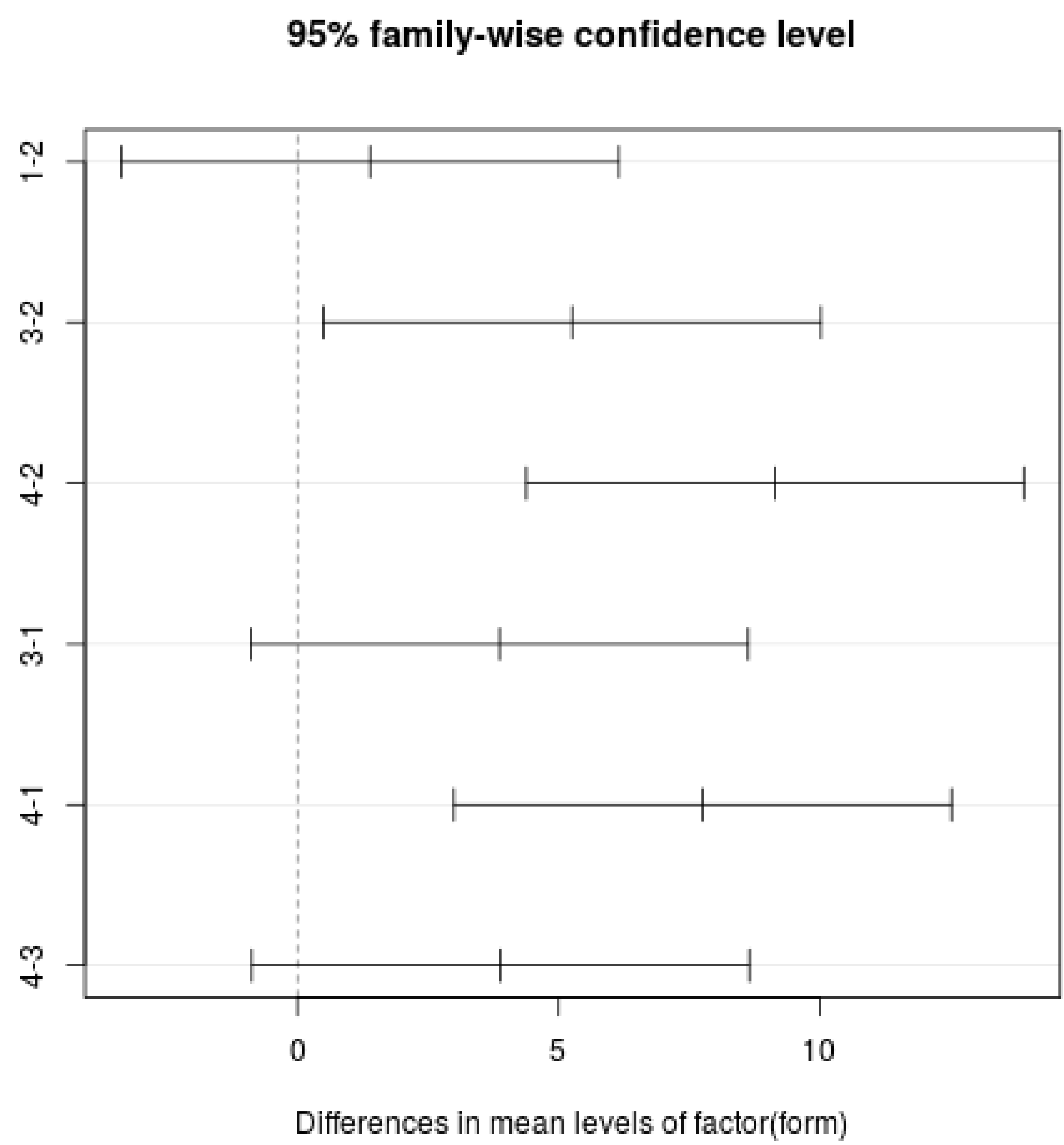
11

$$
\begin{aligned}
Q_{\alpha,I,I(J-1)} &= Q_{.05,5,15} \\
w &= Q_{.05,5,15}\sqrt{MSE/J} \\
w &= 4.37\sqrt{272.8/4} \\
w &= 36.09
\end{aligned}
$$

