Chapter: 8

MEK4560
The Finite Element Method in Solid Mechanics II

(March 12, 2008)

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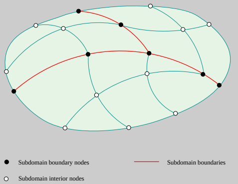

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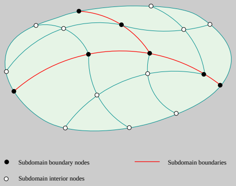

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8. Dynamic analysis, part I

The present and the next chapter is a brief introduction to *dynamic analysis*. The first part discuss:

- *formulation*,
- *discretization and separation of variables*,
- *the mass matrix*,
- *damping*,
- *free vibration*,
- *modal analysis*.

while the second mainly discuss:

- *harmonic loading*
- *direct time integration*.

Dynamic analysis is discussed in Chapter 11 in [Cook et al., 2002][2]. The discusstion of d'Alemberts principle is from [Lanczos, 1986][3].

[2] R. D. Cook, D. S. Malkus, M. E. Plesha, and R. J. Witt. *Concepts and Applications of Finite Element Analysis*. Number ISBN: 0-471-35605-0. John Wiley & Sons, Inc., 4th edition, October 2002.

[3] C. Lanczos. *The Variational Principles of Mechanics*. Dover, 4th edition, 1986.

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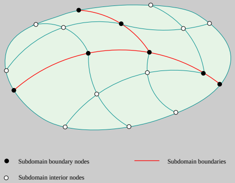

8.1. Introduction

Structural response is time dependant if the loading is time dependant. However, if the loading is periodic and the frequency is smaller than one quarter(one third) of the lowest natural frequency[1] the inertia forces are small and the construction can be analysed using “static methods”:

$$\boldsymbol{K}\boldsymbol{D}(t) = \boldsymbol{R}(t)$$

The inertia forces are important when the load frequencies are higher, or in *free vibrations* analysis, i.e. of the forces are zero.

Dynamic anlysis are mainly:

- *Wave propagation*, loads are sudden and last for a short time, explosion loads, collitions, etc. The response is rich in high frequency response and the time frame is short, the time the wave use to progagate throug the construction.
- *Structural dynamics* the frequency of the load is of the same order ase the lowest natural frequensies of the structure.

In structural dynamics there are main three classes of analysis:

[1]We define this shortly.

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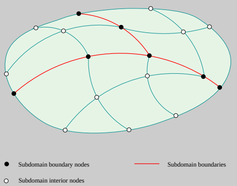

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- In *free vibration* the goal is to compare the natural frequencies of the structure to the frequencies of the loads. In a good design the frequencies should be well separated.
- In *time varying response* the response to suddenly applied loads.
- In *harmonic forced vibrations* the response to periodic load is analysed. Here the initial respons is of no interest, the interest is the "steady state" response.

Time varying response analysis is based on:

- *Modal time integration.*
- *Direct time integration methods.*

Remark 8.1

8.2. d'Alemberts principle

The french mathematician and philosopher d'Alembert (1717-1783) extended the validity of the static *virtual work* principle to dynamic problems. The simple but powerful idea is the following: Rewrite the fundamental law in Newton mechanincs; *the rate of change of linear momentum is equal to the sum of external forces*:

$$\int_V \boldsymbol{F}\, dV + \int_S \boldsymbol{\Phi}\, dS = \frac{d}{dt}\int_V \rho \boldsymbol{v}\, dV$$

as

$$\int_V \boldsymbol{F}\, dV + \int_S \boldsymbol{\Phi}\, dS - \frac{d}{dt}\int_V \rho \boldsymbol{v}\, dV = \boldsymbol{0}$$

Define the vector $\boldsymbol{R}^I$:

$$\boldsymbol{R}^I = -\frac{d}{dt}\int_V \rho \boldsymbol{v}\, dV$$

This can be viewed as a force, $\boldsymbol{R}^I$ is called the inertia force. The balance equation can be formulated as:

$$\boldsymbol{R} + \boldsymbol{R}^I = \boldsymbol{0} \tag{8.1}$$

where $\boldsymbol{R}$ is the sum of external forces. Not that this holds for all times.

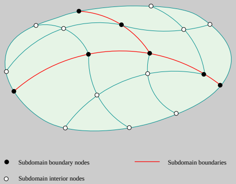
The equation is a *principle*, in static analysis we have static equilibrium if the sum of the forces are zero. The equation Equation 8.1 state that by adding inertia forces the forces are in equilibrium for all times.

Since this is a priciple, it can be used for a system, i.e. dynamic analysis of structures can be based on it.

