16

$$\begin{aligned} f(x) &= \lambda^x e^{-\lambda}/x! \\ l(\lambda) &= \log\lambda \sum_i x_i - n\lambda - \sum_i \log(x_i!) \\ s(\lambda) &= \sum_i x_i/\lambda - n \\ 0 &= \sum_i x_i/\hat{\lambda} - n \\ \hat{\lambda} &= \sum_i x_i/n = \bar{x} \\ &= 3.876667 \end{aligned}$$

```
> obsC = c(6,24,42,59,62,44,41,14,6,2)
> x = 0:9
> lambdahat = sum(obsC*x)/sum(obsC)
> lambdahat
[1] 3.876667
```

$$\begin{aligned} \hat{p}(x) &= \hat{\lambda}^x e^{-\hat{\lambda}}/x! \\ &= 3.88^x e^{-3.88}/x! \\ \chi^2 &= \sum_i (n\hat{p}(x_i) - N_i)^2/(n\hat{p}(x_i) \end{aligned}$$

```
> phat = lambdahat^x*exp(-lambdahat)/factorial(x)
> tphat = phat[1:9]
> tphat[9] = 1-sum(phat[1:8])
> tobsC = obsC[1:9]
> tobsC[9] = obsC[9] + obsC[10]
> n = sum(obsC)
> chisq = sum((n*tphat-tobsC)^2/(n*tphat))
> chisq
[1] 7.810414
> qchisq(0.9, 9-1-1)
[1] 12.01704
```

So we do not reject H_0 at $\alpha = 0.1$ so the Poisson distribution is a good fit.

18

```
> fs = c(12,20,23,15,13)
> xs = c(0.1,0.150, 0.200, 0.250)
> muhat = .173
> sigmahat = .066
> ps = c(0,pnorm(xs, muhat,sigmahat),1)
> phats = ps[2:6]-ps[1:5]
> n = sum(fs)
> chisq = sum((n*phats-fs)^2/(n*phats))
```

```
> chisq
[1] 1.607175
> qchisq(0.95, 5-1-2)
[1] 5.991465
```

So we do not reject H_0 i.e. normality

19

$$
\begin{aligned}
l(\theta_0,\theta_1) &= \sum_i n_i log p_i \\
&= n_1 2\log\theta_1 + n_2 2\log\theta_2 + n_3 2\log(1-\theta_1-\theta_2) + n_4\log 2 + n_4\log\theta_1 + n_4\log\theta_2 + n_5\log 2 + \\
&\quad n_5\log\theta_1 + n_5\log(1-\theta_1-\theta_2) + n_6\log 2 + n_6\log\theta_2 + n_6\log(1-\theta_1-\theta 2) \\
&= (2n_1+n_4+n_5)\log\theta_1 + (2n_2+n_4+n_6)\log\theta_2 + (2n_3+n_5+n_6)\log(1-\theta_1-\theta_2) + (n_3+n_5+n_6)\log 2 \\
&= 171\log\theta_1 + 110\log\theta_2 + 119\log(1-\theta_1-\theta_2) + 131\log 2
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial l(\theta_1,\theta_2)}{\partial\theta_1} &= 171/\theta_1 - 119/(1-\theta_1-\theta_2) \\
0 &= 171/\hat{\theta}_1 - 119/(1-\hat{\theta}_1-\hat{\theta}_2 \\
0 &= 171 - 171\hat{\theta_1} - 171\hat{\theta}_2 - 119\hat{\theta}_1 \\
171 &= 290\hat{\theta_1} + 171\hat{\theta}_2 \\
110 &= 110\hat{\theta_1} + 229\hat{\theta}_2 \\
\hat{\theta}_1 &= 0.4275 \\
\hat{\theta}_2 &= 0.2750 \\
1-\hat{\theta}_1-\hat{\theta}_2 &= 0.2975
\end{aligned}
$$

```
> ns = c(49, 26,14,20,53,38)
> 2*ns[1] + ns[4] + ns[5]
[1] 171
> 2*ns[2] + ns[4] + ns[6]
[1] 110
> 2*ns[3] + ns[5] + ns[6]
[1] 119
> A = matrix(c(290, 110,171, 229),ncol=2)
> b = c(171,110)
> theta = solve(A,b)
> theta1 = theta[1]
> theta2 = theta[2]
> theta3 = 1-sum(theta)
> ps = c(theta1^2, theta2^2, theta3^2, 2*theta1*theta2, 2*theta1*theta3, 2*theta2*theta3)
> n = sum(ns)
> chisq = sum((n*ps-ns)^2/(n*ps))
> chisq
[1] 29.30334
qchisq(0.95, 6-1)
[1] 11.0705
> qchisq(0.95, 6-1-2)
[1] 7.814728
```

