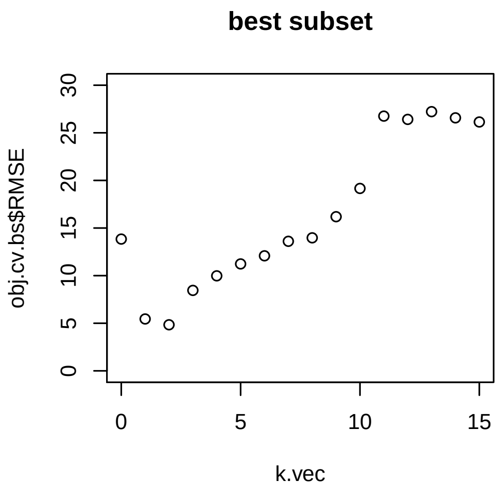
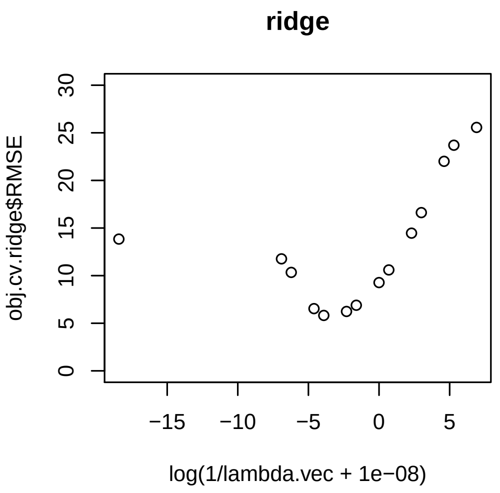
R-code for Extra exercise 3.2
Linear regression by OLS, best subset regression and ridge regression

Part a)

```
library(MASS)
library(leaps)

source("/nr/user/aldrin/UiO/STK4030/Rfunc/div.sim.r")
source("/nr/user/aldrin/UiO/STK4030/Rfunc/cv.k.r")
source("/nr/user/aldrin/UiO/STK4030/Rfunc/lm.best.subset.r")
source("/nr/user/aldrin/UiO/STK4030/Rfunc/calc.err.r")
source("/nr/user/aldrin/UiO/STK4030/Rfunc/calc.res.r")

p<-15

beta0<-3
beta<-c(rep(2,5),rep(1,5),rep(0,5))

Sigma.x<-matrix(0.8,nrow=p,ncol=p)
diag(Sigma.x)<-1

sd.eps<-5

### set k for k-fold cross validation
k.in.cv<-10



n.train<-20

### simulate X in the training data
X.train<-sim.X(n.train,p,Sigma.x)

### Simulate y in the trainig set and in the test set
y.train<-sim.y.given.X(X.train,beta0,beta,sd.eps)
y.test<-sim.y.given.X(X.test,beta0,beta,sd.eps)

### construct the training data
training.data<-data.frame(y=y.train,X=I(X.train))
```

```
ind.cv<-1:nrow(training.data)
ind.cv<-sample(ind.cv)

k.vec<-seq(0,p,1)
obj.cv.bs<-cv.k(data=training.data,k=k.in.cv,ind=ind.cv,lm.method="best.subset",
                 tuning.par=k.vec)
obj.bs<-lm.best.subset(x=training.data$X,y=training.data$y,
                       k.vec=obj.cv.bs$opt.tuning.par)
beta0.hat<-obj.bs$coef[1]
beta.hat<-obj.bs$coef[-1]


lambda.vec<-c(1e20,
              1000,500,100,50,10,5,1,
              0.5,0.1,0.05,0.01,0.005,0.001,
              0)
obj.cv.ridge<-cv.k(data=training.data,k=k.in.cv,ind=ind.cv,lm.method="ridge",
                   tuning.par=lambda.vec)
obj.ridge<-lm.ridge(y~X,data=training.data,
                    lambda=obj.cv.ridge$opt.tuning.par)
beta0.hat<-coef(obj.ridge)[1]
beta.hat<-coef(obj.ridge)[-1]

pdf(file="tuning.pdf")
par(mfrow=c(2,2))
plot(k.vec,obj.cv.bs$RMSE,main="best subset",ylim=c(0,30))
plot(log(1/lambda.vec+0.00000001),obj.cv.ridge$RMSE,main="ridge",ylim=c(0,30))
dev.off()
```


best subset
obj.cv.bs$RMSE
k.vec


ridge
obj.cv.ridge$RMSE
log(1/lambda.vec + 1e−08)

```
### in file div.sim.r

sim.X<-function(n,p,Sigma.x) {
  X<-mvrnorm(n = n, mu=rep(0,p), Sigma=Sigma.x)
  X
}

sim.y.given.X<-function(X,beta0,beta,sd.eps) {
  n<-nrow(X)
  eps<-rnorm(n,mean=0,sd=sd.eps)
  y<-beta0+X%*%beta+eps
  y
}
calc.Ey.given.X<-function(X,beta0,beta) {
  n<-nrow(X)
  Ey<-beta0+X%*%beta
  Ey
}
```

```
lm.best.subset<-function(x,y,k.vec=c(0,ncol(x))) {
###require(leaps)

if (is.vector(x)) {
  x<-matrix(x,ncol=1)
}

leaps.obj<-leaps(x,y,nbest=1)

p<-ncol(x)
k.diff<-length(k.vec)
coef<-matrix(0,nrow=k.diff,ncol=p+1)
which.matrix<-matrix(FALSE,nrow=k.diff,ncol=p+1)

ii<-0
for (k in k.vec) {
  ii<-ii+1
  if (k==0) {
    coef[ii,1]<-mean(y)
  } else {
    which<-leaps.obj$which[k,]
    x.subset<-x[,which]
    lm.obj<-lm(y~x.subset)
    which<-c(TRUE,which)
    which.matrix[ii,]<-which
