Chapter: 9

MEK4560
The Finite Element Method in Solid Mechanics II

(March 26, 2008)

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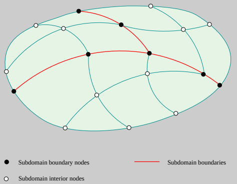

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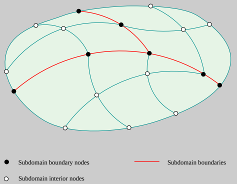

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9. Dynamic analysis, part II

This chapter continue the discussion of *dynamic analysis*. In Chapter Chapter 8 free vibration and modal superposition was discussed. The topic in this Chapter is *direct time integration* of the set of *ordinary differential equation* (ODE) arising from using the Finite element method for the spatial variation of the differential equations.

In the following we discuss the following topics

- *implicit* and *explicit* time integration,
- *stability*
- *consistency*
- *convergence*
- *accuracy*

The discussion is limited to linear equations. Time integration is discussed in the textbook [Cook et al., 2002][2] Chapter 11, in particular from 11.11.

[2] R. D. Cook, D. S. Malkus, M. E. Plesha, and R. J. Witt. *Concepts and Applications of Finite Element Analysis*. Number ISBN: 0-471-35605-0. John Wiley & Sons, Inc., 4th edition, October 2002.

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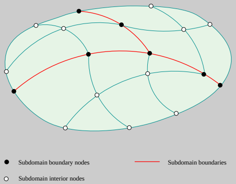

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9.1. Semi discretization

In Finite element methods for dynamic problems it is common to do separation of variables:

$$\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{N}(\boldsymbol{x})\boldsymbol{d}(t) \qquad \text{thus} \qquad \dot{\boldsymbol{u}}(\boldsymbol{x},t) = \boldsymbol{N}(\boldsymbol{x})\dot{\boldsymbol{d}}(t),\ \ddot{\boldsymbol{u}}(\boldsymbol{x},t) = \boldsymbol{N}(\boldsymbol{x})\ddot{\boldsymbol{d}}(t)$$

This result in the set of second order ordinary differential equations

$$\boldsymbol{M}\ddot{\boldsymbol{D}} + \boldsymbol{C}\dot{\boldsymbol{D}} + \boldsymbol{K}\boldsymbol{D} = \boldsymbol{R}(t)$$

and the initial and boundary conditions:

$$\begin{aligned} \boldsymbol{D}(0) &= \boldsymbol{D}_0 \\ \dot{\boldsymbol{D}}(0) &= \boldsymbol{V}_0 \\ \boldsymbol{D}(t) &= \boldsymbol{D}_d(t) \qquad \text{on } S_d \end{aligned}$$

Time integration of this system is used for dynamic analysis of constructions.

Remark 9.1 The Finite Element method can also be used in time. A continuous in time method would couple the equations in time and result in much larger linear systems. An alternative is to use discontinuous in time methods, which are similar to time stepping methods..

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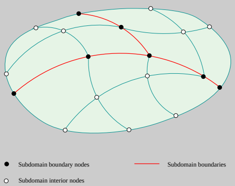

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9.2. Time integration

Model problem: In order to introduce implicit and explicit time integration, stability, consistency, convergence and accuracy a model problem is introduced:

$$\dot{d} + \lambda d = r \qquad \text{with initial condition} \qquad d(0) = d_0$$

This is the simplest ordinary differential equation, but still useful in order to understand the basics of numerical methods. Recall also that modal superposition reduced the coupled dynamical system to a set of uncoupled scalar equations. Se also the figure below:

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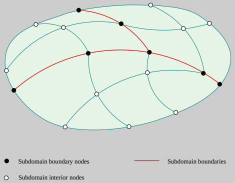

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$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\begin{gathered} \boldsymbol{M}\ddot{\boldsymbol{D}} + \boldsymbol{K}\boldsymbol{D} = \boldsymbol{R}(t) \\ \boldsymbol{D}(0) = \boldsymbol{D}_0 \end{gathered}
\arrow[r, "\textit{Time discretization}"] \arrow[d, "\textit{Modal decomposition}"']
&
\begin{gathered} \boldsymbol{M}\ddot{\boldsymbol{D}}_{n+1} + \boldsymbol{K}\boldsymbol{D}_{n+1} = \boldsymbol{R}(t_{n+1}) \\ \boldsymbol{D}_{n+1} = \boldsymbol{D}_n + \Delta t \ddot{\boldsymbol{D}}_{n+\alpha} \\ \ddot{\boldsymbol{D}}_{n+\alpha} = (1-\alpha)\dot{\boldsymbol{D}}_n + \alpha \dot{\boldsymbol{D}}_{n+1} \end{gathered}
\arrow[d, "\textit{Modal decomposition}"']
\\
\begin{gathered} \ddot{d} + \lambda d = r \\ d(0) = d_0 \end{gathered}
\arrow[r, "\textit{Time discretization}"]
&
\begin{gathered} \dot{d}_{n+1} + \lambda d_{n+1} = r(t) \\ d_{n+1} = d_n + \Delta t \dot{d}_{n+\alpha} \\ \dot{d}_{n+1} = (1-\alpha)\dot{d}_n + \alpha \dot{d}_{n+1} \end{gathered}
\end{tikzcd}
$$