So we reject H_0 at 0.05

24

```
> nsL = c(409, 11,22,7,277)
> nsS = c(512, 4,14,11,220)
> nL = sum(nsL)
> nS = sum(nsS)
> nCs = nsL+nsS
> eL = nL*nCs/(nL+nS)
> eS = nS*nCs/(nL+nS)
> chisq = sum((nsL-eL)^2/eL) + sum((nsS-eS)^2/eS)
> chisq
[1] 23.17859
> qchisq(0.95, (2-1)*(5-1))
[1] 9.487729
```

So we reject H_0 at $\alpha = 0.05$

29

```
> ns = matrix(c(479,173, 119,
+ 214, 47,15,
+ 172, 45,85),ncol=3)
> ns = t(ns)
> njs = matrix(colSums(ns), ncol=3)
> nis = matrix(rowSums(ns))
> n = sum(ns)
> eijs = nis%*%njs/n
> chisq = sum((ns-eijs)^2/eijs)
> chisq
[1] 64.65417
> qchisq(0.99, (3-1)*(3-1))
[1] 13.2767
```

So we reject the independence hypothesis H_0

Exam2007, 3

$$H_0 : p_{ij} = p_{i\cdot} \cdot p_{\cdot j} \quad \forall i, j$$

$$X^2 = 187.79 > 7.814728 = \chi^2_{.05,(2-1)(3-1)}$$

So we reject H_0

```
ns = matrix(c(212, 202, 118, 178,
+ 673, 123, 167, 528),ncol=2)
> ns = t(ns)
es = matrix(c(285.48, 104.84, 91.94, 227.74,
+ 599.52, 220.16, 193.06, 478.26),ncol=2)
> es = t(es)
> (es-ns)^2/es
          [,1]     [,2]     [,3]     [,4]
[1,] 18.913095 90.04259 7.386596 10.86356
[2,]  9.006056 42.87820 3.517682  5.17306
```

Survival of 1.class and death rate of manskap contibutes to the large X^2

Exam2005, 4

$$H_0 : p_{ij} = p_{i\cdot} \cdot p_{\cdot j} \quad \forall i, j$$

$$\chi^2 = 6.6814 > 5.991465 = \chi^2_{.05,(2-1)*(3-1)}$$

So we reject the independence hypothesis H_0 at $\alpha = 0.05$

Exam 1991, 4

b

$$\begin{aligned}
lik(\theta) &= p_{AA}^{X_{AA}} p_{Aa}^{X_{Aa}} p_{aa}^{X_{aa}} \\
&= \theta^{2X_{AA}} (2\theta(1-\theta))^{X_{Aa}} (1-\theta)^{2X_{aa}} \\
l(\theta) &= 2X_{AA} \log\theta + X_{Aa}(\log(2) + \log\theta + \log(1-\theta)) + 2X_{aa}\log(1-\theta) \\
s(\theta) &= 2X_{AA}/\theta + X_{Aa}/\theta - X_{Aa}/(1-\theta) - 2X_{aa}/(1-\theta) \\
0 &= 2X_{AA}/\hat{\theta} + X_{Aa}/\hat{\theta} - X_{Aa}/(1-\hat{\theta}) - 2X_{aa}/(1-\hat{\theta}) \\
0 &= 2X_{AA} - 2X_{AA}\hat{\theta} + X_{Aa} - X_{Aa}\hat{\theta} - X_{Aa}\hat{\theta} - 2X_{aa}\hat{\theta} \\
(2X_{AA} + X_{Aa} + X_{Aa} + 2X_{aa})\hat{\theta} &= 2X_{AA} + X_{Aa} \\
\hat{\theta} &= \frac{2X_{AA} + X_{Aa}}{2X_{AA} + 2X_{Aa} + 2X_{aa}} \\
\hat{\theta} &= \frac{2X_{AA} + X_{Aa}}{n}
\end{aligned}$$