$\bar{x}_{3\cdot}$	$\bar{x}_{1\cdot}$	$\bar{x}_{4\cdot}$	$\bar{x}_{2\cdot}$	$\bar{x}_{5\cdot}$
437.5	462.0	469.3	512.8	532.1

15

```
exe11.5=read.table("http://www.uio.no/studier/emner/matnat/math/STK2120/v16/exe11-05.txt",
                    header=T, sep=",");
aov.fit=aov(Fe~factor(form),data=exe11.5)
summary(aov.fit)
#problem 11.15
tukey.fit = TukeyHSD(aov.fit,ordered=TRUE)
print(tukey.fit)
png('exe1115.png')
plot(tukey.fit)
dev.off()
```



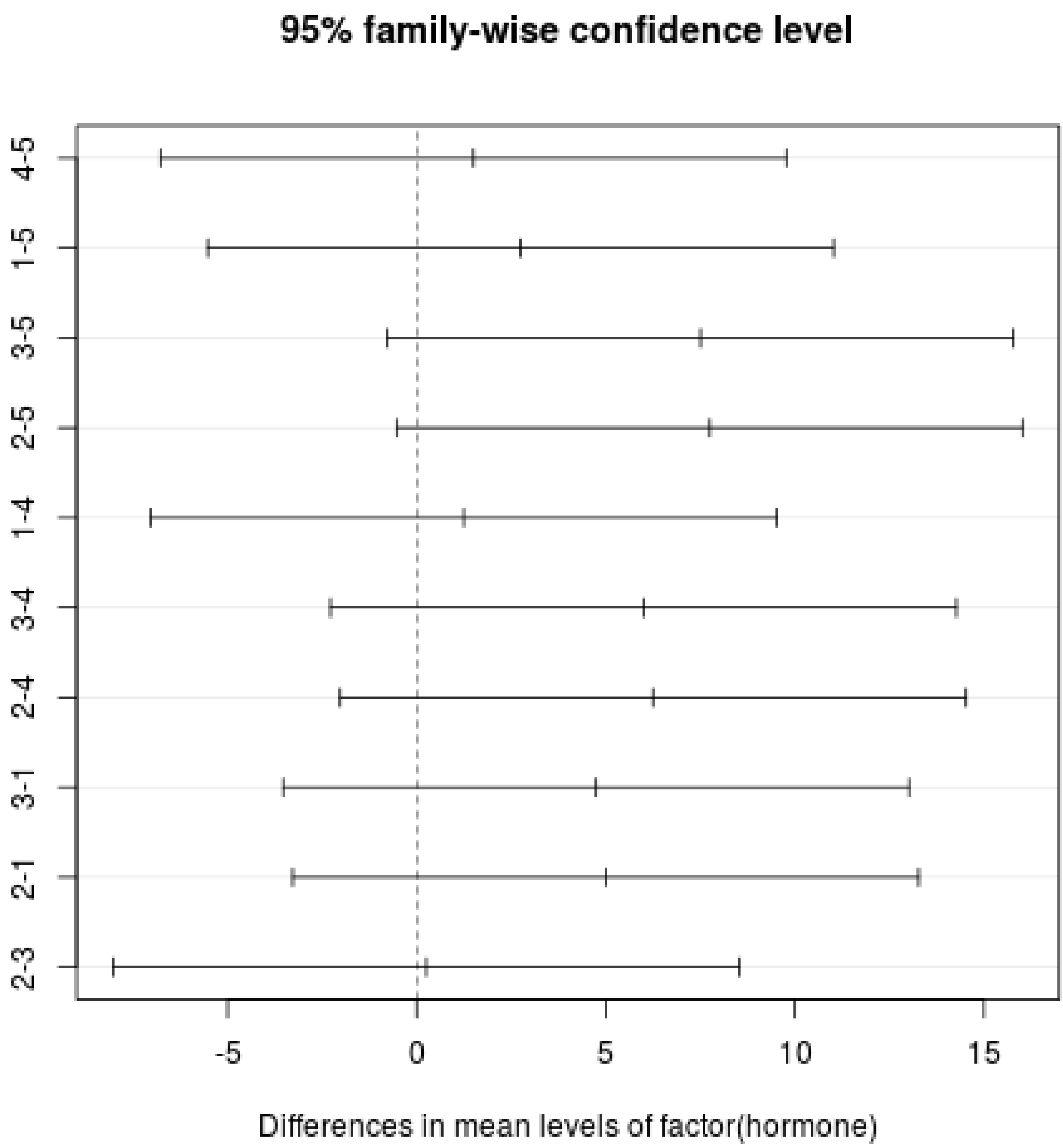
18

```
exe11.18=read.table("http://www.uio.no/studier/emner/matnat/math/STK2120/v16/exe11-18.txt",
                     header=T)
aov.fit=aov(growth~factor(hormone),data=exe11.18)
summary(aov.fit)
tukey.fit = TukeyHSD(aov.fit,ordered=T)
print(tukey.fit)
png("exe1118.png")
plot(tukey.fit)
dev.off()
```