In addition, second order equations can be reduced to first order equations by a change of variables:

$$\boldsymbol{V} = \dot{\boldsymbol{D}} \quad \text{such that} \quad \boldsymbol{M}\dot{\boldsymbol{V}} + \boldsymbol{C}\boldsymbol{V} + \boldsymbol{K}\boldsymbol{D} = \boldsymbol{R}(t)$$

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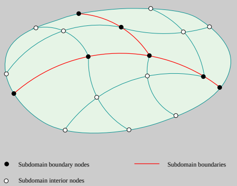


9.3. Stability

In order to introduce the concept of stability we consider the *homogeneous* model equation:

$$\dot{d} + \lambda d = 0 \tag{9.1}$$

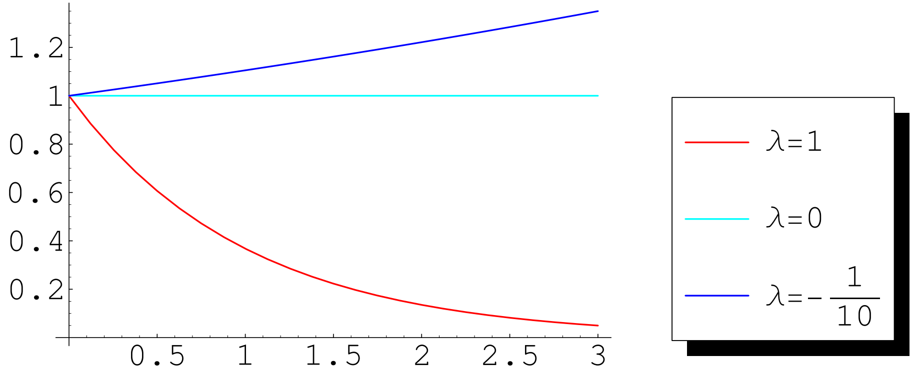
Let Δt be a positive number, $t_n = n\Delta t$, and consider the first order equation with initial condition $d(t_n)$ given at time t_n. The solution at time t_{n+1} is

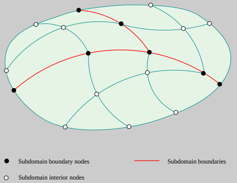
$$d(t_{n+1}) = e^{-\lambda(t_{n+1}-t_n)}\, d(t_n)$$

Thus:

$$\begin{aligned} |d(t_{n+1})| &< |d(t_n)| \qquad \lambda > 0 \\ |d(t_{n+1})| &= |d(t_n)| \qquad \lambda = 0 \end{aligned}$$

The exponential function for different values of λ is shown below:





Consider the Taylor series

$$d(t_{n+1}) = d(t_n) + \Delta t\, \dot{d}(t_n) + \Big(\frac{\Delta t^2}{2}\, \ddot{d}(t_n) + \cdots\Big) \tag{9.2}$$

The time stepping method is derived from the terms to the left of the parenthesis

$$d_{n+1} = d_n + \Delta t\, \dot{d}(t_n) \quad \text{where} \quad \dot{d}(t_n) + \lambda d_n = 0 \tag{9.3}$$

The value of d at a new time step is computed directly using known quantities, an *explicit* method. The error introduced by dropping the remaining terms in the Taylor series are the *local truncation error*. Here it is proportional to Δt^2. This method is called the *forward Euler* method.

Rearranging the formula result in

$$d_{n+1} = \big(1 - \Delta t\ \lambda\big)\ d_n \tag{9.4}$$

We would like

$$|1 - \Delta t\ \lambda| \leq 1 \tag{9.5}$$

i.e.

$$-1 \leq 1 - \Delta t\ \lambda \leq 1. \tag{9.6}$$

The right inequality is satisfied for all Δt, in order to satisfy the left

$$\Delta t \leq \frac{2}{\lambda} \tag{9.7}$$

Thus, for the method to be *stable* the time step must be sufficiently small, i.e. the method is *conditionally stable.*

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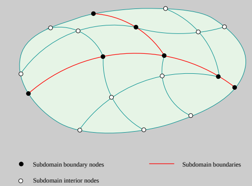

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An alternative method is to evaluate the Taylor series at the time t_{n+1}

$$d(t_{n+1} - \Delta t) = d(t_{n+1}) - \Delta t\ \dot{d}(t_{n+1})\big(+\frac{\Delta t^2}{2}\ \ddot{d}(t_{n+1}) + \cdots\big) \tag{9.8}$$

