35

a

$$\begin{aligned} f_A &= \frac{MSA}{MSE} \\ &= \frac{SSA/(I-1)}{SSE/((I-1)(J-1))} \\ &= \frac{30.6/4}{59.2/12} \\ &= 1.55 < F_{.05,4,12} \end{aligned}$$

We don't reject H_{0A}

b

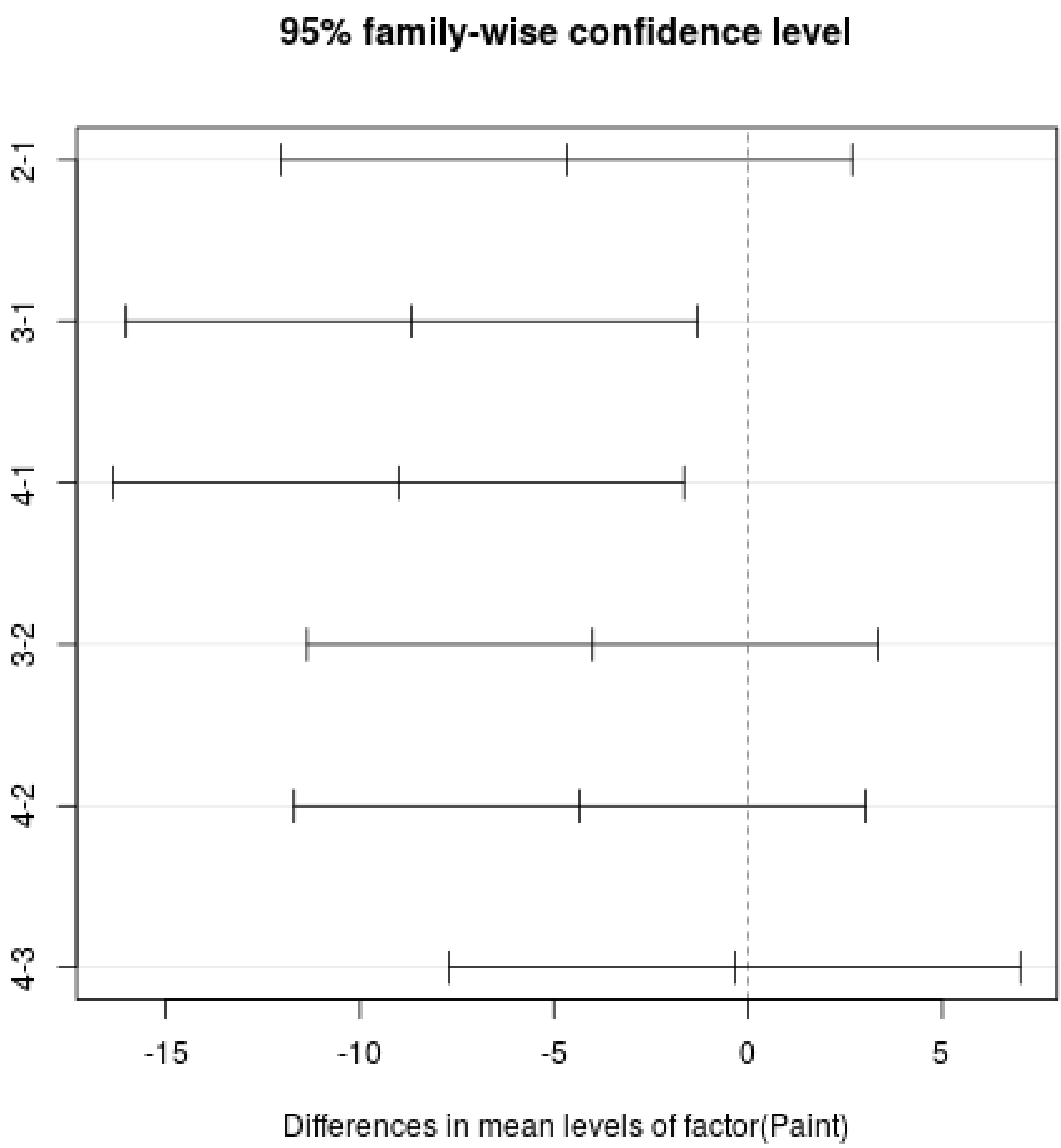
$$\begin{aligned} f_B &= \frac{MSB}{MSE} \\ &= \frac{SSB/(J-1)}{SSE/((I-1)(J-1))} \\ &= \frac{44.1/3}{59.2/12} \\ &= 2.98 < F_{.05,3,12} = 3.49 \end{aligned}$$