    coef[ii,which]<-lm.obj$coef
  }

}

res<-list(coef=coef, which=which.matrix)
return(res)
}
```

```
cv.k<-function(data,k,ind=NULL,lm.method="ridge",tuning.par) {


  n<-nrow(data)
  if (is.null(ind)) {
    ind<-1:n
    ind<-sample(ind)
  }
  data<-data[ind,]

  n.most.parts<-floor(n/k)
  n.last.part<-n-(k-1)*n.most.parts

  ind<-list(k)
  for (i in 1:(k-1)) {
    ind[[i]]<-((i-1)*n.most.parts+1):(i*n.most.parts)
  }
  ind[[k]]<-((k-1)*n.most.parts+1):n

  pred<-matrix(NA,nrow=n,ncol=length(tuning.par))

  for (i in 1:k) {
    training.dat<-data[-ind[[i]],]
    test.dat<-data[ind[[i]],]

    if (lm.method=="ridge") {
      obj<-lm.ridge(y~X,data=training.dat,lambda=tuning.par)
      coef<-coef(obj)
### must use the coef function here, because lm.ridge else returns
### the scaled coefficients
    }

    if (lm.method=="best.subset") {
      obj<-lm.best.subset(x=training.dat$X,y=training.dat$y,k.vec=tuning.par)
      coef<-obj$coef
    }

    X.test<-cbind(1,test.dat$X)
    pred[ind[[i]],]<-X.test%*%t(coef)
  }

  RMSE<-sqrt(apply((data$y-pred)^2,2,mean))
  which.opt<-which.min(RMSE)
  opt.tuning.par<-tuning.par[which.opt]
  opt.RMSE<-tuning.par[which.opt]

  res<-list(opt.tuning.par=opt.tuning.par,
            opt.RMSE=opt.RMSE,
            which.opt=which.opt,
            tuning.par=tuning.par,
            RMSE=RMSE)
  res
}
```

```
calc.err<-function(err,beta0,beta,y.test,Ey.test,beta0.hat,beta.hat,X.test) {

###    y.pred<-predict(obj.lm,newdata=X.test.df)
### equivalent to
y.pred<-beta0.hat+X.test%*%beta.hat


### calculate errors for estimation or prediction of beta, y and f
err$beta.err[i,]<-c(beta0,beta)-c(beta0.hat,beta.hat)
err$pred.err[i,]<-y.test-y.pred
err$f.err[i,]<-Ey.test-y.pred

err
}
```

```
calc.res<-function(err) {

beta.err<-err$beta.err
pred.err<-err$pred.err
f.err<-err$f.err

### calculate bias, squared bias, variance and MSE for betahat
beta.bias<-apply(beta.err,2,mean)
beta.bias.2<-beta.bias^2
beta.var<-apply(beta.err,2,var)
beta.MSE<-apply(beta.err^2,2,mean)

beta.res<-cbind(beta.bias,beta.bias.2,beta.var,beta.MSE)

### converts matrices into vectors
pred.err<-as.vector(pred.err)
f.err<-as.vector(f.err)

### calculate bias, squared bias, variance and MSE for the prediction,
### i.e. for Xbetahat used as a predicton of y
pred.bias<-mean(pred.err)
pred.bias.2<-pred.bias^2
pred.var<-var(pred.err)
pred.MSE<-mean(pred.err^2)

### calculate bias, squared bias, variance and MSE for fhat,
### i.e. for fhat = Xbetahat used as an estimate of f

f.bias<-mean(f.err)
f.bias.2<-f.bias^2
f.var<-var(f.err)
f.MSE<-mean(f.err^2)

pred.res<-c(pred.bias,pred.bias.2,pred.var,pred.MSE)
f.res<-c(f.bias,f.bias.2,f.var,f.MSE)
res<-rbind(beta.res,pred.res,f.res)
colnames(res)<-c("bias","bias2","var","MSE")

tmp<-rep(NA,p+1)
for (i in 1:(p+1)) {
  tmp[i]<-paste("beta",i-1,sep="")
}
rownames(res)<-c(tmp,"pred","f")