This result in the time stepping method

$$d(t_{n+1}) = d(t_n) + \Delta t\ \dot{d}(t_{n+1}) \quad \text{where} \quad \dot{d}(t_{n+1}) + \lambda\ d_{n+1} = 0 \tag{9.9}$$

This is an *implicit* method, in general an equation must be solved in order to compute d at the new time. The method is called the *backward Euler method.* As for the forward Euler method the local truncation error is of order Δt^2. Moreover,

$$\big(1 + \Delta t\ \lambda\big)d(t_{n+1}) = d(t_n). \tag{9.10}$$

Thus $|d(t_{n+1})| \leq |d(t_n)|$ for all Δt and the method is *unconditionally stable.*

It is also possible to evaluate the Taylor series at the point $t_{n+\alpha} = t_n + \alpha\Delta t$ for $\alpha \in [0,1]$, i.e.

$$d(t_{n+1}) = d(t_{n+\alpha}) + (1-\alpha)\Delta t\dot{d}(t_{n+\alpha}) + \frac{1}{2}(1-\alpha)^2\Delta t^2\ddot{d}(t_{n+\alpha}) + O(\Delta t^3) \tag{9.11}$$

and

$$d(t_n) = d(t_{n+\alpha}) - \alpha\Delta t\dot{d}(t_{n+\alpha}) + \frac{1}{2}\alpha^2\Delta t^2\ddot{d}(t_{n+\alpha}) + O(\Delta t^3) \tag{9.12}$$

Subtracting the second equation from the first result in

$$d_{n+1} = d_n + \Delta t\dot{d}(t_{n+\alpha}) + \frac{1}{2}(1-2\alpha)\Delta t^2\ddot{d}(t_{n+\alpha}) + O(\Delta t^3) \tag{9.13}$$

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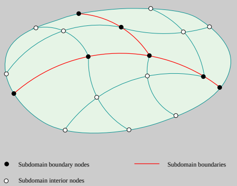

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The local truncation error are proportional to Δt^2, except when $\alpha = 1/2$ when it is proportional to Δt^3. The time stepping method becomes

$$
\begin{aligned}
& d_{n+1} = d_n + \Delta t \dot{d}(t_{n+\alpha}) \\
& \qquad \text{where} \quad \dot{d}(t_{n+\alpha}) = (1-\alpha)\dot{d}(t_n) + \alpha \dot{d}(t_{n+1}) \\
& \qquad \text{and} \quad \dot{d}(t_n) + \lambda d_n = 0
\end{aligned}
\tag{9.14}
$$

Rearranging result in

$$
(1 + \Delta t \alpha \lambda) d_{n+1} = \big(1 - \Delta t \lambda (1-\alpha)\big)\, d_n \tag{9.15}
$$

Different choices of the parameter α result in methods with different properties. The most common choices of α are summarized below:

α	Method	Type
0	Forward Euler	Explicit
$\frac{1}{2}$	The midpoint rule	Implicit
1	Backward Euler	Implicit

We have introduced the terms *implicit* and *explicit*:

Explicit	new d depends on old, known, quantities.
Implicit	new d depends on new values of d.

We also comment on the stability properties of the α method. Since $1 + \Delta t \alpha \lambda > 0$ for all Δt

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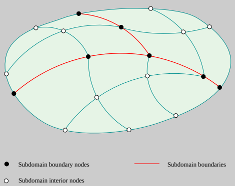

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and α it follows that

$$d_{n+1} = \frac{1-\Delta t(1-\alpha)\lambda}{1+\Delta t\alpha\lambda} d_n = A d_n \tag{9.16}$$

The method is stable for Δt and α satisfying $|A| \leq 1$, i.e.

$$-1 \leq \frac{1-\Delta t(1-\alpha)\lambda}{1+\Delta t\alpha\lambda} \leq 1$$

The right inequality is always satisfied, the first is always satisfied if $\alpha \geq \frac{1}{2}$. For $\alpha < \frac{1}{2}$ it follows that

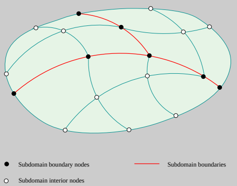
$$\lambda \Delta t \leq \frac{2}{1-2\alpha}$$

For a given α this restrict the time step. Note that a large λ implies a small time step in order to obtain a stable time stepping method.

Stability: Some method are stable for all time steps wile other methods impose restrictions on the time step:

- *Conditionally stable:* There are restrictions on Δt.
- *Unconditionally stable:* No restrictions on Δt. (Due to stability, accuracy will limit the size of the time step.)

Note that for a system conditionally stable for all modes.



Remark 9.2 For a second order problem the least interesting frequencies will put the strictest restrictions on Δt.

For this reason implicit methods are often used in structural analysis.

Remark 9.3 For wave propagation problems explicit methods is often used since a short time interval and high frequency modes is of interest. Combined with a diagonal mass matrix the computational complexity is reasonable even if a lot of time steps is required.