The governing equations derived by considerint the external forces, inner forces, inertia forces and viscous forces for an arbitrary movement. For an arbitrary volume element:

$$\int_V \delta \boldsymbol{u}^T \boldsymbol{F}\, dV + \int_{S_t} \delta \boldsymbol{u}^T \boldsymbol{\Phi}\, dS = \int_V \delta \boldsymbol{\varepsilon}^T \boldsymbol{\sigma}\, dV + \int_V \delta \boldsymbol{u}^T \rho \ddot{\mathbf{u}}\, dV + \int_V \delta \boldsymbol{u}^T L(\cdot,\cdot,\cdot,\dot{\boldsymbol{u}})\, dV \tag{8.2}$$

Here $\delta \boldsymbol{u}$ and $\delta \boldsymbol{\varepsilon}$ are virtual displacements and virtual strains, $\boldsymbol{F}$ are volume forces, $\boldsymbol{\Phi}$ are surface tractions and ρ is the mass density. The damping term is not specified in detail here,



we only note that a fist derivative in time must be present. Damping will be briefely discussed below.

Remark 8.2 The integrals are over space coordinates, as for static analysis, and not over time. The element mehtods is used in space.

An alternative model is *Hamilton's principle*, is similar to *minimum potential energy* for static problem. In this case the integrals are also along the time axis.

The strong form is derived from (8.2). We neglect damping. Since $\delta \boldsymbol{u}$ is arbitrary we obtain

$$\rho \ddot{\boldsymbol{u}} - \nabla \cdot \boldsymbol{\sigma} = \boldsymbol{F} \quad \text{for all } \boldsymbol{x} \text{ and } t > 0 \tag{8.3}$$

In addition boundary conditions are derived and initial condition are specified.

8.3. An example

We neglect damping and external forces, and assume that the displacements are in the axial direction. This result in the wave equation

$$\rho A u_{,tt} - EA u_{,xx} = 0 \quad u(0,t) = u(\pi,t) = 0 \quad \text{for } t > 0, 0 < x < \pi \tag{8.4}$$

The initial conditions are

$$u(x,0) = \sin(x) \quad \text{and} \quad u_{,t}(x,0) = 0 \quad \text{for } 0 < x < \pi. \tag{8.5}$$

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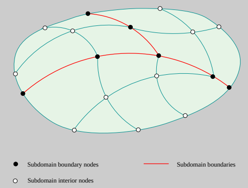

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Here A is the area of the cross section. Let $A = E = \rho = 1$ and assume that

$$u(x,t) = U(x)V(t) \tag{8.6}$$

Substitution into the wave equation result in

$$U\ddot{V} - U_{,xx}V = 0 \tag{8.7}$$

Division by U and V result in

$$\frac{\ddot{V}}{V} = \frac{U_{,xx}}{U} = -c^2. \tag{8.8}$$

since the left hand side depend on t and the right hand side depend on x. The constant c is positive. (Why?)

It is clear that

$$-U_{xx} = c^2 U \tag{8.9}$$

The constants c^2 satisfying the equation are the egenvalues of the negative of the second derivative, and the corresponding functions are eigenfunctions. Note that

$$U(x) = \sin(cx) \tag{8.10}$$

satisfies (8.10). If $c = k$, U also satisfies the boundary conditons. Thus the eigenvalues and the corresponding eigenfunctions are

$$(k^2, \sin(kx)) \quad \text{for } k = 1, 2, \ldots \tag{8.11}$$

Note that it infinitly many eigenvalues and eigefunctions, and that the eigenvalues are real, positive and grow towards infinity.

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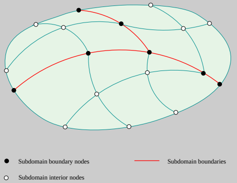

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Let $V(t) = \cos(t)$, it satifies the equation for V, in addition

$$u(x,t) = \cos(t)\sin(x) \tag{8.12}$$

satisfies the wave equation together with the initial and boundary conditions.

8.4. Separation of variables

In a dynamic model the displacement $\boldsymbol{u}$ is a function of space and time, $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, t)$. This is approximated using finite elements in space and degrees of fredom depending on time:

$$\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{N}(\boldsymbol{x})\boldsymbol{d}(t) \quad \dot{\boldsymbol{u}}(\boldsymbol{x},t) = \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{d}}(t) \quad \ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \boldsymbol{N}(\boldsymbol{x})\ddot{\boldsymbol{d}}(t)$$

Note, the basis functions, $\boldsymbol{N} = \boldsymbol{N}(\boldsymbol{x})$, depend on the space coordinates, $\boldsymbol{x}$. The nodal degrees of freedom, $\boldsymbol{d} = \boldsymbol{d}(t)$, are functions of time, t.

Using the finite elment method in space in Equation 8.2 result in:

$$\delta \boldsymbol{d}^T \left[\int_V \boldsymbol{B}^T \boldsymbol{\sigma}\, dV + \int_V \rho \boldsymbol{N}^T \boldsymbol{N}\, dV\, \ddot{\boldsymbol{d}} + \int_V [\ldots]\, dV\, \dot{\boldsymbol{d}} - \int_V \boldsymbol{N}^T \boldsymbol{F}\, dV - \int_{S_t} \boldsymbol{N}^T \boldsymbol{\Phi}\, dS \right] = 0 \tag{8.13}$$

Since the degrees of freedom are continuous functions of time, this is a semi-discretization.

