

Homework assignment 1

Student name

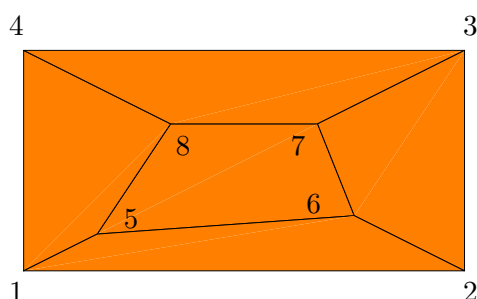
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1 Exercise

Part 1.1

A patch of membrane elements together with the coordinates of the nodes are shown in the figure below:

Node	x	y
1	0	0
2	0.24	0
3	0.24	0.12
4	0	0.12
5	0.04	0.02
6	0.18	0.03
7	0.16	0.08
8	0.08	0.08



The strain energy is given by

$$U(\mathbf{u}) = \int_A h \boldsymbol{\varepsilon}^T \mathbf{E} \boldsymbol{\varepsilon} dA \quad \text{where} \quad \mathbf{u} = \begin{Bmatrix} u \\ v \end{Bmatrix} \quad \text{og} \quad \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix}$$

The material is isotropic

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \mathbf{E} \boldsymbol{\varepsilon}$$

Find a displacement field for a patch test when we have

- the rigid-body motion in the x -direction is $u_0 = 10^{-3}$,
- the rigid-body motion in the y -direction is $v_0 = 2 \times 10^{-3}$,
- the rigid-body motion around the z -axis is $\theta_0 = 3 \times 10^{-3}$,
- constant axial strain $\varepsilon_{xx0} = 4 \times 10^{-3}$,
- constant axial strain $\varepsilon_{yy0} = 5 \times 10^{-3}$ and

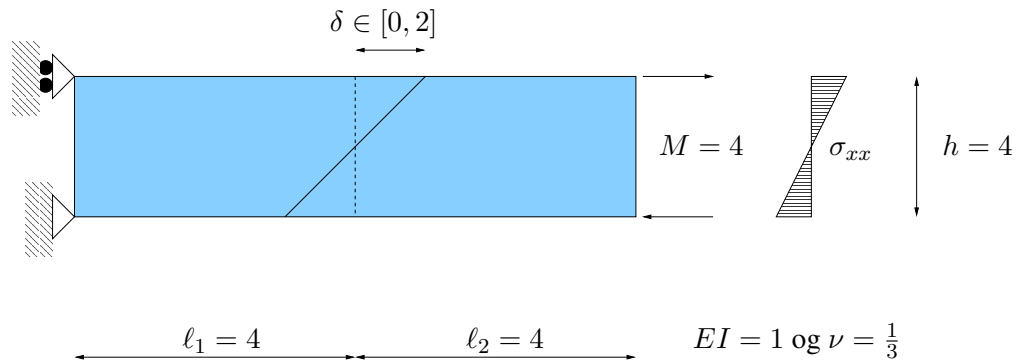
- constant shear strain $\gamma_{xy0} = 6 \times 10^{-3}$.

Part 1.2

Model the patch test problem in ANSYS, and check if the PLANE42 and the PLANE82 elements satisfy the test. Make stress plots of the results.

Part 1.3

The figure show a cantilever loaded with a moment at the end. This can be viewed as a higher order patch test.



Analyze the problem using two PLANE42 og PLANE82 elements. Vary the δ between 0 og 2, i.e. move two nodes. Compare the answers to the exact stress variation.

2 Solution

A ANSYS input file

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