a

F test significant with $\alpha = 0.05$

b



None of the hormones give significant differences.

24

Source	df	Sum of Squares	Mean Square	f
Groups			76.09	
Error				
Total		1123.14		

Source	df	Sum of Squares	Mean Square	f
Groups	I-1	SSTr	76.09	f
Error	n-I	SSE	MSE	
Total	n-1	1123.14		

Source	df	Sum of Squares	Mean Square	f
Groups	2	152.18	76.09	5.56
Error	71	970.96	13.6754	
Total	73	1123.14		

$$5.56 \geq F_{.01,2,71} \approx 4.94$$

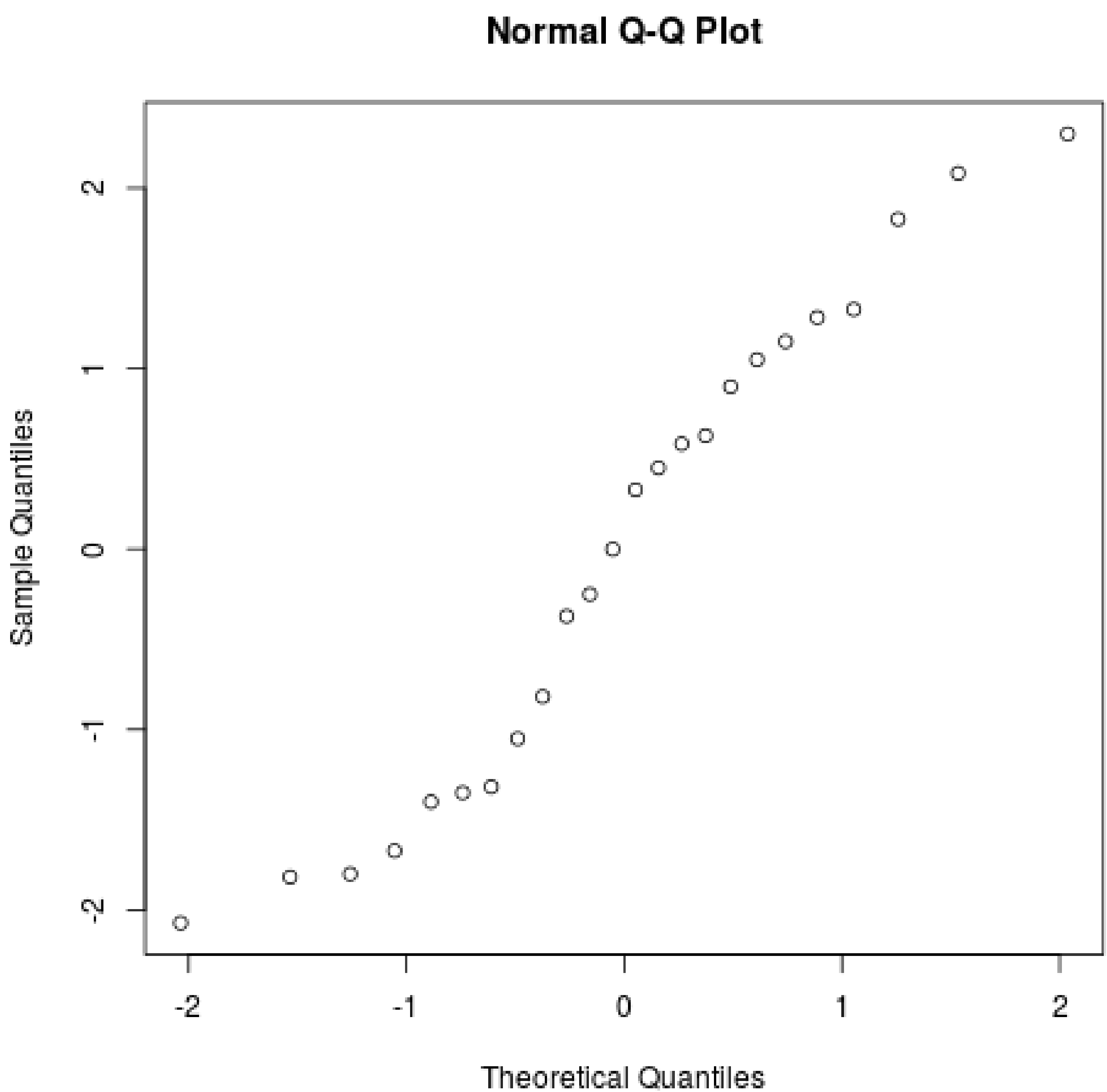
So we reject H_0 at level .01

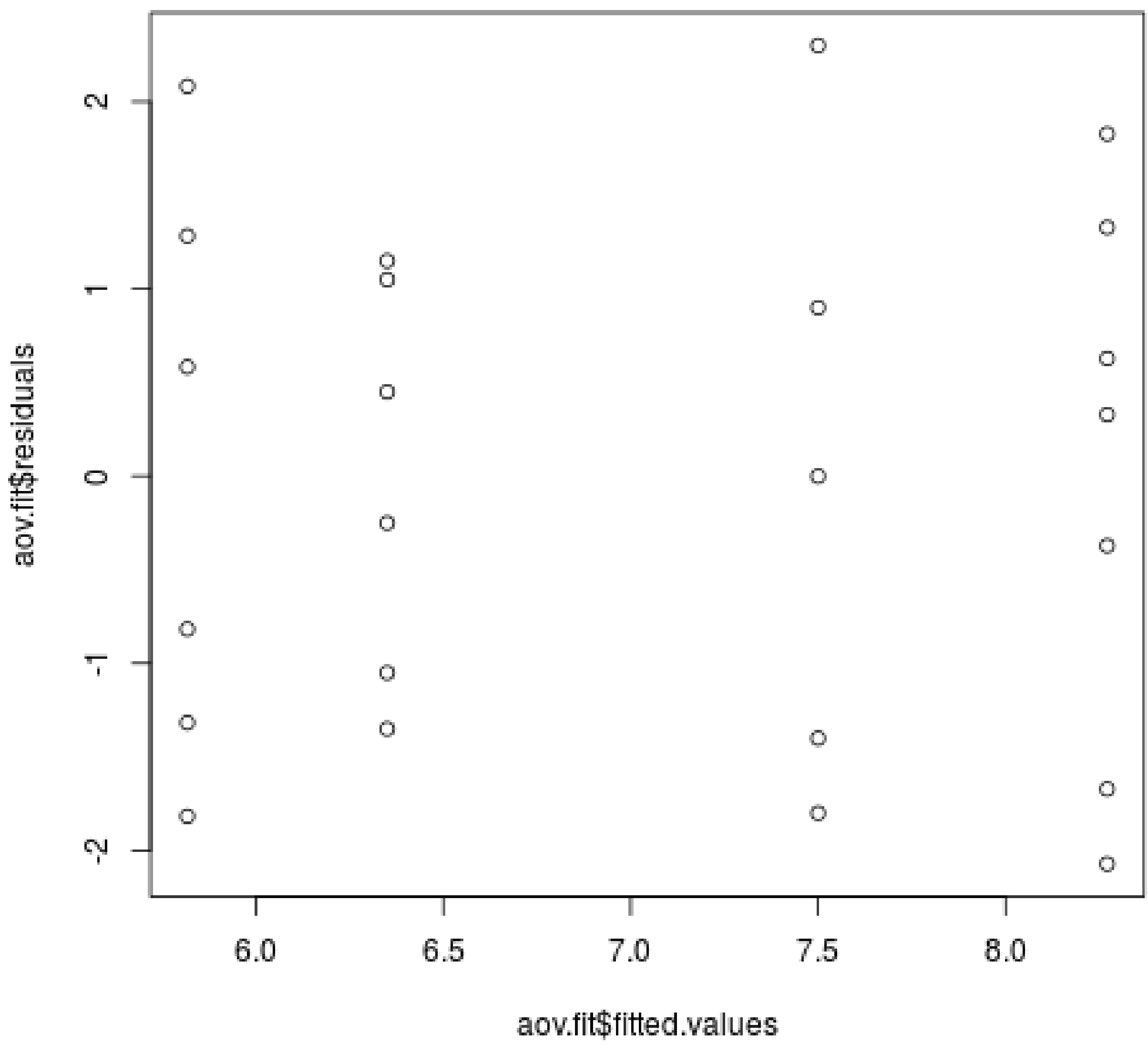
27

```
exe11.27=read.table(
          "http://www.uio.no/studier/emner/matnat/math/STK2120/v16/exe11-27.txt",
           header=T)
aov.fit=aov(folacin~factor(brand),data=exe11.27)
summary(aov.fit)

#                Df Sum Sq Mean Sq F value Pr(>F)
#factor(brand)    3  23.50   7.832   3.749 0.0276 *
#Residuals       20  41.78   2.089
png("exe1127b.png")
qqnorm(aov.fit$residuals)
dev.off()
png("exe1127b2.png")
plot(aov.fit$fitted.values,aov.fit$residuals)
dev.off()
tukey.fit = TukeyHSD(aov.fit ,ordered=T)
png("exe1127c.png")
print(tukey.fit)
plot(tukey.fit)
dev.off()
```