9.4. Convergence

Let $\tau(t)$ denote the local truncation error.

Teorem 9.1 *If*

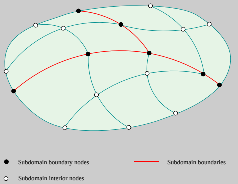
1. $|A| \le 1$, *stability*
2. $|\tau(t)| \le c\Delta t^k$, $t \in [0, T]$ *and* $k > 0$, *consistency*

is satisfied, then $e(t_n) \to 0$ *when* $\Delta t \to 0$. *Here*

$$e(t_n) = d(t_n) - d_n$$

This is the *Lax equivalence theorem*:

$$\textit{consistency} + \textit{stability} \to \textit{convergence}$$



9.5. Structural analysis

The system of differential equations are

$$\boldsymbol{M}\ddot{\boldsymbol{D}} + \boldsymbol{C}\dot{\boldsymbol{D}} + \boldsymbol{K}\boldsymbol{D} = \boldsymbol{R}(t)$$

with initial conditions

$$\boldsymbol{D}(0) = \boldsymbol{D}_0 \qquad \text{and} \qquad \dot{\boldsymbol{D}}(0) = \boldsymbol{V}_0$$

The following (family of) time stepping algorithm(s) is introduced

$$\begin{aligned}
\boldsymbol{M}\boldsymbol{A}_n + \boldsymbol{C}\boldsymbol{V}_n + \boldsymbol{K}\boldsymbol{D}_n &= \boldsymbol{R}_n \\
\boldsymbol{D}_{n+1} &= \boldsymbol{D}_n + \Delta t \boldsymbol{V}_n + \frac{1}{2}\Delta t^2 \left[(1-2\beta)\boldsymbol{A}_n + 2\beta \boldsymbol{A}_{n+1}\right] \\
\boldsymbol{V}_{n+1} &= \boldsymbol{V}_n + \Delta t \left[(1-\gamma)\boldsymbol{A}_n + \gamma \boldsymbol{A}_{n+1}\right]
\end{aligned}$$

This is Newmarks formulas. Two parameters are introduced, β and γ. The can be chosen from accuracy and/or stability requirements. Note that the equations for $\boldsymbol{D}_{n+1}$ and $\boldsymbol{V}_{n+1}$ are derived from Taylor series.

Stability: The method is unconditionally stable if

$$2\beta \geq \gamma \geq \frac{1}{2}$$

It is conditionally stable if

$$\gamma \geq \frac{1}{2}, \qquad \beta \leq \frac{1}{2}\gamma \qquad \text{and} \qquad \omega \Delta t \leq \Omega_{\text{critical}}$$

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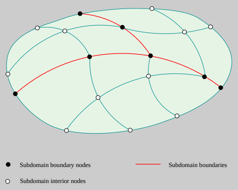

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where

$$\Omega_{\text{critical}} = \frac{\xi(\gamma - \frac{1}{2}) + [\frac{1}{2}\gamma - \beta + \xi^2(\gamma - \frac{1}{2})^2]^{\frac{1}{2}}}{(\frac{1}{2}\gamma - \beta)}$$

Stability must be satisfied for all modes. Note that for $\gamma = \frac{1}{2}$ damping do not influence on stability.

Convergence: We have second order convergence, $k = 2$, if $\gamma = \frac{1}{2}$ and first oder convergence, $k = 1$, otherwise.

Accuracy: Consider a second order equation modeling a damped one degree of freedom system,

$$\ddot{d} + 2\xi\omega\dot{d} + \omega^2 d = 0, \qquad d(0) = d_0 \qquad \text{and} \qquad \dot{d}(0) = v_0,$$

The solution is given by

$$d(t) = e^{-\xi\omega\, t}\,(d_0 \cos\omega_d\, t + c \sin\omega_d\, t)$$

where

$$c = \frac{v_0 + \xi\omega d_0}{\omega_d} \qquad \text{and} \qquad \omega_d = \sqrt{1 - \xi^2}\,\omega$$

Here ξ damping relative to critical damping while $\omega = \frac{2\pi}{T}$ is the undamped eigenfrequency. The corresponding values for the discrete problem is denoted

$$\bar{\xi} \qquad \text{and} \qquad \bar{T}$$

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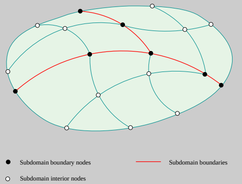