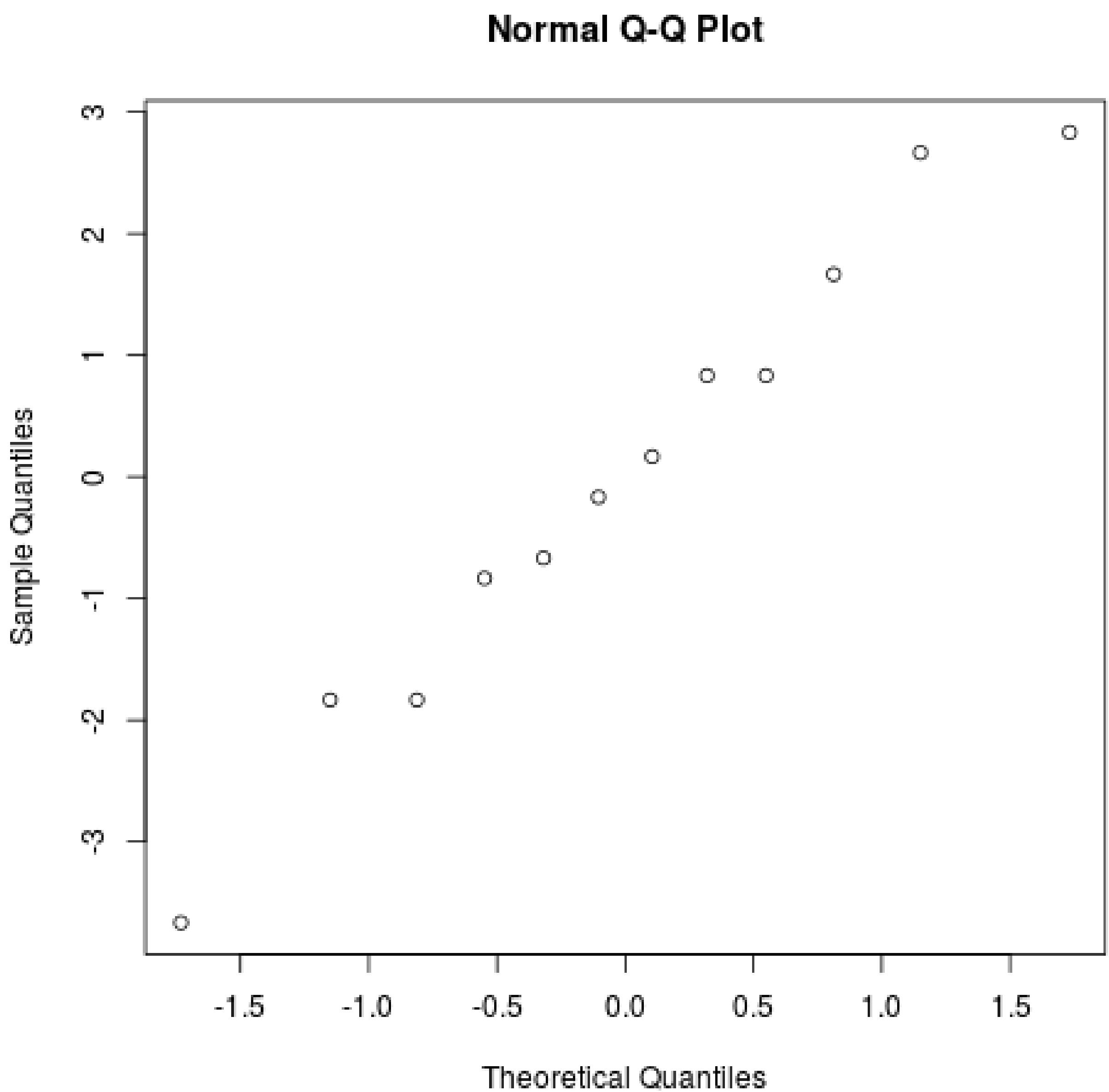
We don't reject H_{0B}

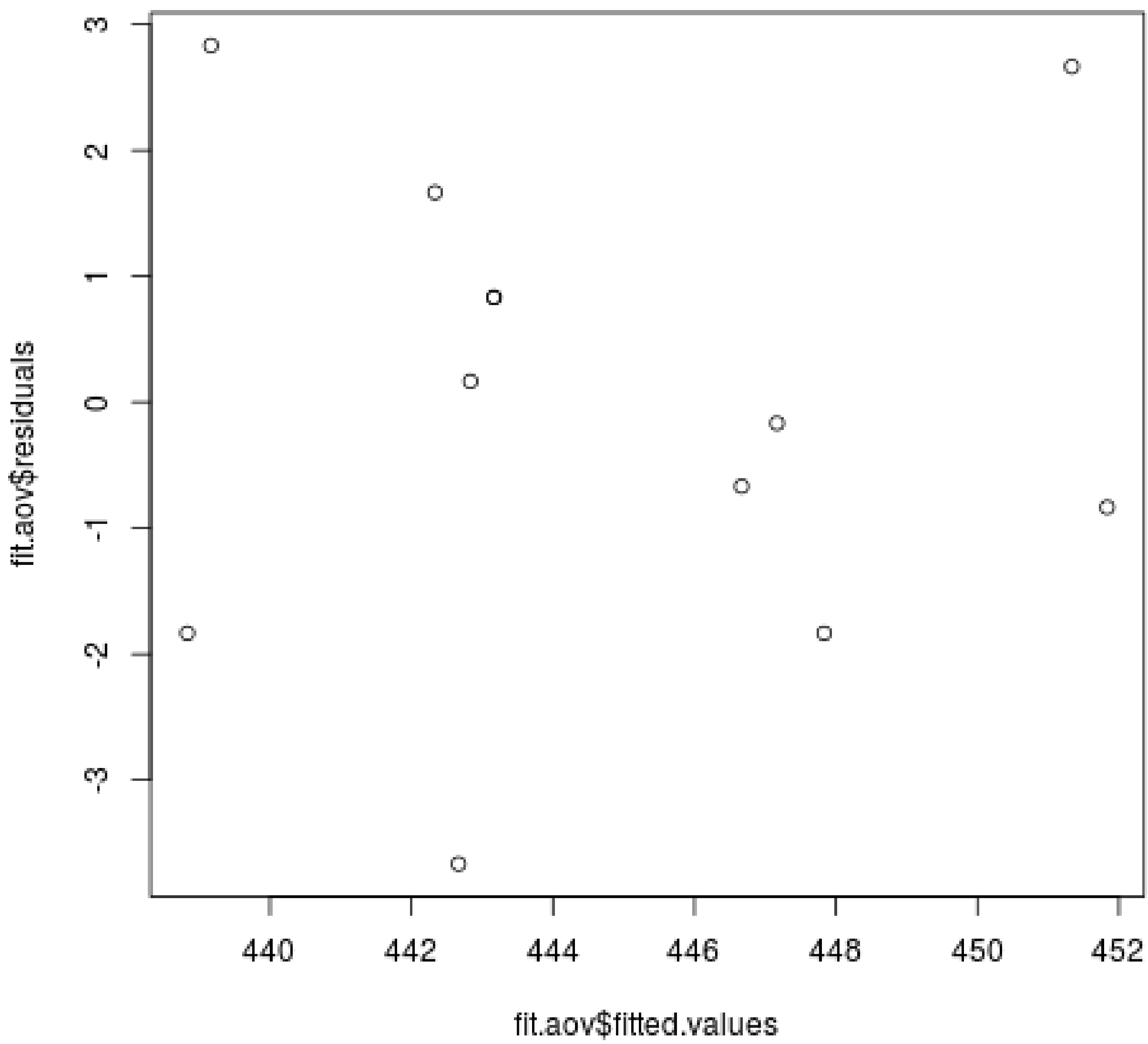
38

```
exe11.38=read.table(
    "http://www.uio.no/studier/emner/matnat/math/STK2120/v16/exe11-38.txt",
    header=T,sep=",");
fit.aov=aov(Response~factor(Paint)+factor(Roller),data=exe11.38);
summary(fit.aov)

#                 Df Sum Sq Mean Sq F value Pr(>F)
#factor(Paint)     3 159.58   53.19   7.848 0.0169 *
#factor(Roller)    2  38.00   19.00   2.803 0.1381
#Residuals         6  40.67    6.78

                                    #b) Paint is significant for \alpha=0.05.
                                    #c) Roller is not.


tukey.Paint=TukeyHSD(fit.aov,"factor(Paint)")
png('exe1138tuk.png')
plot(tukey.Paint)
dev.off()
print(tukey.Paint)

                                    #Paint Brand 1 seems preferrable
#e
png('exe1138norm.png')
qqnorm(fit.aov$residuals)
```

```
dev.off()
                                        #Normalitity is reasonable
png('exe1138res.png')
plot(fit.aov$fitted.values, fit.aov$residuals)
dev.off()
                                        #Constant variance is reasonable
```



