Homework assignment 3

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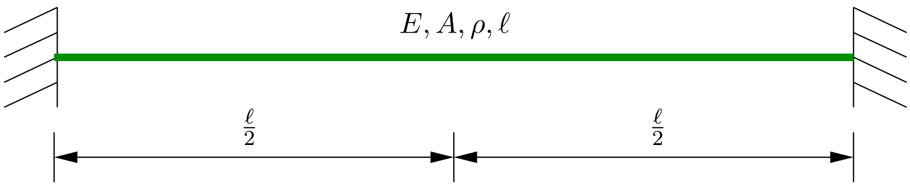
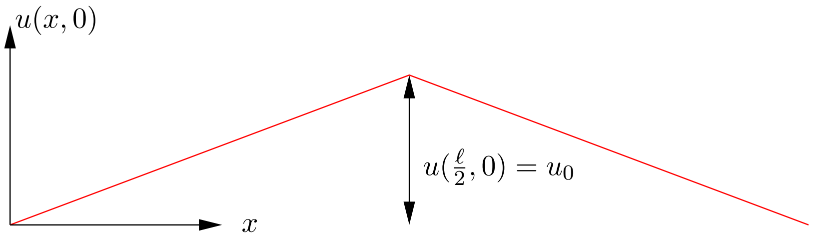
3 Exercise

Part 3.1
Consider the bar construction below. The data is given in the figure, E is the module of elasticity, A is the cross sectional area, ρ is the density and ℓ is the length of the bar.





Initial deformation, $u(x, 0)$

a) Use separation of variables to find the solution of the bar problem (EA and ρ are constants)

$$EA\frac{\partial^2 u(x,t)}{\partial x^2} = \rho A\frac{\partial^2 u(x,t)}{\partial t^2}, \quad u(0,t) = u(\ell,t) = 0,$$

The initial displacement is given in the figure and $\frac{\partial u(x,0)}{\partial t} = 0$.

b) Derive an expression for the axial force, $N(x,t) = EA\frac{\partial u(x,t)}{\partial x}$.

c) What is the wave speed?

d) Plot the displacement, $u(x,t)$, as a function of x and t. (Choose $E = A = \rho = \ell = 1, t \in 0, 1/3, 2/3, 1$).

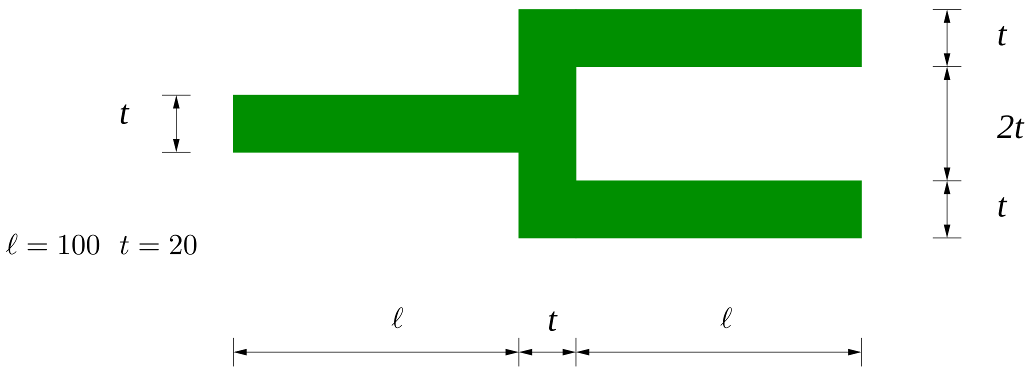
Part 3.2
Solve the bar problem above using the Finite Element method. As above let $E = A = \rho = \ell = 1$. (The coefficients below are adjusted for the time stepping algorithm in ANSYS, other programs may have other time stepping methods.)

a) Use default values for γ and β and compute the solution at time $t = 1$. Experiment with different choices of Δt and number of elements. Compare the results to the analytical solution.

b) Using $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{5}$ the time stepping method is conditionally stable. For which values of Δt is the method stable?

c) Use the integration parameters from b) and compute the Finite Element solution at $t = 1$. What happen if Δt is larger than the critical value? (The simulation may have to be continued to a time t larger than 1 in order to observe the effect.)

Part 3.3
Consider the geometry depicted in the Figure below.



- Determine the ratio of the module of elasticity , E, to the density, ρ, such that the lowest eigenfrequency is $f_1 = 440Hz$. Assume the construction to be free, i.e. no fixed boundary conditions. The material is isotropic with, $\nu = 0.3$.
- Determine the ratio of E to ρ such that the eigenfrequency is $f_1 = 440Hz$ when the left boundary is fixed.

Use membrane elements with depth one. First compute an approximations using beam theory.

Solution

A ANSYS input fil

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