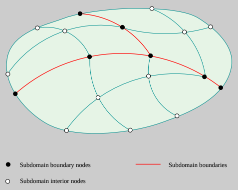
Using reduced integration, evaluating at $s = 0$ the condition $a_2 + \frac{a_3}{R} = 0$ is satisfied even if a_4, a_5 and a_6 are nonzero, thus using reduced integration on the membrane term membrane locking is avoided.

Other curved elements: Exact integration may be appropriate, depending on the choice of basis functions.

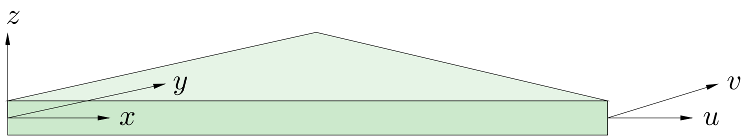
7.3. Plane shell elements

A shell can be approximated using flat elements. If the number of elements are increased a curved surface can be approximated to any desired accuracy. A flat shell element consist of two parts: a *membrane* and a *plate* part as indicated below.

Department of Mathematics
University of Oslo



Mek 4560
Torgeir Rusten

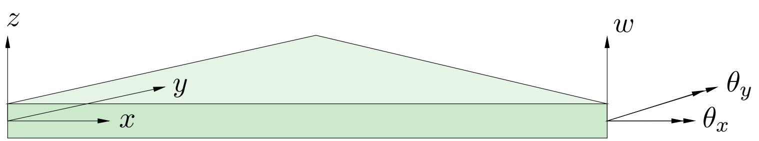


Membrane

Shell =

+

Plate



Note that the membrane and the plate element is established in a local coordinate system.

The two formulations:

$$\boldsymbol{k}_m \boldsymbol{d}_m = \boldsymbol{r}_m^e \qquad \text{and} \qquad \boldsymbol{k}_p \boldsymbol{d}_p = \boldsymbol{r}_p^e$$

result in the equations:

$$\begin{bmatrix} \boldsymbol{k}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{k}_p \end{bmatrix} \begin{Bmatrix} \boldsymbol{d}_m \\ \boldsymbol{d}_p \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{r}_m^e \\ \boldsymbol{r}_p^e \end{Bmatrix}$$

There is no coupling between the membrane and the plate part in the local coordinate system.

The element has five degrees of freedom in each node.

In relation to shells it may be convenient to add a sixth degree of freedom, θ_z.

It is customary in finite element codes that all the nodes has six degrees of freedom in a shell formulation

$$\begin{bmatrix} \boldsymbol{k}_m & \mathbf{0} & 0 \\ \mathbf{0} & \boldsymbol{k}_p & 0 \\ \mathbf{0} & \mathbf{0} & 0 \end{bmatrix} \begin{Bmatrix} \boldsymbol{d}_m \\ \boldsymbol{d}_p \\ \theta_z \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{r}_m^e \\ \boldsymbol{r}_p^e \\ 0 \end{Bmatrix}$$

Note that no stiffness is related to the sixth degree of freedom θ_z. This might cause some trouble in the linear solver if some elements are planar.

The degrees of freedom are usually ordered consecutively for each node:

$$\boldsymbol{d}_i^T = \begin{Bmatrix} u & v & w & \theta_x & \theta_y & \theta_z \end{Bmatrix}$$

I.e. two groups of vectors. The local stiffness matrix is transformed to global coordinates:

$$\begin{Bmatrix} \boldsymbol{d}_u \\ \boldsymbol{d}_\theta \end{Bmatrix}_l^i = \begin{bmatrix} \boldsymbol{T}_3 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{T}_3 \end{bmatrix} \begin{Bmatrix} \boldsymbol{d}_u \\ \boldsymbol{d}_\theta \end{Bmatrix}_g^i$$

where $\boldsymbol{T}_3$ is a transformation matrix from global to local coordinates. The transformation matrix $\boldsymbol{T}$ transform the displacement for the element:

$$\boldsymbol{d}_l = \boldsymbol{T}\boldsymbol{d}_g$$

The global stiffness matrix and load vector becomes:

$$\boldsymbol{k}_g = \boldsymbol{T}^T \boldsymbol{k}_l \boldsymbol{T} \qquad \text{and} \qquad \boldsymbol{r}_g^e = \boldsymbol{T}^T \boldsymbol{r}_l^e$$

Remark 7.1 If all the element connected to a node are not in the same plane, all six degrees of freedom is assigned stiffness even if the local system has five degrees of freedom.