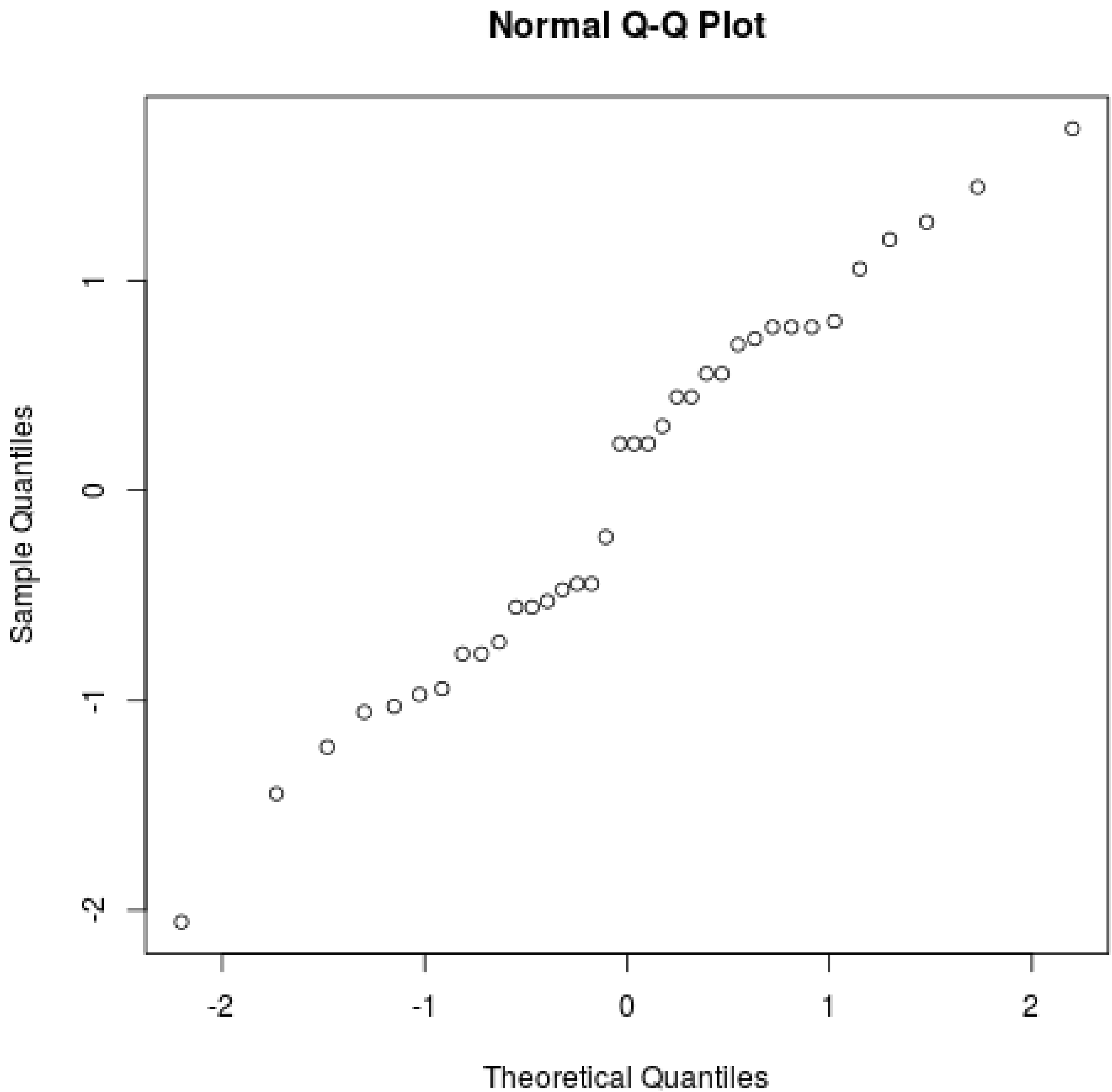

Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
3
2
1
0
-1
-2
-3
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5