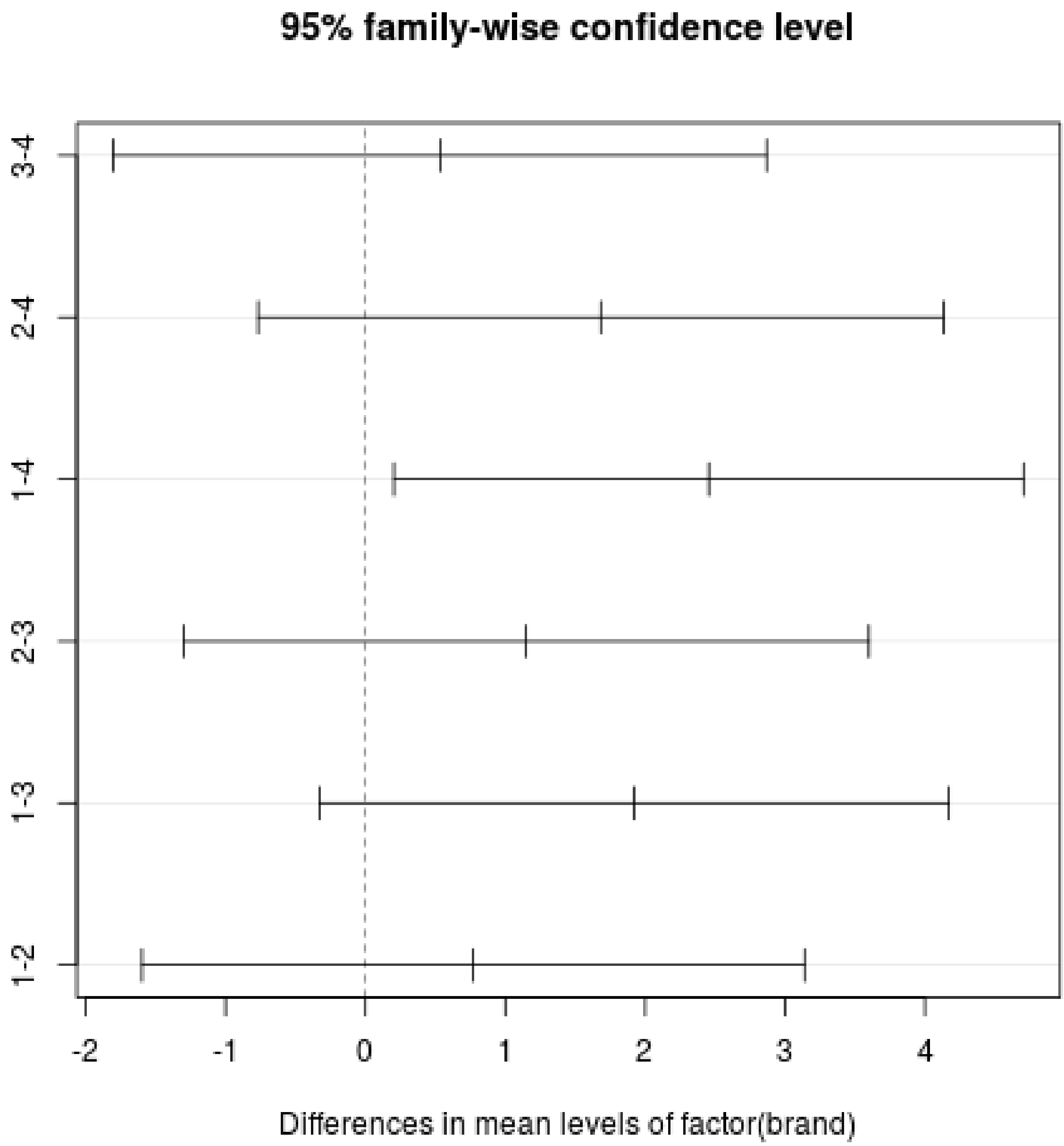
b


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
-2
-1
0
1
2



Normality is plausible. Constant variance is plausible.

c



Only brand 1 and 4 are significantly different

30

```
mu = c(0,0,-1,1)
s = 1;
I = 4;
J = 8;
testa = power.anova.test(groups=I, n=J, between.var=var(mu), within.var=s^2, power=NULL)
print(1-testa$power)
```

```
#b
beta=0.05
testb = power.anova.test(groups=I, n=NULL, between.var=var(mu), within.var=s^2, power=1-bet
print(testb$n)


I=5; J=10;s=1;mu = c(0,0,0,0,1);
testc = power.anova.test(groups=I, n=J, between.var=var(mu), within.var=s^2, power=NULL)
print(1-testc$power)
```

a) $\beta = 0.11$, b) $J > 9.36$, c) $\beta = 0.44$

Ekstraoppgave 2

a

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A \cap B) \leq P(A_1) + P(A_2)$$