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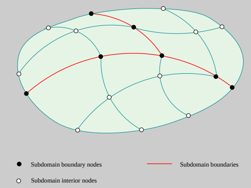

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Since $\delta \boldsymbol{d}$ in Equation 8.13 are arbitrary, Equation 8.13 take the form of a system of coupled, second order, ordinary differential equations:

$$\boldsymbol{M}\ddot{\boldsymbol{d}} + \boldsymbol{C}\dot{\boldsymbol{d}} + \boldsymbol{K}\boldsymbol{d} = \boldsymbol{R} \tag{8.14}$$

The additional matrices introduced in dynamic analysis are:

$$\boldsymbol{M} = \int_V \rho \boldsymbol{N}^T \boldsymbol{N}\, dV \tag{8.15}$$

the *mass matrix*, and

$$\boldsymbol{C} = \int_V [\quad]\, dV \tag{8.16}$$

the *demping matrix*. Some examples of the damping matrix is discussed below.

Before disussing the solution of the set of oridinary differential equations we make some comments on the mass matrix and the damping matrix.

8.5. The mass matrix

The computation of the mass matrix is similar to the computation of the stiffness matrix. The basis functions are the same and the assembly process is identical. The mass matrix is symmetric, positive definite, and sparse. In general the nonzero structure is identical to the stiffness matrix.

A mass matrix computed using the same integration rule as the stiffness matrix i called a *consistent mass matrix*. The term used to compte the mass matrix do not involve derivatives

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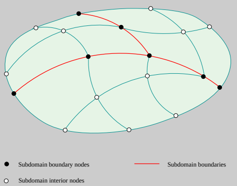

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of the functions. It turns out that with a suitable choice of integration rule it is possible to make the mass matrix diagonal.

Consider linear basis functions on a triangle and use an integration rule involving the nodal points. Then the element matrix becomes diagonal, thus the system mass matrix is also diagonal. This is called *reduced mass matrix*. A reduced mass matrix may also be obtained by adding off diagonal elements to the diagonal.

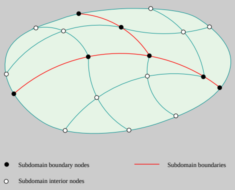
A diagonal mass matrix is more economical in computational time and storage. Note however that the properties and accuracy of integration methods may change. For further details see [Cook et al., 2002].

8.6. Rayleigh damping

Damping in constructions are not viscous but due to mechanisms that are not easy to model. It is customary in structural analysis to add the effect of damping without a good model.

Damping in structural analysis can be included using:

- *a damping model* based on the physical phenomenon
- *spectral damping* introduce viscous damping as a precentage of critical damping of the construction.



Rayleigh damping is frequently used. Here a the damping matrix is a linear combination of the global mass and stiffness matrices:

$$\boldsymbol{C} = \alpha \boldsymbol{K} + \beta \boldsymbol{M}$$

The mass matrix damp the low frequencies while the stiffness matrix damp the high frequencies. The relation between critical damping and the coefficients (α, β), is found from:

$$\xi = \frac{1}{2}\left(\alpha\omega + \frac{\beta}{\omega}\right)$$

The damping coefficients, (α, β), are found using two critical damping conditions at two different frequencies:

$$\alpha = \frac{2\left(\xi_2\omega_2 - \xi_1\omega_1\right)}{\omega_2^2 - \omega_1^2}$$
$$\beta = \frac{2\omega_1\omega_2\left(\xi_1\omega_2 - \xi_2\omega_1\right)}{\omega_2^2 - \omega_1^2}$$

The figure below show the contribution from each of the factores together with the total effect of damping as a function of frequency and critical damping:

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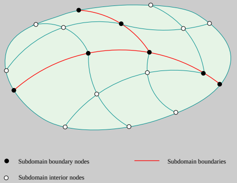

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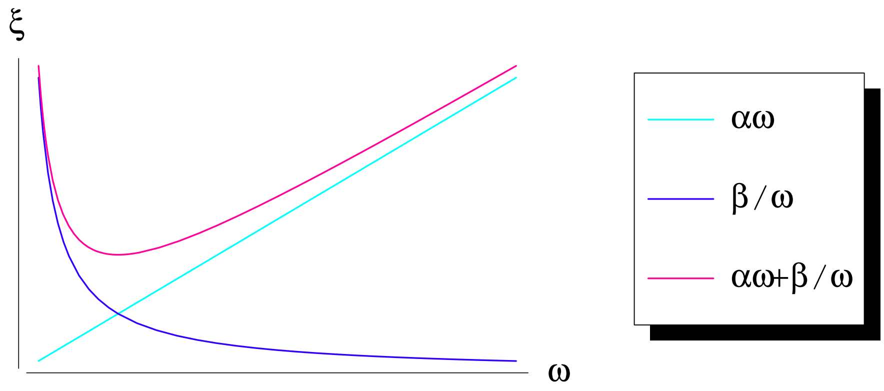

$\beta \boldsymbol{M}$ damp the lowest moedes, while $\alpha \boldsymbol{K}$ damp the higher modes.

