Extra 10

a

b

7.6

a

```
> exe7.06=scan("http://www.uio.no/studier/emner/matnat/math/STK2120/v16/exe7-06.txt")
Read 31 items
> ys = log(exe7.06)
> muHat = mean(ys)
> sigmaHat = sd(ys)
> muHat
[1] 5.101652
> sigmaHat
[1] 0.4960617
```

b

$$E(X) = e^{\mu+\sigma^2/2}$$
$$E(\hat{X}) = e^{\hat{\mu}+\hat{\sigma}^2/2}$$
$$E(\hat{X}) = e^{5.101652+0.4960617^2/2} = 185.8037$$

c

```
B=10000
n=31
theta.star=rep(NA,B)
mu = 5.101652
s = 0.4960617
theta = exp(mu+s^2/2)
for (b in 1:B)
{
    x.star = rlnorm(n,mu,s)
    y.star = log(x.star)
    theta.star[b]=exp(mean(y.star)+var(y.star)/2)
}
bias.theta = mean(theta.star)-theta
sd.theta = sd(theta.star)

print(bias.theta)
print(sd.theta)

[1] 1.03993
[1] 17.74444
```

d

```
exe7.06=scan(
    "http://www.uio.no/studier/emner/matnat/math/STK2120/v16/exe7-06.txt")
B=10000
```

```
n=length(exe7.06)
theta.star=rep(NA,B)
mu = 5.101652
s = 0.4960617
theta = exp(mu+s^2/2)

for (b in 1:B)
{
    x.star=sample(exe7.06,n,replace=T)
    y.star = log(x.star)
    theta.star[b]=exp(mean(y.star)+var(y.star)/2)
}
bias.theta = mean(theta.star)-theta
sd.theta = sd(theta.star)

print(bias.theta)
print(sd.theta)

[1] -0.09091476
[1] 17.54799
```

e

The estimated standard errors is approximately equal, but the parametric bootstrap yields a slightly bigger bias.

Storvik 6

a

```
x <- rnorm(15,2,1)
plot(ecdf(x),verticals=TRUE)
lines(sort(x),pnorm(sort(x),2,1),lty=2)
```

b

$$E(I(A)) = 1P(A) + 0(1 - P(A)) = P(A)$$

c

$$\begin{aligned}
E(\hat{F}(x)) &= \frac{1}{n}\sum_i E(I(X_i \leq x)) \\
&= \frac{1}{n}\sum_i P(X_i \leq x) \\
&= \frac{1}{n}\sum_i F(x) \\
&= F(x)
\end{aligned}$$

d

$$\begin{aligned}V(\hat{F}(x)) &= \frac{1}{n^2}\sum_i V(I(X_i \le x)) \\ &= \frac{1}{n}V(I(X \le x)) \\ &= \frac{1}{n}\left[E(I(X \le x)^2) - E(I(X \le x))^2\right] \\ &= \frac{1}{n}\left[P(X \le x) - P(X \le x)^2\right] \\ &= \frac{1}{n}\left[F(x) - F(x)^2\right] \\ &= \frac{1}{n}\left[F(x)(1 - F(x))\right]\end{aligned}$$

e

$$F(x) = 1/n\sum_i I(x \ge x_i) = 1/n\sum_i I(x_i \le x)$$

```
x <- rnorm(15,2,1)
mu.hat = mean(x); sigma.hat = sqrt(var(x))
plot(sort(x),pnorm(sort(x),mu.hat,sigma.hat),type="l")
lines(sort(x),pnorm(sort(x),2,1), lty=2)
```

Storvik 7

a

$$\begin{aligned}E^{\hat{F}}[\hat{\theta}(X^*] &= \frac{1}{n}\sum_j E^{\hat{F}}[X_j^*] \\ &= E^{\hat{F}}[X^*] \\ &=_{6e} 1/n\sum_i x_i \\ &= \bar{x}\end{aligned}$$

b

$$\begin{aligned}\theta(\hat{F}) &= E^{\hat{F}}(X) = \bar{x} \\ E^{\hat{F}}(\theta(\hat{X})) - \theta(\hat{F}) &= \bar{x} - \bar{x} = 0\end{aligned}$$

c

$$
\begin{aligned}
V^{\hat{F}}(X) &=_{6e} \sum_i (x_i - E^{\hat{F}}(X))^2 p(x_i) \\
&= \sum_i (x_i - \bar{x})^2 \frac{1}{n}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\theta}(\mathbf{x}^*) &= \bar{\mathbf{x}^*} \\
V(\hat{\theta}(\mathbf{x}^*)) &= V(\bar{\mathbf{x}^*}) \\
&= \frac{1}{n^2} \sum_i V(x_i^*) \\
&= \frac{1}{n} V(x^*) \\
&= \frac{1}{n^2} \sum_i (x_i - \bar{x})^2
\end{aligned}
$$