b

$$\begin{aligned} P(\cup_{i=1}^{t+1} A_i) &= P((\cup_{i=1}^{t} A_i) \cup A_{t+1}) \\ &\leq P((\cup_{i=1}^{t} A_i)) + P(A_{t+1}) \\ &\leq \sum_{i=1}^{t} P(A_i) + P(A_{t+1}) \\ &= \sum_{i=1}^{t+1} P(A_i) \end{aligned}$$

c

$$\begin{aligned} (\cap_i A_i)' &= \cup_i A_i' \\ P((\cap_i A_i)') &= P(\cup_i A_i') \\ 1 - P(\cap_i A_i) &= P(\cup_i A_i') \\ P(\cap_i A_i) &= 1 - P(\cup_i A_i') \\ P(\cap_i A_i) &\geq 1 - \sum_i P(A_i') \\ P(\cap_i A_i) &\geq 1 - \sum_i (1 - P(A_i)) \end{aligned}$$

d

$$\begin{aligned} P(L_i < \theta_i < U_i) &= 1 - \alpha_i \\ P(\cap_i (L_i < \theta_i < U_i)) &\geq 1 - \sum_i (1 - (1 - \alpha_i)) \\ &\geq 1 - \sum_i \alpha_i \end{aligned}$$

e

$$L_{ij} = \bar{X}_{i\cdot} - \bar{X}_{j\cdot} - t_{\alpha/(2k),n-1}\sqrt{MSE(\frac{1}{J_i} + \frac{1}{J_j})}$$