8.7. Free vibrations

In free vibrations the goal is to find the eigenfrequencies of the construction. It is customary to neglect damping. This simplify the analysis and frequently damping do not change the eigenfrequencises significantly. The eigenfrequencies are found using an eigenvalue analysis.

The eigenvalue problem are a generalized eigevalue problem: Find the eigenvalue ω^2 and the eigenfunctions $\bar{\boldsymbol{D}}$ such that

$$\left(\boldsymbol{K} - \omega^2 \boldsymbol{M}\right) \bar{\boldsymbol{D}} = \boldsymbol{0}$$

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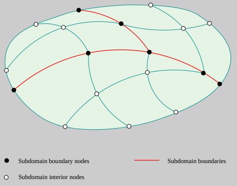

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Since K and M are symmetric we know that the eigenvalues are positive. Usualy a small number of the smallest eigenvalues, lowest frequencies, are computed.

The eigenvalues and vectors can be rearranged as

$$K\boldsymbol{\Phi} = M\boldsymbol{\Phi}\Omega \tag{8.17}$$

where the columns of $\boldsymbol{\Phi}$ are the eigenvectors

$$\boldsymbol{\Phi} = \begin{bmatrix} \bar{\boldsymbol{D}}_1 & \bar{\boldsymbol{D}}_2 & \dots & \bar{\boldsymbol{D}}_n \end{bmatrix}$$

and Ω is a diagonal matrix with the eigenvalue along the diagonal. The eigenvalues are orthogonal with respect to the mass matrix, thus

$$\boldsymbol{\Phi}^T M\boldsymbol{\Phi} = I. \tag{8.18}$$

The eigenvalues are linearly independant, thus there is a vector $\boldsymbol{z}$ such that

$$\boldsymbol{D} = \boldsymbol{\Phi}\boldsymbol{Z} \tag{8.19}$$

Using this in the system of differential equations

$$\boldsymbol{M}\ddot{\boldsymbol{D}} + \boldsymbol{K}\boldsymbol{D} = \boldsymbol{M}\boldsymbol{\Phi}\ddot{\boldsymbol{z}} + \boldsymbol{K}\boldsymbol{\Phi}\boldsymbol{z} = \boldsymbol{M}\boldsymbol{\Phi}\ddot{\boldsymbol{z}} + \boldsymbol{M}\boldsymbol{\Phi}\Omega\boldsymbol{z} \tag{8.20}$$

Multiplying from the rith with $\boldsymbol{\Phi}^T$ result in the system

$$\ddot{\boldsymbol{z}}_j + \omega_j^2 \boldsymbol{z}_j = 0 \tag{8.21}$$

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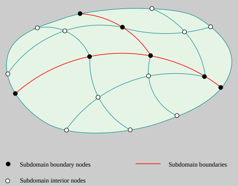

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for $j = 1, 2, \ldots$. This explains why ω_j are the eigenfrequencise of the system.

Can the eigenvalues and eigenfrequencies be computed accurately? How many can be computed to a reasonable accuracy?

If the element model represent the constructions volume and geometry sufficiently accurate and a consistent mass matrix is used, the computed natural frequencies are upper bounds for the exact natural frequencies. The order of convergence is given by

$$\|\omega - \omega_e\| = O(h^{2(p+1-m)})$$

where h measure the mess size, p is the order of the polynomial and m the highest derivative in the weak form.

8.8. Modal time integration

In a time history analysis the dynamical system (8.14) for the displacements $\boldsymbol{D}$, given the initial conditions $\boldsymbol{D}$ og $\dot{\boldsymbol{D}}$ at time $t = 0$. Here the loads are nozero and damping is frequently considered.

In a modal superpostion analysis the systemEquation 8.14 is transformed as for the free vibration system above. The vector $\boldsymbol{Z}$ is a vector of amplitude values, or generalized displacements. Arguing as above

$$\ddot{Z}_i + 2\xi\omega_i\dot{Z}_i + \omega_i^2 Z_i = Q_i \qquad \text{hvor} \qquad Q_i = \bar{\boldsymbol{D}}_i^T \boldsymbol{R} \tag{8.22}$$

This is a set of uncoupled*ordinary differential equations*, Q_i is a known fucntion of t.

