

49

a

Source	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Curing Time		30763.0		
Mix		34185.6		
Residual		97436.8		
Total		205966.6		
Source	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Curing Time	$(I - 1)$	30763.0	$SSA/(I - 1)$	MSA/MSE
Mix	$(J - 1)$	34185.6	$SSB/(J - 1)$	MSB/MSE
Interaction	$(I - 1)(J - 1)$	<i>SSAB</i>	$SSAB/(I - 1)(J - 1)$	$MSAB/MSE$
Error	$IJ(K - 1)$	97436.8	$SSE/IJ(K - 1)$	
Total	$IJK - 1$	205966.6		
Source	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>
Curing Time	2	30763.0	15368.0	3.79
Mix	3	34185.6	11395.2	2.81
Interaction	6	43608.2	7268.0	1.79
Error	24	97436.8	4059.6	
Total	35	205966.6		

b

$$f_{AB} = 1.79 < F_{.05,6,24} = 2.51$$

So we do not reject H_{0AB} , so we conclude that there is no interaction

c

$$f_A = 3.79 > F_{.05,2,24} = 3.40$$

so we reject H_{0A}

d

$$f_B = 2.81 < F_{.05,3,24} = 3.01$$

so we do not reject H_{0B}

e

$$\begin{aligned} w &= Q_{\alpha,I,IJ(K-1)}\sqrt{MSE/JK} \\ &= Q_{.05,3,4}\sqrt{4059.6/(4 * 3)} \\ &= 64.93 \end{aligned}$$

$\bar{x}_{3..}$	$\bar{x}_{1..}$	$\bar{x}_{2..}$
3960.02	4010.88	4029.10

53

```

exe11.53=read.table(
  "http://www.uio.no/studier/emner/matnat/math/STK2120/v16/exe11-53.txt",
  header=T, sep=",")

# Toveis variananalyse (jf. side 600 i D&B):
fit.exe11.53=aov(lifetime~factor(pen)+factor(surface)+ factor(pen):factor(surface),
  data=exe11.53)
summary(fit.exe11.53)

#
#factor(pen)          Df Sum Sq Mean Sq F value Pr(>F)
#factor(surface)     2  2888  1444.0  2.109  0.164
#factor(pen):factor(surface) 6  8100  1350.0  1.972  0.149
#Residuals          12  8216  684.7
#

```

No significant effects

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a

$$\begin{aligned}
& E(\bar{X}_{i..} - \bar{X}_{...}) \\
&= \sum_j \sum_k E(X_{ijk})/(JK) - \sum_i \sum_j \sum_k E(X_{ijk})/(IJK) \\
&= \sum_j \sum_k (\mu + \alpha_i + \beta_j + \gamma_{ij})/(JK) - \sum_i \sum_j \sum_k (\mu + \alpha_i + \beta_j + \gamma_{ij})/(IJK) \\
&= \sum_j (\mu + \alpha_i + \beta_j + \gamma_{ij})/J - \sum_i \sum_j (\mu + \alpha_i + \beta_j + \gamma_{ij})/(IJ) \\
&= \mu + \alpha_i + \sum_j (\beta_j + \gamma_{ij})/J - \mu - \sum_i \alpha_i - \sum_i \sum_j (\beta_j + \gamma_{ij})/(IJ) \\
&= \mu + \alpha_i + \sum_j \beta_j/J + \sum_j \gamma_{ij}/J - \mu - \sum_i \alpha_i - \sum_i (\sum_j \beta_j/IJ + \sum_j \gamma_{ij}/IJ) \\
&= \mu + \alpha_i - \mu = \alpha_i
\end{aligned}$$

b

$$\begin{aligned}
E(\hat{\gamma}_{ij}) &= E(\bar{X}_{ij.} - \bar{X}_{i..} - \bar{X}_{.j.} + \bar{X}_{...}) \\
&= E(\bar{X}_{ij.} - \bar{X}_{.j.}) - E(\bar{X}_{i..} + \bar{X}_{...}) \\
&= \sum_k E(X_{ijk})/K - \sum_i \sum_k E(X_{ijk})/IK - \alpha_i \\
&= \sum_k (\mu + \alpha_i + \beta_j + \gamma_{ij})/K - \sum_i \sum_k (\mu + \alpha_i + \beta_j + \gamma_{ij})/IK - \alpha_i \\
&= (\mu + \alpha_i + \beta_j + \gamma_{ij}) - (\mu + \beta_j) - \alpha_i \\
&= \gamma_{ij}
\end{aligned}$$

Ekamen 1

Fixed Effects model with interaction.				
Source	df	SS	MS	F
Kjonn	1	2.133	*	0.6410
Signaltype	*	97.267	*	14.6119
Interaksjon	*	*	*	3.4952
Error	*	79.880	*	
Total	29	202.547		

Source	df	SS	MS	F
Kjonn	1	2.133	2.133	0.6410
Signaltype	2	97.267	48.634	14.6119
Interaksjon	2	23.267	11.634	3.4952
Error	24	79.880	3.328	
Total	29	202.547		

b

$$f_{AB} = 3.4952 \geq F_{.05,2,24} = 3.4$$

we reject H_{0AB}

$$f_A = 0.64 < F_{.05,1,24} = 4.26$$

$$f_B = 14.6119 \geq F_{.05,2,24} = 3.4$$

Signaltype is a significant factor.

Eksamen 2

a

$$X_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$$

$E(MSE) = \sigma^2$ so $MSE = 5.99$ is an estimate for the variance.

b

f_A : is speed a significant factor?

f_B : is Formulat a significant factor?

f_{AB} : is the interaction between speed and formulat a significant factor?

Yes, yes, no ($\alpha = 0.05$)

A simplified model is two-factor anova without interaction.