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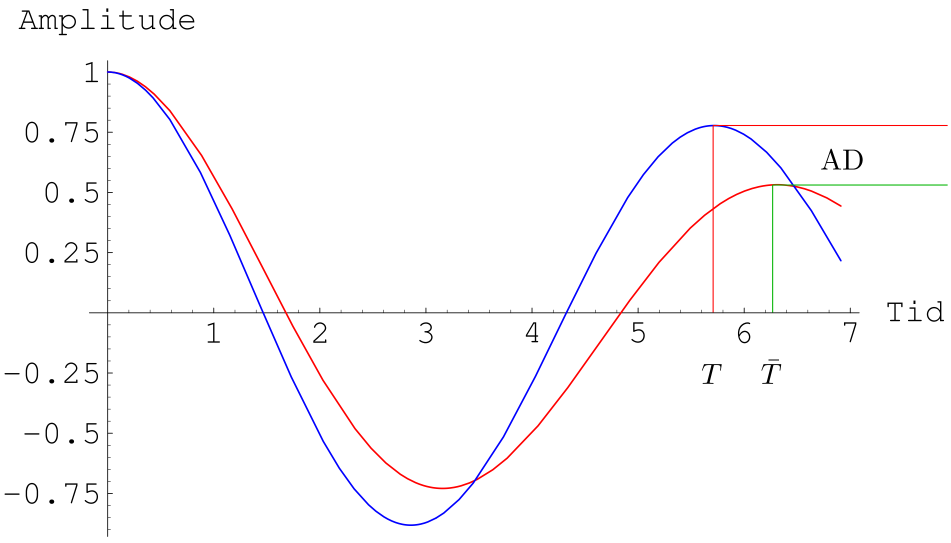
For *Newmark's* method

$$\bar{\xi} = \xi + \frac{1}{2}\left(\gamma - \frac{1}{2}\right)\left(\Omega + O(\Omega^2)\right) \qquad \text{and} \qquad \frac{\bar{T} - T}{T} = O(\Omega^2)$$

$\Omega = \omega \Delta t$. The reduction in amplitude can also be analyzed (AD = *amplitude decay*). It turns out that

$$\text{AD} \approx 2\pi\bar{\xi}$$



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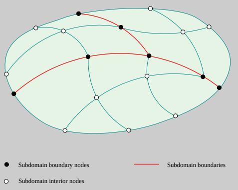

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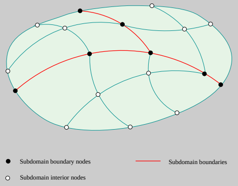
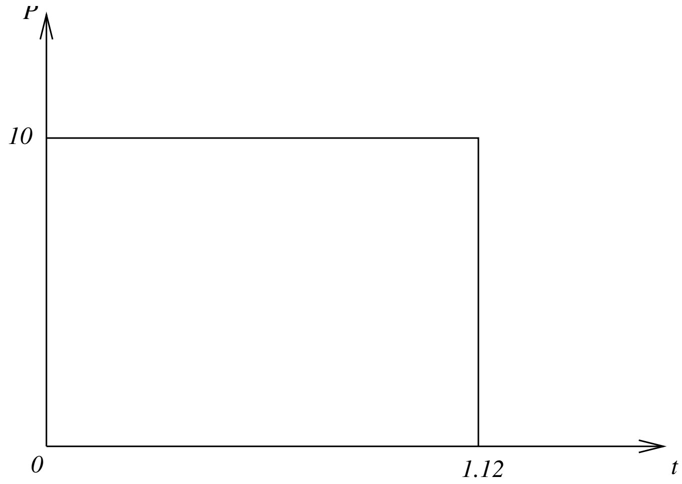


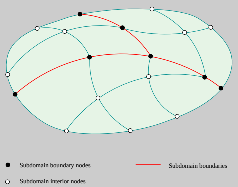
9.6. Dempet fjærsystem (`MATRIX27 & MASS21`)

Betrakt systemet i Kapittel 8.9 igjen, men denne gang med dempning. Dempningen er gitt ved $\xi_1 = 0.01$ og $\xi_2 = 0.005$. Lasten $P(t)$ $(=R_2)$ virker på systemet som vist i figuren nedenfor.



Problem: Bruk ANSYS til å beregne og plotte responsen (r_1 og r_2) ved $\Delta t, 2\Delta t, \ldots 60\Delta t$ hvor $\Delta t = 0.14$.