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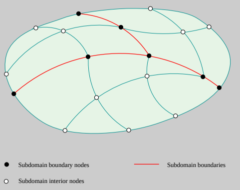

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For at det dynamiske problemet skal være komplett trenger vi initialverdier på $\dot{Z}_i$ og Z_i:

$$\boldsymbol{Z}|_{t=0} = \boldsymbol{\Phi}^T \boldsymbol{M}\boldsymbol{D}|_{t=0} \qquad \text{og} \qquad \dot{\boldsymbol{Z}}|_{t=0} = \boldsymbol{\Phi}^T \boldsymbol{M}\dot{\boldsymbol{D}}|_{t=0}$$

Når vi har funnet løsningen for $\boldsymbol{Z}$ kan nodeforskyvningene finnes fra Equation 8.19.

Det finnes mange måter å løse de dynamisk ligningene i Equation 8.22 på. Dette kommer vi tilbake til i Kapittel 9.

I noen tilfeller kan vi finne en eksakt løsning for $Z_i(t)$. Dersom Q_i er stykkevis lineær så kan den eksakte løsningen for Z_i og $\dot{Z}_i$ finnes som en kombinasjon av $e^{-\xi\omega t}\sin\omega t$ og $e^{-\xi\omega t}\cos\omega t$.

Initial verdi problem: Dersom vi kun har initialverdier og ingen last kan løsningen av

$$4\,z(t) + \frac{2\,z'(t)}{5} + z''(t) = 0, \qquad z(0) = 1, \qquad z'(0) = 0$$

finnes eksakt

$$z(t) = \frac{33\,\cos(\frac{3\sqrt{11}\,t}{5}) + \sqrt{11}\,\sin(\frac{3\sqrt{11}\,t}{5})}{33\,e^{\frac{t}{5}}}$$

Løsningen er skissert i figuren under:

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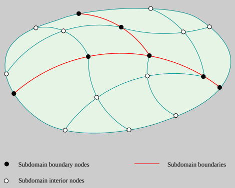

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8.9. Bjelke problemet

I dette eksempelet skal vi igjen se på bjelkeproblemet. Vi skal etablere uttrykkene for *stivhets-matrisen* og *massematrisen* slik at vi kan foreta en *egenfrekvensanalyse*. Stivhetsmatrisen kjenner vi fra tidligere:

$$\boldsymbol{k} = \frac{EI}{\ell^3} \begin{bmatrix} 12 & -6\,\ell & -12 & -6\,\ell \\ -6\,\ell & 4\,\ell^2 & 6\,\ell & 2\,\ell^2 \\ -12 & 6\,\ell & 12 & 6\,\ell \\ -6\,\ell & 2\,\ell^2 & 6\,\ell & 4\,\ell^2 \end{bmatrix}$$

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Masse matrisen etableres fra uttrykket:

$$\boldsymbol{m} = \int_\ell \bar{\rho} \boldsymbol{N}^T \boldsymbol{N} \, dx$$

hvor $\bar{\rho}$ er masse per lengdeenhet. Dette gir oss følgende uttrykk dersom en benytter de kjente bjelkefunksjonene:

$$\boldsymbol{m} = \frac{\rho A \ell}{420} \begin{bmatrix} 156 & -22\,\ell & 54 & 13\,\ell \\ -22\,\ell & 4\,\ell^2 & -13\,\ell & -3\,\ell^2 \\ 54 & -13\,\ell & 156 & 22\,\ell \\ 13\,\ell & -3\,\ell^2 & 22\,\ell & 4\,\ell^2 \end{bmatrix}$$

Vi skal nå finne egenfrekvensene og egensvingeformene til en fritt opplagt bjelke. Antall ligninger blir nå redusert til to, og det fri svinge problemet er redusert til:

$$\left(\frac{EI}{\ell} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} - \omega^2 \frac{\rho A \ell^3}{420} \begin{bmatrix} 4 & -3 \\ -3 & 4 \end{bmatrix} \right) \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Dette systemet har to egenverdier og to tilhørende egenvektorer:

$$\begin{Bmatrix} \omega_1^2 \\ \omega_2^2 \end{Bmatrix} = \frac{EI}{\rho A \ell^4} \begin{Bmatrix} 120 \\ 2520 \end{Bmatrix}$$

Egenvektorene er lik de vi fant for knekningsproblemet:

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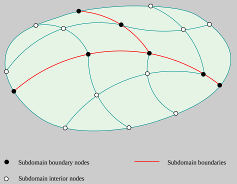

