1a

$$H_0 : \mu_i = \mu_j \, \forall \, i, j \in \{1, 2, 3, 4, 5\}$$

Reject H_0 if:

$$f \geq F_{\alpha,(I-1),I(J-1)} = F_{.05,4,30} = 2.69$$

$$\begin{aligned} f &= \frac{MSTr}{MSE} && (1) \\ &= \frac{123.50}{22.16} && (2) \\ &= 5.57 \geq 2.69 && (3) \end{aligned}$$

So H_0 is rejected

1b

$$\begin{aligned} &F_{.01,4,30} = 4.02 < f = 5.57 < F_{.001,4,30} = 6.12 && (4) \\ \Rightarrow\, &0.001 < p < 0.01 && (5) \end{aligned}$$

2

With μ_i = true average lumen output for brand i bulbs

$$\begin{aligned} &H_0 : \mu_1 = \mu_2 = \mu_3 && (6) \\ &H_1 : \exists i, j \in \{1, 2, 3\} \; s.t. \; \mu_i \neq \mu_j && (7) \end{aligned}$$

ANOVA test:

$$\begin{aligned} f &= \frac{MSTr}{MSE} && (8) \\ &= \frac{SSTr/(I-1)}{MSE/I(J-1)} && (9) \\ &= \frac{591.2/2}{4773.3/21} && (10) \\ &= 1.30 && (11) \end{aligned}$$

Looking up in the table we find:

$$\begin{aligned} 1.30 < F_{.10,2,21} = 2.57 \;\Rightarrow && (12) \\ p > 0.10 > 0.05 && (13) \end{aligned}$$

So we reject H_0.

4

Reject H_0 if:

$$f \geq F_{\alpha,(I-1),I(J-1)} = F_{.01,2,10} = 5.49$$

$$\begin{aligned}
f &= \frac{MSTr}{MSE} && (14)\\
&= \frac{SSTr/(I-1)}{MSE} && (15)\\
&= \frac{J\sum_i(\bar{x}_{i\cdot} - \bar{x}_{\cdot\cdot})^2/(I-1)}{\sum_i s_i^2/I} && (16)\\
&= \frac{10((1.63-1.54)^2 + (1.56-1.54)^2 + (1.42-1.54)^2/2}{(0.27^2 f + 0.24^2 + 0.26^2)/3} && (17)\\
&= \frac{0.1143}{0.0660} && (18)\\
&= 1.73 && (19)
\end{aligned}$$

So $f < F_{.01,2,10}$ and we fail to reject H_0

5

```
exe11.5=read.table("http://www.uio.no/studier/emner/matnat/math/STK2120/v
                  header=T, sep=",");
aov.fit=aov(Fe~factor(form),data=exe11.5)
summary(aov.fit)
```

6

Source	df	Sum of Squares	Mean Square	f
Brand				
Error			14713.69	
Total		310500.76		

Source	df	Sum of Squares	Mean Square	f
Brand	I-1	SSTr	SSTr/(I-1)	MSTr/MSE
Error	I(J-1)	SSE	14713.69	
Total	IJ-1	310500.76		

Source	df	Sum of Squares	Mean Square	f
Brand	3	75081.72	25027.24	1.70
Error	16	235419.04	14713.69	
Total	19	310500.76		

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$1.70 < F_{.10,3,16} = 2.46$$

So we fail to reject H_0

9

$$x_{ij} - \bar{x}_{\cdot\cdot} = (x_{ij} - \bar{x}_{i\cdot}) + (x_{i\cdot} - \bar{x}_{\cdot\cdot}) \tag{20}$$

$$(x_{ij} - \bar{x}_{\cdot\cdot})^2 = ((x_{ij} - \bar{x}_{i\cdot}) + (x_{i\cdot} - \bar{x}_{\cdot\cdot}))^2 \tag{21}$$

$$\sum_i \sum_j (x_{ij} - \bar{x}_{\cdot\cdot})^2 = \sum_i \sum_j ((x_{ij} - \bar{x}_{i\cdot}) + (x_{i\cdot} - \bar{x}_{\cdot\cdot}))^2 \tag{22}$$

$$SST = \sum_i \sum_j (x_{ij} - \bar{x}_{i\cdot})^2 + (x_{i\cdot} - \bar{x}_{\cdot\cdot})^2 + 2(x_{ij} - \bar{x}_{i\cdot})(x_{i\cdot} - \bar{x}_{\cdot\cdot}) \tag{23}$$

$$SST = SSE + SSTr + \sum_i \sum_j 2(x_{ij} - \bar{x}_{i\cdot})(x_{i\cdot} - \bar{x}_{\cdot\cdot}) \tag{24}$$