Remark 7.2 If all elements connected to a node is in the same plane the rotation around the normal to the plane will have no stiffness, i.e. the stiffness matrix $\boldsymbol{k}_g$ is singular:

How to avoid this?

1. Have five degrees of freedom at the node..
2. Fix this degree of freedom.
3. Add an artificial stiffness.
   [Zienkiewicz and Taylor, 2000][2] propose the relations

$$\begin{Bmatrix} M_{z1} \\ M_{z2} \\ M_{z3} \end{Bmatrix} = \frac{1}{2}\alpha E A t \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{Bmatrix} \theta_{z1} \\ \theta_{z2} \\ \theta_{z3} \end{Bmatrix}$$

[2] O. C. Zienkiewicz and R. L. Taylor. *The finite element metod*, volume 1, The Basis. Butterworth-Heinemann, fifth edition, 2000.

Department of Mathematics University of Oslo

Mek 4560
Torgeir Rusten

for three node shell elements. Here E is the module of elasticity, A is the element area, t is the thickness and α is a constant. In ([Cook et al., 2002][1] a value of 0.3 or less is indicated. Mood show the effect.)

4. Use membrane elements with drilling degrees of freedom, θ_z.

   This also has some problems. For doubly curved shells modeled as planar elements the rotation θ_z represent a problem. It is coupled to the bending rotations θ_x, θ_y, through the neighbor elements. This is correct for piecewise planar shells, however for smooth surfaces it result in excessive bending stiffness. This particularly problematic for bending dominated analysis and coarse element models.

Remark 7.3 Rectangular elements has another problem related to doubly curved shells, the four nodes are not necessarily in a plane.

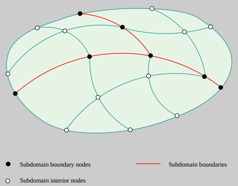
7.4. Thick shell elements

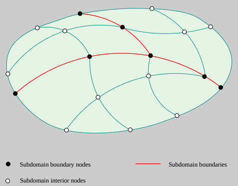
Shell models can be derived based on a three dimensional model, called continuum based shell formulation, curved isoparametric elements or *degenerated volumelement*.

Here we consider a curved shell formulation based on a 20 node volume element.

The formulation reduce the volume element using a three step kinematic reduction:

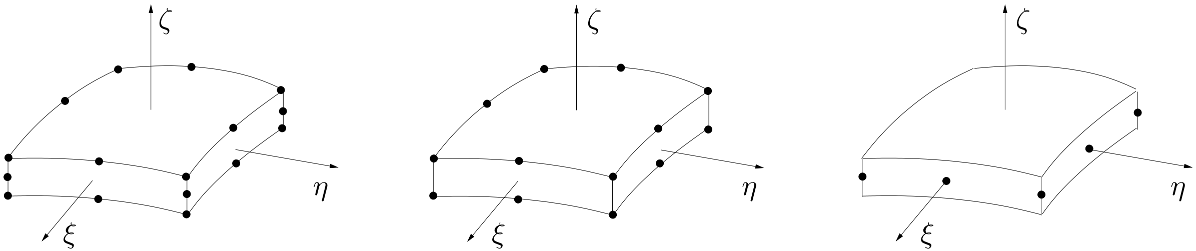
[1] R. D. Cook, D. S. Malkus, M. E. Plesha, and R. J. Witt. *Concepts and Applications of Finite Element Analysis*. Number ISBN: 0-471-35605-0. John Wiley & Sons, Inc., 4th edition, October 2002.





1. Start with a 20 node volume element where the thickness is small compared to the other dimensions.
2. The mid-plane nodes are eliminated, thus lines through the thickness is straight but not necessarily normal to the middle plane, Mindlin-Reissner assumptions.
3. The displacements for nodes on a thickness-direction line are equal and placed in the mid-plane. Each mid-plane node has five degrees of freedom.