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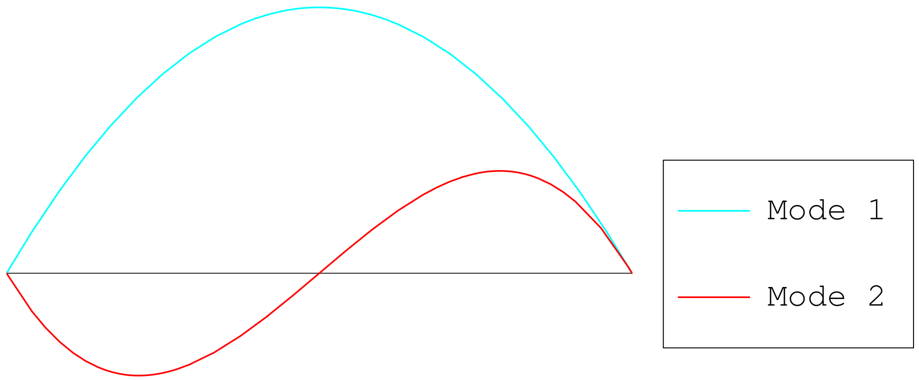

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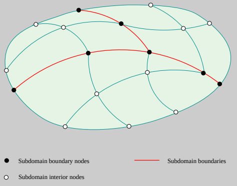

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8.10. Fjærsystem (MATRIX27 & MASS21)

Problem: Betrakt det udempede systemet hvor den dynamiske likevektsligningen er gitt ved

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{r}_1 \\ \ddot{r}_2 \end{bmatrix} + \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

(se også fig. 4.2 s. 102 i [Bergan et al., 1985][1])
Bruk ANSYS til å beregne systemets to egenverdier. Kontroller disse med håndberegninger.

Løsning:

```
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/TITLE, Modal egenverdianalyse av et fjærsystem

/PREP7
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ET,2,MASS21,,,4             ! og massmatrise
ET,3,MASS21,,,4

r,1,4,,,,, $ rmore,-4       ! Fjær 1
$ rmore $ rmore
$ rmore $ rmore
$ rmore $ rmore
$ rmore $ rmore,,,,4
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[1] P. G. Bergan, P. K. Larsen, and E. Mollestad. *Svingning av Konstruksjoner*. Tapir Forlag, Trondheim, 1985.

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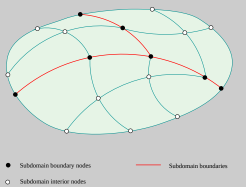

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r,2,2,,,,, $ rmore,-2       ! Fjær 2
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$ rmore $ rmore
$ rmore $ rmore
$ rmore $ rmore,,,,4

r,3,2 $ r,4,1

n,1 $ n,2,1 $ n,3,2         ! 3 knutepunkt
type,1 $ real,1 $ e,1,2     ! Fjærene
type,1 $ real,2 $ e,2,3
type,2 $ real,3 $ e,2       ! Massene
type,3 $ real,4 $ e,3
fini

/solu
antype,modal                ! Modalanalyse
modopt,reduc,2              ! 2 første egenfrekvensene/-formene
d,1,all $ d,2,uy,,,,,rotx,roty,rotz,uz
d,3,uy,,,,,rotx,roty,rotz,uz
m,2,ux,3,1
!f,2,fx,1
solve
save
fini
/solu
expass,on
mxpand,4                    ! 'expand' de 4 modene
solve
fini
/post1
set,list
fini
exit
```


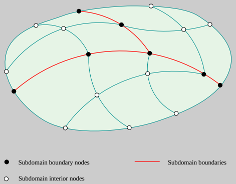

Svar/kommentarer: De to egenverdiene er hhv. $\omega_1 = \sqrt{2}$ og $\omega_2 = \sqrt{5}$. ANSYS gir også de eksakte analytiske løsningene ($f_1 = \frac{2\pi}{\omega_1} = 0.22508$ og $f_2 = \frac{2\pi}{\omega_2} = 0.35588$).

Reduce metoden brukes til å løse for egenverdiene.

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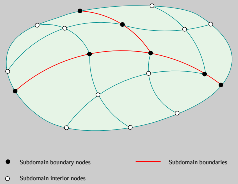

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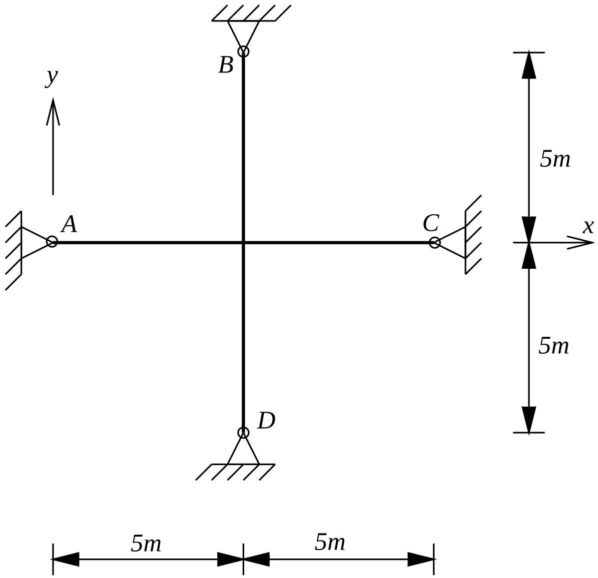
8.11. Ramme (BEAM3)



Problem: Figuren viser en ramme som er fastinnspent på endene A, B, C og D. Alle elementer har et kvadratisk tverrsnitt $(0.125 \times 0.125 m^2)$ og materialloven er gitt ved $E = 2.07 \times 10^{11} \frac{N}{m^4}$ og $\rho = 8000 \frac{kg}{m^3}$.
Foreta en modalanalyse med ANSYS for å bestemme de fire første naturlige egenfrekvenser og tilhørende svingeformer.