Løsning:




```
/BATCH,LIST
/FILNAM,ex482
/TITLE, Full transientanalyse av et dempet fjaersystem

/PREP7
ET,1,MATRIX27,,,4           ! Element med symmetrisk stivhets-
ET,2,MASS21,,,4             ! og massene
ET,3,MASS21,,,4
R,1,4,,,,, $ RMORE,-4       ! Fjær 1
$ RMORE $ RMORE
$ RMORE $ RMORE
$ RMORE $ RMORE
$ RMORE $ RMORE,,,,4
R,2,2,,,,, $ RMORE,-2       ! Fjær 2
$ RMORE $ RMORE
$ RMORE $ RMORE
$ RMORE $ RMORE
$ RMORE $ RMORE,,,,4
R,3,2 $ R,4,1
N,1 $ N,2,1 $ N,3,2         ! 3 kn. pkt.
TYPE,1 $ REAL,1 $ E,1,2     ! Fjærene
TYPE,1 $ REAL,2 $ E,2,3
TYPE,2 $ REAL,3 $ E,2       ! Massene
TYPE,3 $ REAL,4 $ E,3
FINI
/SOLU
ANTYPE,TRANS                ! Transient analyse
TRNOPT,FULL                 ! Full transient analyse
D,1,ALL $ D,2,UY $ D,3,UY
TINTP,,0.25,0.5             ! Numerisk integrasjonsparametre
F1=12.896 $ F2=101.94       ! Regn ut Rayleigh demnings koeffisientene
PI=ACOS(-1)                 ! pi
*DIM,COEFF,,2,2
*DIM,RHS,,2,1 $ *DIM,X,,2,1
COEFF(1,1)=1/(4*PI*F1),1/(4*PI*F2)
COEFF(1,2)=PI*F1,PI*F2
RHS(1,1)=0.01,0.005
```

```
*MOPER,X(1,1),COEFF(1,1),SOLVE,RHS(1,1)
ALPHAD,X(1) $ BETAD,X(2)     ! Masse- og stivhetsdempningskoeffisient
DELTIM,0.14                  ! Tidsdiskretisering
KBC,1                        ! Konstant last
OUTRES,ALL,ALL
TIME,1.12
F,3,FX,10
LSWRITE
FDELE,ALL,FX
TIME,8.4
LSWRITE
SAVE
LSSOLVE,1,2
FINI

/POST26
FILE,,RST
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NSOL,3,3,U,X,UX3
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FINI
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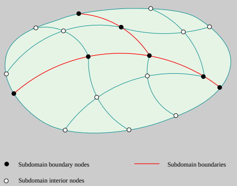

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Svar/kommentarer: Rayleighdempning som er uttrykt ved

$$\boldsymbol{C} = \alpha \boldsymbol{K} + \beta \boldsymbol{M} \qquad \text{hvor} \qquad \alpha = \frac{2\left(\xi_2\omega_2 - \xi_1\omega_1\right)}{\omega_2^2 - \omega_1^2}$$

$$\beta = \frac{2\omega_1\omega_2\left(\xi_1\omega_2 - \xi_2\omega_1\right)}{\omega_2^2 - \omega_1^2}$$

er benyttet her. α og β bestemmes ved bruk av ξ_1 og ξ_2 ved to forskjellige frekvenser (her brukes de to egenfrekvensene) - dette gjøres ved å løse et 2×2 matrisesystem. Et alternativ ville være å bruke kommandoen `MDAMP` og unngå beregningen av α og β.

Figuren nedenfor viser tydelig hvordan responsen til systemet dempes i løpet av kort tid.

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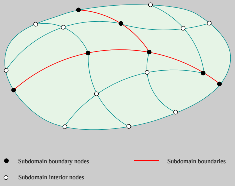

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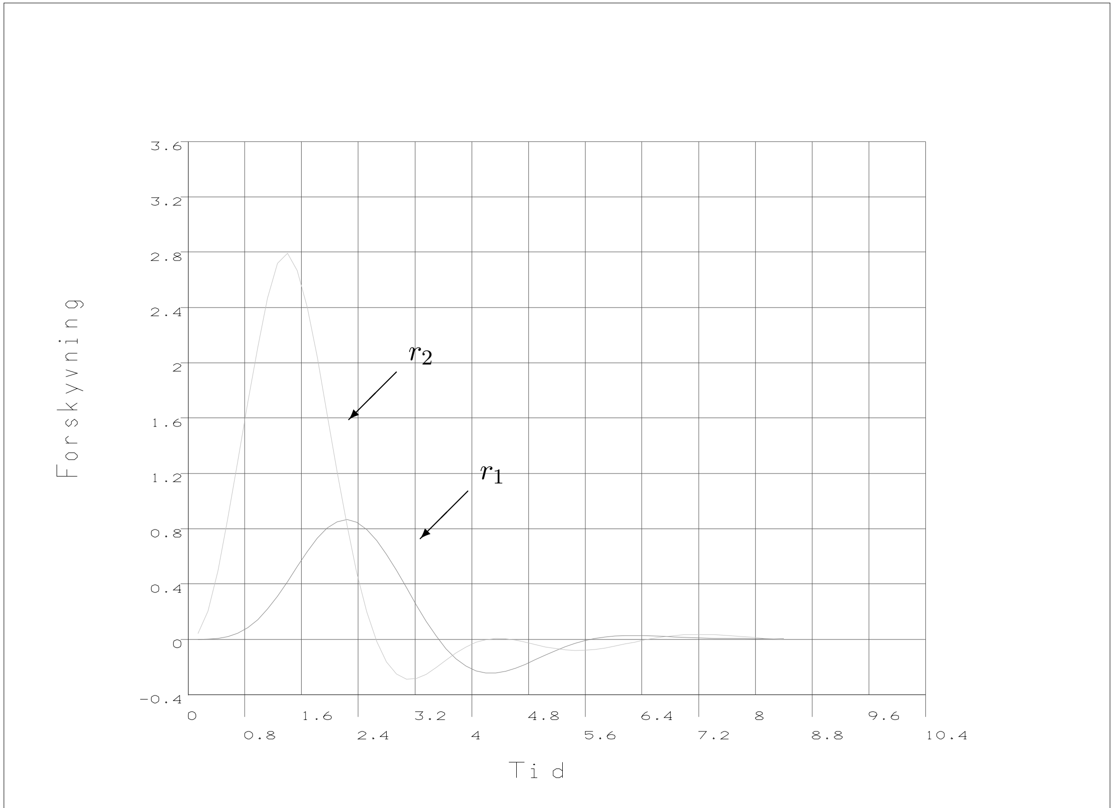

Responsplott for et dempet fjærsystem

9.7. Udempet fjærsystem (MATRIX27 & MASS21)

Betrakt systemet med den dynamiske ligningen som er vist i Kapittel 8.9. Responsen til systemet skal beregnes i ANSYS v.h.a. numerisk integrasjon (jfr.[Crisfield, 1991][3]).

Bruk Newmarks metode med $\gamma = \frac{1}{2}$ og $\beta = \frac{1}{4}$ (d.v.s. konstant gjennomsnittsakselerasjon) til å beregne responsen r_1 og r_2 ved tidene $\Delta t, 2\Delta t, \ldots 12\Delta t$ hvor $\Delta t = 0.28$ og $\Delta t = 0.07$.

Problem: Plott responsen r_1 mot tiden t for analysene i dette avsnitt og i Kapittel 8.9. Kommenter eventuelle forskjeller mellom de to responskurvene.

Løsning:

```
/BATCH,LIST
/FILNAM,ex481
/TITLE, Dynamisk transient analyse av et fjærsystem

/PREP7
ET,1,MATRIX27,,,4          ! Element med symmetrisk stivhets-
ET,2,MASS21,,,4            ! og massene
ET,3,MASS21,,,4

R,1,4,,,,,, $ RMORE,-4     ! Fjær 1
$ RMORE $ RMORE
```

[3] M.A. Crisfield. *Non-Linear Finite Element Analysis of Solids and Structures, Volume 1: Essentials*. Wiley, 1991.

```
$ RMORE $ RMORE
$ RMORE $ RMORE
$ RMORE $ RMORE,,,,4

R,2,2,,,,, $ RMORE,-2       ! Fjær 2
$ RMORE $ RMORE
$ RMORE $ RMORE
$ RMORE $ RMORE
$ RMORE $ RMORE,,,,4

R,3,2 $ R,4,1

N,1 $ N,2,1 $ N,3,2         ! 3 kn. pkt.
TYPE,1 $ REAL,1 $ E,1,2     ! Fjærene
TYPE,1 $ REAL,2 $ E,2,3
TYPE,2 $ REAL,3 $ E,2       ! Massene
TYPE,3 $ REAL,4 $ E,3
FINI

/SOLU
ANTYPE,TRANS                ! Transient analyse
TRNOPT,FULL                 ! Full transient analyse
D,1,ALL $ D,2,UY
D,3,UY
TINTP,,0.25,0.5             ! Numerisk integrasjonsparametre
DELTIM,0.28                 ! Tidsdiskretisering
KBC,1                       ! Konstant last
OUTRES,ALL,ALL
!LSWRITE
TIME,3.36                   ! 12X0.28
KBC,1                       ! Konst. last (fra t=0)
F,3,FX,10
SAVE
SOLVE
FINI