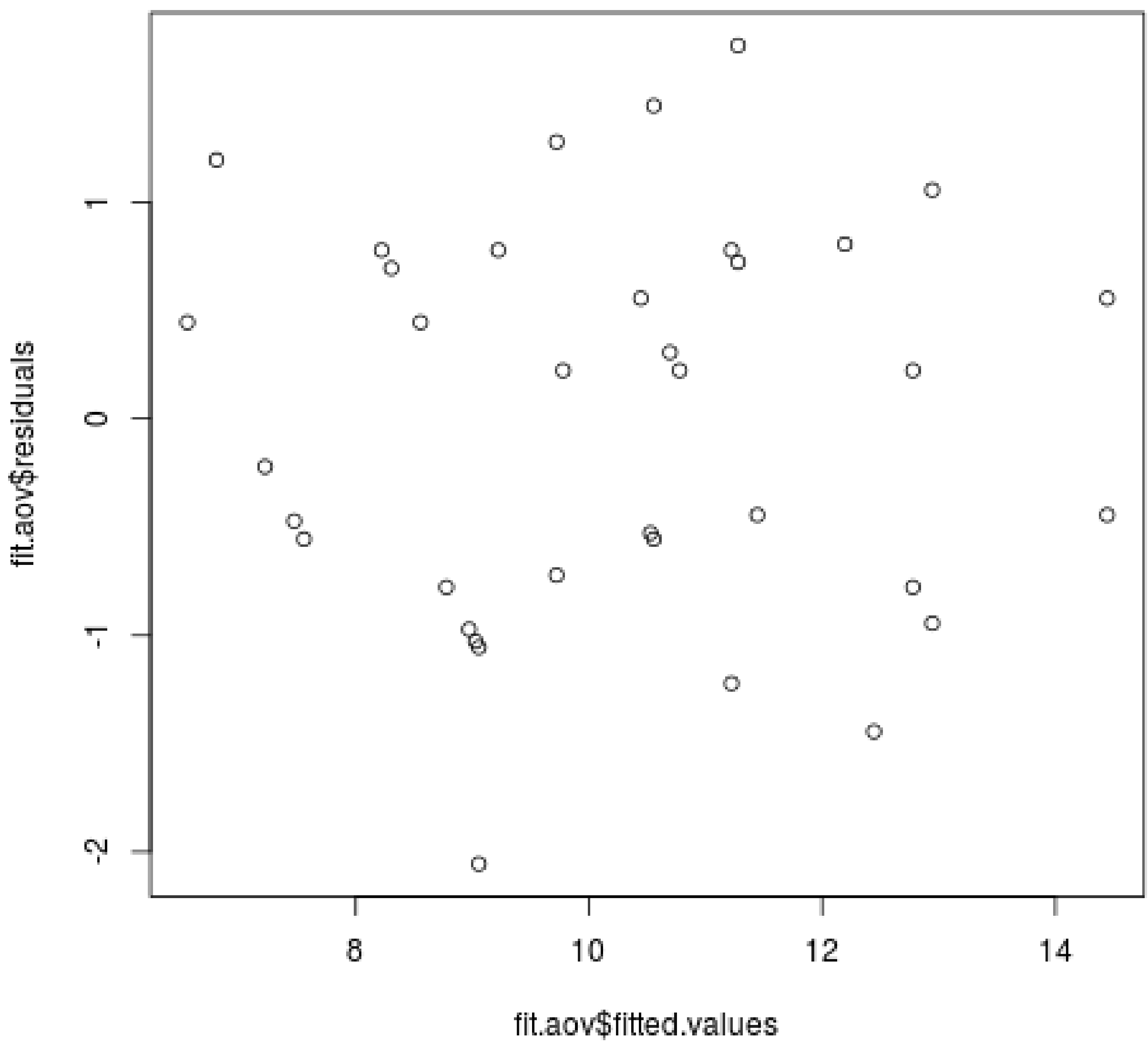


42

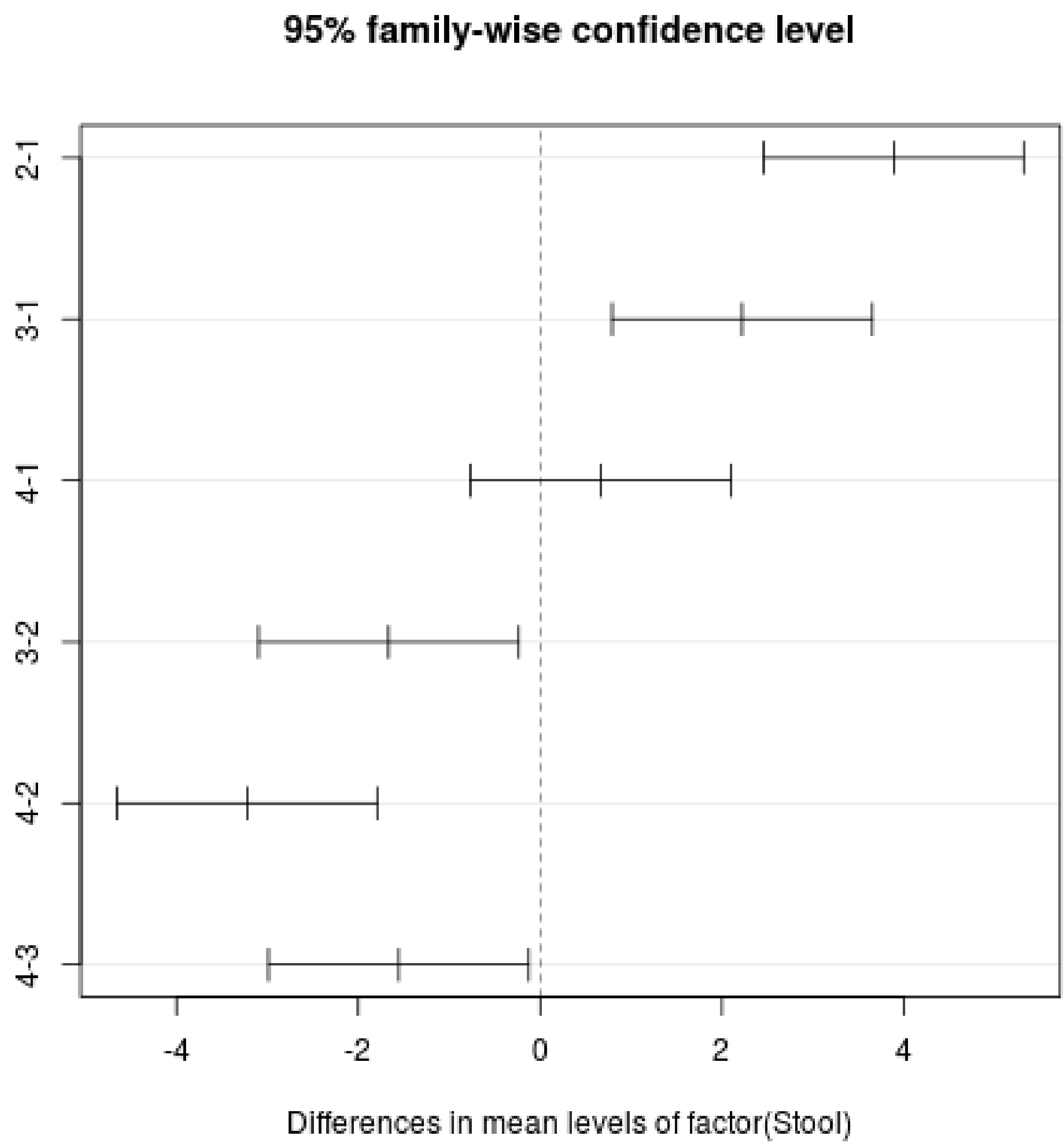
```
exe11.42=read.table(
    "http://www.uio.no/studier/emner/matnat/math/STK2120/v16/exe11-42.txt",
    header=T,sep=",")

fit.aov=aov(response~factor(subject)+factor(Stool),data=exe11.42);
summary(fit.aov)
#                  Df Sum Sq Mean Sq F value    Pr(>F)
#factor(subject)   8  66.50   8.312   6.866 0.000106 ***
#factor(Stool)     3  81.19  27.065  22.356 3.93e-07 ***
#Residuals        24  29.06   1.211
#---
```

```
png('exe1142norm.png')
qqnorm(fit.aov$residuals)
dev.off()

png('exe1142res.png')
plot(fit.aov$fitted.values, fit.aov$residuals)
dev.off()

tukey.Stool=TukeyHSD(fit.aov,"factor(Stool)")

png('exe1142tuk.png')
plot(tukey.Stool)
dev.off()
print(tukey.Stool)
```


Normal Q-Q Plot
Sample Quantiles
Theoretical Quantiles
-2
-1
0
1
2
-2
-1
0
1



fit.aov$residuals
fit.aov$fitted.values
-2
-1
0
1
8
10
12
14


95% family-wise confidence level
2-1
3-1
4-1
3-2
4-2
4-3
-4
-2
0
2
4
Differences in mean levels of factor(Stool)