$$SST = SSE + SSTr + 2\sum_i (x_{i\cdot} - \bar{x}_{\cdot\cdot}) \sum_j (x_{ij} - \bar{x}_{i\cdot}) \tag{25}$$

$$SST = SSE + SSTr \tag{26}$$

$$\tag{27}$$

10a

$$\begin{aligned} E(\bar{X}_{\cdot\cdot}) &= (1/I)E(\sum_i \bar{X}_{i\cdot}) \\ &= (1/I)\sum_i E(\bar{X}_{i\cdot}) \\ &= (1/I)\sum_i \mu_i \\ &= \mu \end{aligned}$$

10b

$$\begin{aligned} E(\bar{X}_{i\cdot}^2) &= V(\bar{X}_{i\cdot}) + E(\bar{X}_{i\cdot})^2 \\ &= \sigma^2/J + \mu_i^2 \end{aligned}$$

10c

$$\begin{aligned} E(\bar{X}_{\cdot\cdot}^2) &= V(\bar{X}_{\cdot\cdot}) + E(\bar{X}_{\cdot\cdot})^2 \\ &= \sigma^2/IJ + \mu^2 \end{aligned}$$

10d

$$
\begin{aligned}
E(SSTr) &= E(\sum_i x_{i\cdot}^2/J - x_{\cdot\cdot}^2/IJ) \\
&= E(J\sum_i \bar{X}_{i\cdot}^2 - IJ\bar{X}_{\cdot\cdot}^2) \\
&= J\sum_i E(\bar{X}_{i\cdot}^2) - IJE(\bar{X}_{\cdot\cdot}^2) \\
&= J\sum_i(\sigma^2/J + \mu_i^2) - IJ(\sigma^2/IJ + \mu^2) \\
&= I\sigma^2 + J\sum_i \mu_i^2 - \sigma^2 - IJ\mu^2 \\
&= (I-1)\sigma^2 + J(\sum_i \mu_i^2 - I\mu^2) \\
&= (I-1)\sigma^2 + J(\sum_i \mu_i^2 - 2I\mu^2 + I\mu^2) \\
&= (I-1)\sigma^2 + J(\sum_i \mu_i^2 - 2\mu\sum_i \mu_i + I\mu^2) \\
&= (I-1)\sigma^2 + J\sum_i(\mu_i^2 - 2\mu\mu_i + \mu^2) \\
&= (I-1)\sigma^2 + J\sum_i(\mu_i - \mu)^2
\end{aligned}
$$

$$
\begin{aligned}
E(MSTr) &= E(SSTr)/(I-1) \\
&= \sigma^2 + J/(I-1)\sum_i(\mu_i - \mu)^2
\end{aligned}
$$

10e

We have $\sum_i(\mu_i - \mu)^2 \geq 0$ with equality when H_0 is true. So $E(MSTr) \geq \sigma$,with equality when H_0 is true.

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$$\begin{aligned}
T &= \frac{\bar{X}_{2\cdot} - \bar{X}_{1\cdot}}{S_p\sqrt{2/J}} \\
T^2 &= \frac{(\bar{X}_{2\cdot} - \bar{X}_{1\cdot})^2}{(S_1^2 + S_2^2)/2 \cdot 2/J} \\
&= \frac{(\bar{X}_{2\cdot} - \bar{X}_{1\cdot})^2}{(S_1^2 + S_2^2)/J}
\end{aligned}$$

$$\begin{aligned}
F &= \frac{MSTr}{MSE} \\
&= \frac{SSTr/(I-1)}{MSE} \\
&= \frac{J\sum_i(\bar{X}_{i\cdot} - \bar{X}_{\cdot\cdot})^2}{MSE} \\
&= \frac{2J(\bar{X}_{1\cdot}/2 - \bar{X}_{2\cdot}/2)^2}{MSE} \\
&= \frac{J(\bar{X}_{1\cdot} - \bar{X}_{2\cdot})^2}{2MSE} \\
&= \frac{J(\bar{X}_{1\cdot} - \bar{X}_{2\cdot})^2}{2(S_1^2 + S_2^2)/2} \\
&= \frac{(\bar{X}_{1\cdot} - \bar{X}_{2\cdot})^2}{(S_1^2 + S_2^2)/J}
\end{aligned}$$

0.0.1 b

For $t_0 > 0$ and $f_0 = t^2$:

$$\begin{aligned}
P((T < -t_0) \cup (T > t_0)) &= P(T^2 > t_0^2) \\
&= P(F > f_0)
\end{aligned}$$