The kinematic reduction is shown on the figure below.



Geometry: For a typical node, i, a thickness direction vector is established

$$\boldsymbol{V}_{3i} = t_i \begin{Bmatrix} \ell_{3i} \\ m_{3i} \\ n_{3i} \end{Bmatrix} \qquad \text{where} \qquad \begin{Bmatrix} \ell_{3i} \\ m_{3i} \\ n_{3i} \end{Bmatrix} = \frac{1}{t_i} \begin{Bmatrix} x_j - x_k \\ y_j - y_k \\ z_j - z_k \end{Bmatrix}$$

The relation between the local coordinate system (ξ, η, ζ) and the global coordinates (x, y, z) is

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum N_i(\xi, \eta) \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix} + \sum \frac{\zeta}{2} N_i(\xi, \eta) t_i \begin{Bmatrix} \ell_{3i} \\ m_{3i} \\ n_{3i} \end{Bmatrix}$$

The mid-plane coordinates are given by

$$x_i = \frac{1}{2}(x_j + x_k)$$

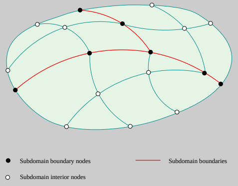
In a Finite element program an alternative is to specify

$$x_i, y_i, z_i, t_i \quad \text{and} \quad \boldsymbol{V}_{3i}$$

We also need two mutually orthogonal vectors, orthogonal to $\boldsymbol{V}_{3i}$. They are tangent vectors to the shell mid-plane. The vectors are used to define the directions of the nodal rotations (α_i, β_i). Details on how to do this is found in the textbook. Note: in general these directions vary from node to node.

Using the tangent vectors we define the matrix

$$\boldsymbol{\mu}_i = \begin{bmatrix} -\frac{\boldsymbol{V}_{2i}}{\|\boldsymbol{V}_{2i}\|} & \frac{\boldsymbol{V}_{1i}}{\|\boldsymbol{V}_{1i}\|} \end{bmatrix}$$



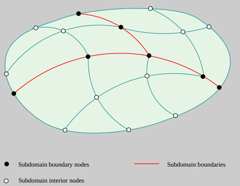
The Jacobi-matrix: The Jacobi matrix of the mapping is used in integration and differentiation. The first column is given by

$$
\begin{aligned}
x_{,\xi} &= \sum N_{i,\xi}(x_i + \frac{\zeta}{2} t_i \ell_{3i}) \\
x_{,\eta} &= \sum N_{i,\eta}(x_i + \frac{\zeta}{2} t_i \ell_{3i}) \\
x_{,\zeta} &= \sum N_i(\frac{1}{2} t_i \ell_{3i})
\end{aligned}
$$

Displacements: The displacements on a point on vector $\boldsymbol{V}_{3i}$ can be established in a local coordinate system and transform it to a global system. This is similar to pates/beams and the result is

$$
\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = N_i \left[\begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \frac{\zeta}{2} t_i \boldsymbol{\mu}_i \begin{Bmatrix} \alpha_i \\ \beta_i \end{Bmatrix} \right]
$$

Department of Mathematics
University of Oslo



Contents
◁◁ ▷▷
◁ ▷
Page 19 of 26
Go Back
Close
Quit

Strains: The strains are found by differentiating the displacements. This can be written

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \boldsymbol{H} \begin{Bmatrix} u_{,x} \\ u_{,y} \\ u_{,z} \\ v_{,x} \\ \vdots \\ w_{,z} \end{Bmatrix} = \boldsymbol{H}\boldsymbol{g}_x$$

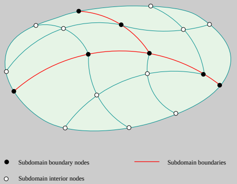
$\boldsymbol{H}$ is a rectangular matrix, see Chapter 6 in the textbook, and

$$\boldsymbol{g}_x = \begin{Bmatrix} u_{,x} \\ u_{,y} \\ u_{,z} \\ v_{,x} \\ \vdots \\ w_{,z} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{J}^{-1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}^{-1} \end{bmatrix} \begin{Bmatrix} u_{,\xi} \\ u_{,\eta} \\ u_{,\zeta} \\ v_{,\xi} \\ \vdots \\ w_{,\zeta} \end{Bmatrix} = \bar{\boldsymbol{J}}^{-1}\boldsymbol{g}_\xi$$

Since the orientation is arbitrary, all six strain components are included.