46

a

$$
\begin{aligned}
y_{ij} &= x_{ij} + d \\
\bar{y}_{..} &= \bar{x}_{..} + d \\
SST_y &= \sum_i \sum_j (y_{ij} - \bar{y}_{..})^2 \\
&= \sum_i \sum_j ((x_{ij} + d) - (\bar{x}_{..} + d))^2 \\
&= \sum_i \sum_j (x_{ij} - \bar{x}_{..})^2 \\
&= SST_x
\end{aligned}
$$

$$
\begin{aligned}
SSE_y &= \sum_i \sum_j (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 \\
&= \sum_i \sum_j ((x_{ij} + d) - (\bar{x}_{i.} + d) - (\bar{x}_{.j} + d) + (\bar{x}_{..} + d))^2 \\
&= \sum_i \sum_j (x_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{x}_{..})^2 \\
&= SSE_x
\end{aligned}
$$

b

$$
\begin{aligned}
y_{ij} &= cx_{ij} \\
\bar{y}_{..} &= c\bar{x}_{..} \\
SST_y &= \sum_i \sum_j (y_{ij} - \bar{y}_{..})^2 \\
&= \sum_i \sum_j (cx_{ij} - c\bar{x}_{..})^2 \\
&= c^2 \sum_i \sum_j (x_{ij} - \bar{x}_{..})^2 \\
&= c^2 SST_x
\end{aligned}
$$

$$
\begin{aligned}
SSE_y &= \sum_i \sum_j (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 \\
&= \sum_i \sum_j (cx_{ij} - c\bar{x}_{i.} - c\bar{x}_{.j} + c\bar{x}_{..})^2 \\
&= c^2 \sum_i \sum_j (x_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{x}_{..})^2 \\
&= c^2 SSE_x
\end{aligned}
$$

$$
\begin{aligned}
F_{Ay} &= MSA_y/MSE_y \\
&= c^2 MSA_x/(c^2 MSE_x) \\
&= MSA_x/MSE_x = F_{Ax}
\end{aligned}
$$

47

$$
\begin{aligned}
E(\bar{X}_{i.} - \bar{X}_{..}) &= \\
E(\sum_j X_{ij}/J - \sum_i \sum_j X_{ij}/IJ) &= \\
\sum_j (\mu + \alpha_i + \beta_j)/J - \sum_i \sum_j (\mu + \alpha_i + \beta_j)/(IJ) &= \\
\mu + \alpha_i + \sum_j \beta_j/J - \sum_i (\mu + \alpha_i + \sum_j \beta_j/J)/I &= \\
\mu + \alpha_i - \sum_i (\mu + \alpha_i)/I &= \\
\mu + \alpha_i - \mu + \sum_i \alpha_i/I &= \\
\alpha_i &
\end{aligned}
$$

Extra 3

a

$$\begin{aligned}
T &= \frac{\bar{D}}{S/\sqrt{J}} \\
T^2 &= \frac{\bar{D}^2}{S^2/J} \\
T^2 &= \frac{\bar{D}^2}{1/(J-1)\sum_j (D_j - \bar{D})^2/J}
\end{aligned}$$

$$\begin{aligned}
F_A &= MSA/MSE \\
&= (SSA/(I-1))/(SSE/(I-1)(J-1)) \\
&= \frac{(\bar{X}_{1\cdot} - \bar{X}_{\cdot\cdot})^2 + (\bar{X}_{2\cdot} - \bar{X}_{\cdot\cdot})^2}{\sum_j ((X_{j1} - \bar{X}_{1\cdot} - \hat{X}_{\cdot j} + \bar{X}_{\cdot\cdot})^2 + (X_{j2} - \bar{X}_{2\cdot} - \hat{X}_{\cdot j} + \bar{X}_{\cdot\cdot})^2)/(J-1)}
\end{aligned}$$

$$\begin{aligned}
\bar{D} &= \sum_j (X_{1j} - X_{2j})/J \\
&= \bar{X}_{1\cdot} - \bar{X}_{2\cdot} \\
\bar{D}^2 &= (\bar{X}_{1\cdot} - \bar{X}_{2\cdot})^2 = 4(\bar{X}_{1\cdot} - \bar{X}_{\cdot\cdot})^2 = 4(\bar{X}_{2\cdot} - \bar{X}_{\cdot\cdot})^2 = 2\left[(\bar{X}_{1\cdot} - \bar{X}_{\cdot\cdot})^2 + (\bar{X}_{2\cdot} - \bar{X}_{\cdot\cdot})^2\right] \\
&= 2SSA/J
\end{aligned}$$