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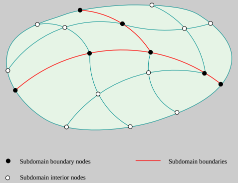

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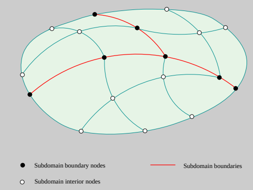

Plot svingeformene i samme figur.

Løsning:

```
/batch,list
/filnam,ex432
/title, Modalanalyse av et rammesystem

/prep7
et,1,beam3,,,,,,1             ! Bjelkeelement
r,1,0.015625,2.04e-5,0.125    ! A,I og høyde
mp,ex,1,2.07e11               ! E
mp,dens,1,8e3                 ! Massetetthet
k,1                           ! Keypoints
k,2,10
k,3,5,5
k,4,5,-5
l,1,2
l,3,4
lesize,all,,,10               ! 10 elementer pr. bjelke
lmesh,all
nummrg,all
eplo
fini

/solu
antype,modal                  ! Modalanalyse
modopt,subsp,4                ! 4 første modene
subopt,10,6
d,node(0,0,0),ux,,,,,uy       ! Grensebetingelser
d,node(10,0,0),ux,,,,,uy
d,node(5,5,0),ux,,,,,uy
d,node(5,-5,0),ux,,,,,uy
solve
```