The gradient in the local coordinate system is found from the expression for displacements

Department of
Mathematics
University of Oslo



Mek 4560
Torgeir Rusten

above

$$
\boldsymbol{g}_\xi = \begin{Bmatrix} u_{,\xi} \\ u_{,\eta} \\ u_{,\zeta} \\ v_{,\xi} \\ \vdots \\ w_{,\zeta} \end{Bmatrix} = \begin{bmatrix} N_{i,\xi} & 0 & 0 & -\frac{\zeta}{2}t_i\ell_{2i}N_{i,\xi} & \frac{\zeta}{2}t_i\ell_{1i}N_{i,\xi} \\ N_{i,\eta} & 0 & 0 & -\frac{\zeta}{2}t_i\ell_{2i}N_{i,\eta} & \frac{\zeta}{2}t_i\ell_{1i}N_{i,\eta} \\ 0 & 0 & 0 & -\frac{1}{2}t_i\ell_{2i}N_i & \frac{1}{2}t_i\ell_{1i}N_i \\ 0 & N_{i,\xi} & 0 & -\frac{\zeta}{2}t_i m_{2i}N_{i,\xi} & \frac{\zeta}{2}t_i m_{1i}N_{i,\xi} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -\frac{1}{2}t_i n_{2i}N_i & \frac{1}{2}t_i n_{1i}N_i \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{Bmatrix} = \boldsymbol{G}_i\boldsymbol{d}_i
$$

Using this the stresses can be expressed as

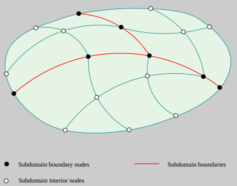
$$
\boldsymbol{\varepsilon} = \boldsymbol{H}\bar{\boldsymbol{J}}^{-1}\boldsymbol{G}_i\boldsymbol{d}_i = \boldsymbol{B}_i\boldsymbol{d}_i
$$

Stress-Strain relation: The stress strain relations can be given as

$$
\boldsymbol{\sigma} = \boldsymbol{E}\boldsymbol{\varepsilon} \qquad \text{or} \qquad \boldsymbol{\sigma}' = \boldsymbol{E}'\boldsymbol{\varepsilon}'
$$

$\boldsymbol{\sigma}$ are stresses in the global coordinate system (x, y, z), while $\boldsymbol{\sigma}'$ are in the local system given by $[\boldsymbol{V}_1, \boldsymbol{V}_2, \boldsymbol{V}_3]$. If the material is isotropic in the local coordinate system, the stress-strain

Department of
Mathematics
University of Oslo



Mek 4560
Torgeir Rusten

relations is given by

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{31} \end{Bmatrix} = \begin{bmatrix} E' & \nu E' & 0 & 0 & 0 & 0 \\ \nu E' & E' & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G^* & 0 \\ 0 & 0 & 0 & 0 & 0 & G^* \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{23} \\ \varepsilon_{31} \end{Bmatrix}$$

where

$$E' = \frac{E}{(1-\nu^2)}, \qquad G = \frac{E}{2(1+\nu)} \qquad \text{and} \qquad G^* = \frac{5G}{6}$$

The factor 5/6 account for variation in shear strains throughout the thickness, they are close to parabolic, not constant as we have assumed.

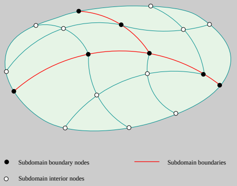
Note that $\sigma_{33} = 0$, i.e. we have assumed plane stress in the local coordinate system.

The stress strain relations $\boldsymbol{E}$ is established using $\boldsymbol{E}'$ and a coordinate transform, see Chapter 8.2 in the textbook. The transformation of the stress strain relation can be written

$$\boldsymbol{E} = \boldsymbol{T}_\varepsilon^T \boldsymbol{E}' \boldsymbol{T}_\varepsilon$$

In case of numerical integration the transformation is applied for each integration point.