$$\begin{aligned}
D_j - \bar{D} &= (X_{1j} - X_{2j}) - (\bar{X}_{1\cdot} - \bar{X}_{2\cdot}) \\
&= 2(X_{1j} - X_{\cdot j}) - 2(\bar{X}_{1\cdot} - \bar{X}_{\cdot\cdot}) \\
&= 2(X_{1j} - X_{\cdot j}) - 2(\bar{X}_{1\cdot} - \bar{X}_{\cdot\cdot}) \\
(D_j - \bar{D})^2 &= 4((X_{1j} - X_{\cdot j}) - (\bar{X}_{1\cdot} - \bar{X}_{\cdot\cdot}))^2 \\
&= 4((X_{2j} - X_{\cdot j}) - (\bar{X}_{2\cdot} - \bar{X}_{\cdot\cdot}))^2 \\
&= 2((X_{1j} - X_{\cdot j}) - (\bar{X}_{1\cdot} - \bar{X}_{\cdot\cdot}))^2 + 2((X_{2j} - X_{\cdot j}) - (\bar{X}_{2\cdot} - \bar{X}_{\cdot\cdot}))^2 \\
\sum_j (D_j - \bar{D})^2 &= 2SSE
\end{aligned}$$

$$\begin{aligned}
\frac{\bar{D}^2}{S^2/J} &= \frac{2SSA}{1/(J-1)2SSE/J} \\
&= \frac{MSA}{MSE/J}
\end{aligned}$$

Eksamensoppgave 3

a

Source	*df*	*SS*	*MS*	*F*
Treatment		37.784		
Residual				
Total		48.736		

Source	df	SS	MS	F
Treatment	$(I-1)$	37.784	$MSTr = SSTr/(I-1)$	$f = MSTr/MSE$
Residual	$(n-I)$	$SSE = SST - SSE$	$MSE = SSE/(n-I)$	
Total	$n-1$	48.736		

Source	df	SS	MS	F
Treatment	3	37.784	12.595	25.450
Residual	23	10.952	0.476	
Total	26	48.736		

$$F_{.05,3,23} = 3.028$$

We reject H_0

b

(i) Reject H_{12} (H_{ij}) if:

$$\frac{X_{1\cdot} - X_{2\cdot}}{\sqrt{MSE(1/9+1/7)}} \geq t_{\alpha,23}$$
$$\frac{X_{i\cdot} - X_{j\cdot}}{\sqrt{MSE(1/9+1/7)}} \geq t_{\alpha,23}$$

(ii) We use the results from exercise 2 with $k = I(I-1)/2 = 4*3/2 = 6$. We then get that $t_{\alpha/(2k),23} = t_{\alpha/12,23}$ corrects for the k different two sided t-tests.
(iii)

$$T_{ij}:$$
$$\mid \bar{y}_{\cdot i} - \bar{y}_{\cdot j} \mid < t_{.05/12,23}\sqrt{MSE(1/n_i + 1/n_j)}$$
$$\mid \bar{y}_{\cdot i} - \bar{y}_{\cdot j} \mid / \sqrt{1/n_i + 1/n_j} < 2.886 * 0.69 = 1.99$$

$$\mid \bar{y}_{\cdot 1} - \bar{y}_{\cdot 2} \mid / \sqrt{1/n_1 + 1/n_2} = 2.95 > 1.99$$
$$\mid \bar{y}_{\cdot 1} - \bar{y}_{\cdot 3} \mid / \sqrt{1/n_1 + 1/n_3} = 3.11 > 1.99$$
$$\mid \bar{y}_{\cdot 1} - \bar{y}_{\cdot 4} \mid / \sqrt{1/n_1 + 1/n_4} = 2.24 > 1.99$$

$$\mid \bar{y}_{\cdot 2} - \bar{y}_{\cdot 3} \mid / \sqrt{1/n_2 + 1/n_3} = 5.36 > 1.99$$
$$\mid \bar{y}_{\cdot 2} - \bar{y}_{\cdot 4} \mid / \sqrt{1/n_2 + 1/n_4} = 4.90 > 1.99$$

$$\mid \bar{y}_{\cdot 3} - \bar{y}_{\cdot 4} \mid / \sqrt{1/n_3 + 1/n_4} = 1.17 < 1.99$$

All except 3 and 4 are significantly different