/POST26
```

```
FILE,,RST
NSOL,2,2,U,X,UX2
NSOL,3,3,U,X,UX3
PRVAR,2,3
/GRID,1
/AXLAB,X,Tid
/AXLAB,Y,Forskyvning
/TITLE,Responsplott for et udempet fjærsystem
PLVAR,2,3
FINI
```

Svar/kommentarer: Her løses ligningene direkte. Responsen er som ventet den samme som i Kapittel 8.9. Dette er fordi alle egensvingeformene fra den modale analysen ble brukt i modalsuperposisjonsanalysen (jfr. [Cook et al., 2002, Bathe, 1982][2,1]).

[2] R. D. Cook, D. S. Malkus, M. E. Plesha, and R. J. Witt. *Concepts and Applications of Finite Element Analysis*. Number ISBN: 0-471-35605-0. John Wiley & Sons, Inc., 4th edition, October 2002.

[1] K. J. Bathe. *The Finite Element Procedures in Engineering Analysis*. Prentice-Hall, Englewood Cliffs, New Jersey, 1982.

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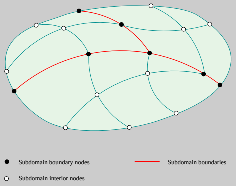

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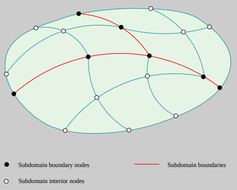
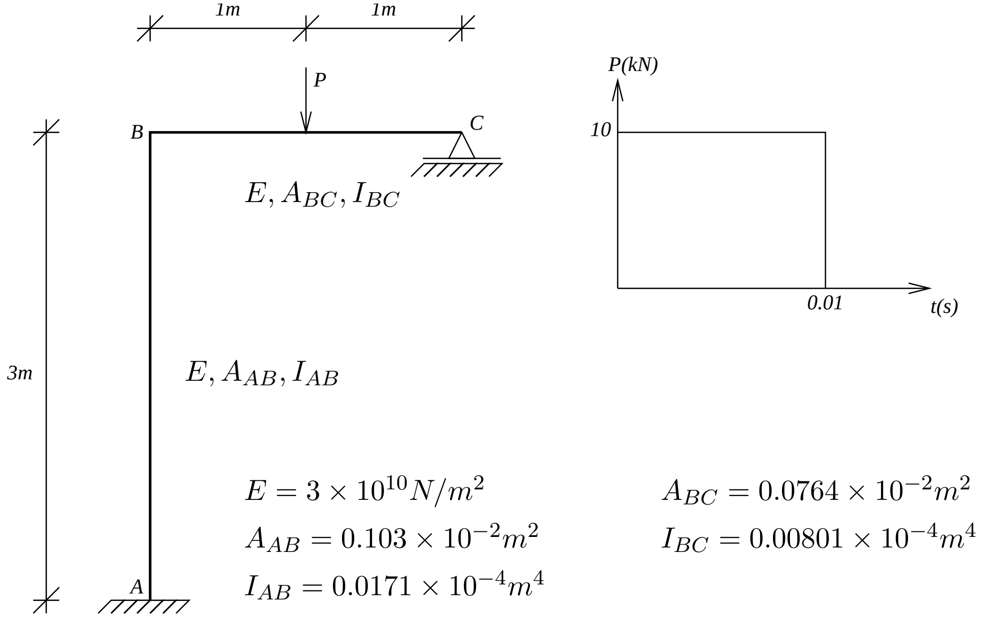


Øving 9.1

Denne oppgaven er en fortsettelse av oppgave 8.1. Hele teksten fra den oppgaven er også inkludert i denne oppgaven.



$E = 3 \times 10^{10} N/m^2$

$A_{AB} = 0.103 \times 10^{-2} m^2$

$I_{AB} = 0.0171 \times 10^{-4} m^4$

$A_{BC} = 0.0764 \times 10^{-2} m^2$

$I_{BC} = 0.00801 \times 10^{-4} m^4$

Rammen i figuren skal modelleres med BEAM3 elementer. AB og BC skal deles inn i to elementer hver.

a) Foreta en modalanalyse av rammen. Resultater for alle svingeformer (lik antall frihetsgrader) skal lagres, men utskrift av kun de første 4 skal vises i besvarelsen.

b) Foreta modalsuperposisjonsanalyser ved bruk av:

  $i)$ alle svingeformer fra a).

  $ii)$ alle svingeformer fra a) untatt de siste to.

  I hver av analysene skal punkt B ha en initiell horisontal hastighet på $1m/s$ og en initiell forskyvning på $0.1m$.

  Responsen til tiden $t = 0.03s$ skal bestemmes for den horisontale forskyvningen av punkt B.

c) Dempningen ved den første og den siste egenfrekvensen er hhv. $\xi_1 = 0.1$ og $\xi_2 = 0.25$. Ved bruk av tallene fra b) skal følgende utføres:

  $i)$ en full transient analyse uten dempning.

  $ii)$ en full transient analyse med Rayleigh dempning.

  Bruk Newmark integrasjon: $\beta = \frac{1}{4}$ og $\gamma = \frac{1}{2}$.

  Plott responsen fra $i)$ og $ii)$ i samme figur.

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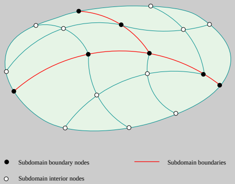

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A. References

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[Cook et al., 2002] Cook, R. D., Malkus, D. S., Plesha, M. E., and Witt, R. J. (2002). *Concepts and Applications of Finite Element Analysis*. Number ISBN: 0-471-35605-0. John Wiley & Sons, Inc., 4th edition.

[Crisfield, 1991] Crisfield, M. (1991). *Non-Linear Finite Element Analysis of Solids and Structures, Volume 1: Essentials*. Wiley.

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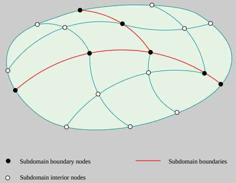

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