$$U_{ij} = \bar{X}_{i\cdot} - \bar{X}_{j\cdot} + t_{\alpha/(2k),n-1}\sqrt{MSE(\frac{1}{J_i} + \frac{1}{J_j})}$$

$$P(\cap_{i=1}^{I} \cap_{j=i+1}^{I} (L_{ij} < \mu_i - \mu_j < U_{ij})) \geq 1 - \sum_{i=1}^{I}\sum_{j=i+1}^{I} \alpha/k$$
$$= 1 - \alpha$$

f

```
tdistc<-function(I,J)
{
  alpha=0.05
  k = I*(I-1)/2;
  percentile = alpha/(2*k)
  df = I*J-I
  return(qt(1-percentile,df)*sqrt(2))
}

qdistc<-function(I,J)
{
  alpha=0.05
  nmeans = I
  df= I*(J-1)
  return(qtukey(alpha, nmeans,df))
}
Is = c(4,6,10)

tdistc(Is,4)
#[1] -4.458565 -4.780554 -5.100622
tdistc(Is,6)
#[1] -4.139572 -4.509653 -4.894409
tdistc(Is,10)
#[1] -3.948445 -4.343618 -4.764907
qdistc(Is,4)
#[1] 0.7501078 1.2241162 1.8142894
> qdistc(Is,6)
#[1] 0.7537133 1.2350366 1.8326608
> qdistc(Is,10)
#[1] 0.7562362 1.2427368 1.8456548
```

Eksamensoppgave 1

a

$$H_0 : \alpha_i = 0 \quad \forall i$$

$$f = 2.92 < F_{.05,2,15} = 3.68$$

We fail to reject H_0

b

$$\begin{aligned} E(\hat{C}) &= E(\sum_i c_i \bar{Y}_{i\cdot}) \\ &= \sum_i \left[c_i E(\bar{Y}_{i\cdot})\right] \\ &= \sum_i \left[c_i(\mu + \alpha_i)\right] \\ &= \mu \sum_i c_i + \sum_i c_i \alpha_i \\ &= C \end{aligned}$$

$$\begin{aligned} H_0 &: \\ &= 0 \end{aligned}$$

$$\begin{aligned} E(\hat{C}^2) &= Var(\hat{C}) + E(\hat{C})^2 \\ &= Var(\hat{C}) + C^2 \end{aligned}$$

c

$$\begin{aligned} Var(\hat{C}) &= \sum_i (c_i^2 Var(\bar{Y}_{i\cdot})) \\ &= \sum_i (c_i^2 \sigma^2 / J) \\ &= \sigma^2 (\sum_i (c_i^2 / J) \\ &= \sigma^2 K_C \end{aligned}$$

Since the numerator has $E = 0$ when H_0 is true and $E > 0$ for H_0 is false, we can reject H_0 for high F_C,

d

$\hat{C}/(\sqrt{K_C}\sigma)$ is normally distributed with $E(\hat{C}/(\sqrt{K_C}\sigma)) = 0$ under H_0 and $Var(\hat{C}/(\sqrt{K_C}\sigma)) = 1$. So it is standard normal, so

$$\left[\hat{C}/(\sqrt{K_C}\sigma)\right]^2 = SS_C/\sigma^2 \sim \chi_1^2$$

We have that $SS_C = \psi_C(\bar{Y}_1., \bar{Y}_2., , , \bar{Y}_I.)$ while $SS_W = \psi_W(S_1^2, S_2^2, , , S_I^2)$ and S_i^2 and $\bar{Y}_{i\cdot}$ are independent. So SS_C and SS_W are independent.

Since SS_C and SS_W are independent, $SS_C/\sigma^2 \sim \chi_1^2$ and $SS_W/\sigma^2 = SSE/\sigma^2 \sim \chi^2_{I(J-1)}$,then

$$F_C = \frac{SS_C/\sigma^2}{SS_W/\sigma^2} \sim F_{1,I(J-1)}$$

e

$$H_0 : \alpha_1 - (1/2)\alpha_2 - (1/2)\alpha_3 = 0$$
$$f = 5.362 > F_{.05,1,15} = 4.54$$

So we reject H_0