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```
fini

/solu
expass,on
mxpand,4                    ! 'expand' de 4 modene
solve
fini

/post1                      ! For plott av svingeformene
/show,ex432pl
/window,1,ltop
/window,2,rtop
/window,3,lbot
/window,4,rbot
/edge,all,1
/triad,off
!/show,me3plt
/plopts,frame,on
/plopts,info,off
/window,all,off
*do,i,1,4
  /window,i,on
  set,1,i
  pldisp,1
  /noerase
  /window,i,off
*enddo
/reset
fini
/exit
```

Svar/kommentarer: Eksakte egenfrekvenser for de 4 første svingeformene er 11.336, 17.687, 17.687 og 17.715. Mens ANSYS gir følgende verdier (i figuren) som er ca. 2% overvurdering av egenverdiene.

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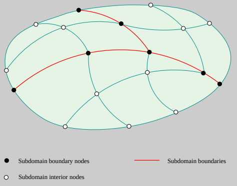

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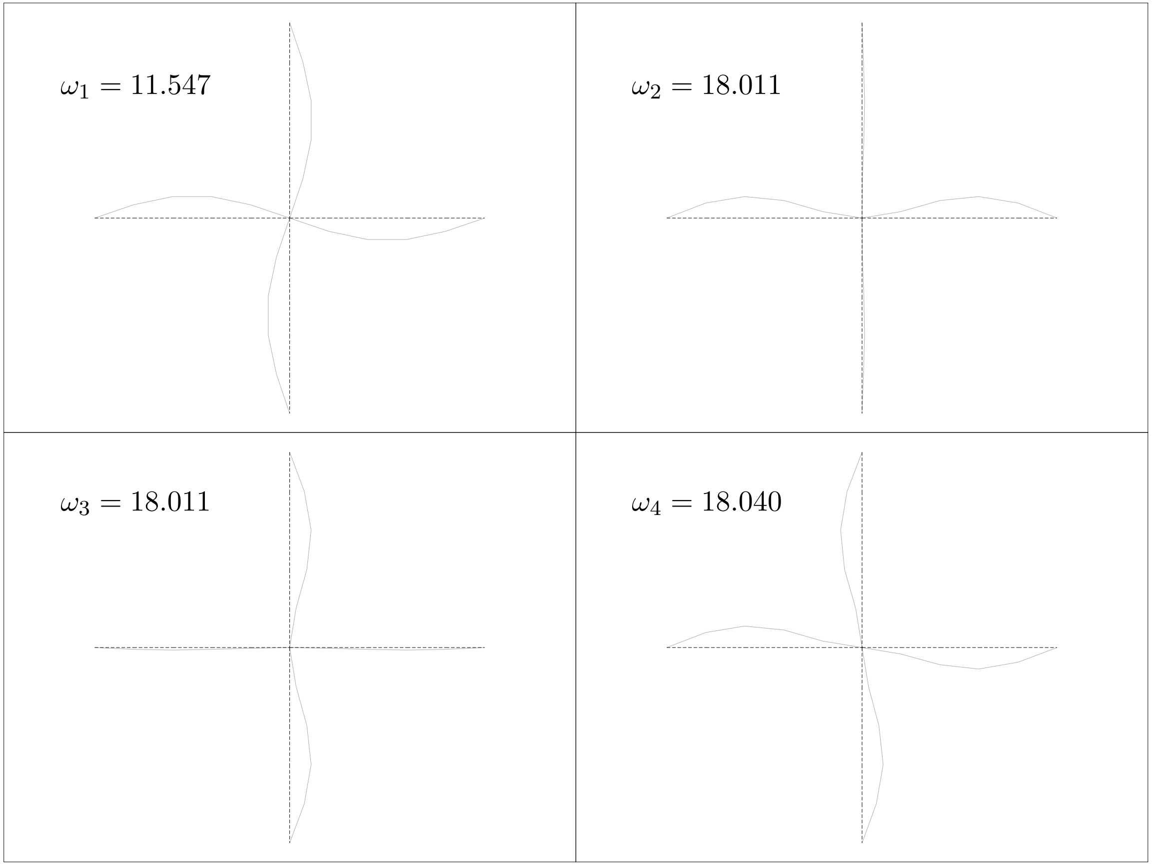
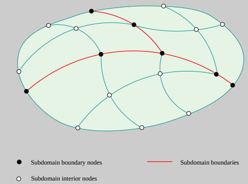
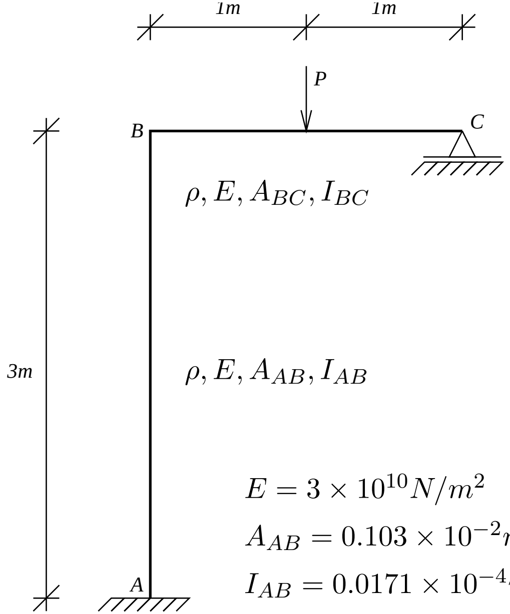


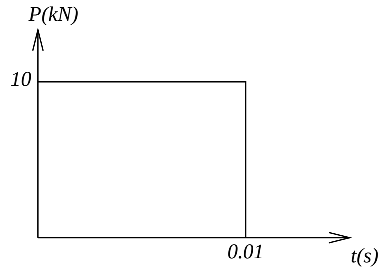
Svingeformer for et rammesystem



Øving 8.1

Rammen i figuren skal modelleres med BEAM3 elementer. AB og BC skal deles inn i to elementer hver.





$$E = 3 \times 10^{10} N/m^2 \qquad A_{BC} = 0.0764 \times 10^{-2} m^2$$

$$A_{AB} = 0.103 \times 10^{-2} m^2 \qquad I_{BC} = 0.00801 \times 10^{-4} m^4$$

$$I_{AB} = 0.0171 \times 10^{-4} m^4 \qquad \rho = 650\, kg/m^3$$

a) Foreta en modalanalyse av rammen. Resultater for alle svingeformer (lik antall frihetsgrader) skal lagres, men utskrift av kun de første 4 skal vises i besvarelsen.

b) Foreta modalsuperposisjonsanalyser ved bruk av:

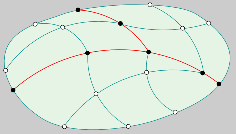
i) alle svingeformer fra a).

ii) alle svingeformer fra a) untatt de siste to.

I hver av analysene skal punkt B ha en initiell horisontal hastighet på $1m/s$ og en initiell forskyvning på $0.1m$.

Responsen til tiden $t = 0.03s$ skal bestemmes for den horisontale forskyvningen av punkt B.

Sett initielle hastigheter og initielle forskyvninger i oppgaven til $u(t = 0) = \dot{u}(t = 0) = 0$.



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A. References

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[Cook et al., 2002] Cook, R. D., Malkus, D. S., Plesha, M. E., and Witt, R. J. (2002). *Concepts and Applications of Finite Element Analysis*. Number ISBN: 0-471-35605-0. John Wiley & Sons, Inc., 4th edition.

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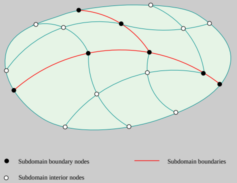

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