Department of Mathematics
University of Oslo



Contents
Page 22 of 26
Go Back
Close
Quit

Stiffness matrix: The element stiffness matrix for an N node element can be expressed

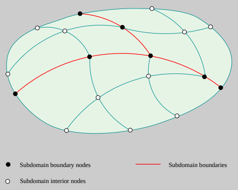
$$\boldsymbol{k} = \int_{\square} \boldsymbol{B}^T \boldsymbol{E} \boldsymbol{B} \det \boldsymbol{J} \, d\square$$

The efficiency of the computations can be improved, see the textbook Chapter 16.5.

Comments:

- The membrane and bending deformations are coupled for curved elements. Thus *membrane locking* might occur.
- This is a thick shell formulation, similar to a thick plate model, thus *shear locking* might occur.
- The *locking* problems can be eliminated by reduced integration, possibly combined with stabilization.
- The element has five degrees of freedom per node, see the comments above on five degrees of freedom.

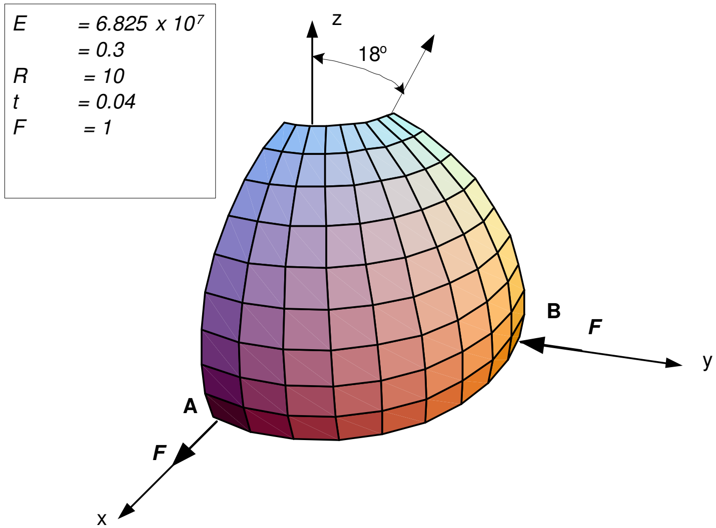
Department of Mathematics
University of Oslo



Mek 4560
Torgeir Rusten

Øving 7.1

Figuren under viser en fjerdedel av et kuleskall.

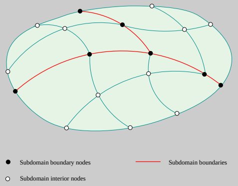


- Modeller kuleskallet i ANSYS. Benytt `SHELL63`.
- Sett på symmetribetingelser på sidekantene som går fra A og B mot hullet i toppen.
- Lag et 2×2 elementnett og se om rotasjonsfrihetsgraden, θ_z lokalt, påvirker resultet. (KEYOPT(3)=$0, 1, 2$)).

Contents
Page 24 of 26
Go Back
Close
Quit

- Benytt både trekant- og firkantelementet. Har firkantelementet problemer med at de fire nodene ikke ligger i et plan?
- Hvilken respons er dominerende for elementet, membran eller bøyning?
- Hvordan virker `SHELL93` elementet for dette problemet?

Department of Mathematics
University of Oslo

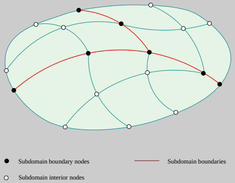


A. References

[Cook et al., 2002] Cook, R. D., Malkus, D. S., Plesha, M. E., and Witt, R. J. (2002). *Concepts and Applications of Finite Element Analysis*. Number ISBN: 0-471-35605-0. John Wiley & Sons, Inc., 4th edition.

[Zienkiewicz and Taylor, 2000] Zienkiewicz, O. C. and Taylor, R. L. (2000). *The finite element metod*, volume 1, The Basis. Butterworth-Heinemann, fifth edition.

Department of Mathematics University of Oslo



Mek 4560
Torgeir Rusten