res
}
```

```
Part b)

library(MASS)
library(leaps)

source("/nr/user/aldrin/UiO/STK4030/Rfunc/div.sim.r")
source("/nr/user/aldrin/UiO/STK4030/Rfunc/cv.k.r")
source("/nr/user/aldrin/UiO/STK4030/Rfunc/lm.best.subset.r")
source("/nr/user/aldrin/UiO/STK4030/Rfunc/calc.err.r")
source("/nr/user/aldrin/UiO/STK4030/Rfunc/calc.res.r")

p<-15

beta0<-3
beta<-c(rep(2,5),rep(1,5),rep(0,5))

Sigma.x<-matrix(0.8,nrow=p,ncol=p)
diag(Sigma.x)<-1

sd.eps<-5

### set k for k-fold cross validation
k.in.cv<-10


### number of observations in training sets and test sets
n.train1<-20
n.train2<-100
n.train3<-1000
n.test<-1000


### number of simulations
n.sim1<-1000
n.sim2<-300
n.sim3<-100
###n.sim<-1000

### number of methods investigated
n.met<-3


### The simulation results will be put in this object,
### which is a list of lists,
### and the element of the inner list will be a matrix
res<-list()
for (j in 1:3) {
  res[[j]]<-list()
  for (no.met in 1:n.met) {
    res[[j]][[no.met]]<-NULL
  }
}

### Varies the number of observations in the training data
for (j in 1:3) {
  print(paste("j=",j))
  if (j==1) {
    n.train<-n.train1
    n.sim<-n.sim1
  }
  if (j==2) {
    n.train<-n.train2
    n.sim<-n.sim2
  }
  if (j==3) {
    n.train<-n.train3
    n.sim<-n.sim3
  }


### Construct matrices where we will store errors
  beta.err<-matrix(NA,n.sim,p+1)
  pred.err<-matrix(NA,n.sim,n.test)
  f.err<-matrix(NA,n.sim,n.test)

  err<-list()
  for (no.met in 1:n.met) {
    err[[no.met]]<-list(beta.err=beta.err,pred.err=pred.err,f.err=f.err)
  }

### loop over the simulations
  for (i in 1:n.sim) {
  print(paste("j=",j," i=",i))


### construction of the matrix X in the test set
```

```
  X.test<-sim.X(n.test,p,Sigma.x)

### Copy the matrix X into a data frame
### The I function is used to get correct variable names in the prediction
### Not used now
###  X.test.df<-data.frame(X=I(X.test))

### construction of E(Y|X) in the test set
  Ey.test<-calc.Ey.given.X(X.test,beta0,beta)

### simulate X in the training data
    X.train<-sim.X(n.train,p,Sigma.x)

### Simulate y in the trainig set and in the test set
    y.train<-sim.y.given.X(X.train,beta0,beta,sd.eps)
    y.test<-sim.y.given.X(X.test,beta0,beta,sd.eps)

### construct the training data
    training.data<-data.frame(y=y.train,X=I(X.train))

### For the same random order of observations for all methods
### when performing cross validation
  ind.cv<-1:nrow(training.data)
  ind.cv<-sample(ind.cv)

### Estimate the model by OLS
    met.no<-1
    obj.lm<-lm(y~X,data=training.data)
    beta0.hat<-obj.lm$coef[1]
    beta.hat<-obj.lm$coef[-1]
    err[[met.no]]<-calc.err(err[[met.no]],
                            beta0,beta,y.test,Ey.test,
                            beta0.hat,beta.hat,X.test)

### Estimate the model by best subset regression and k-fold cross validation
    met.no<-2
    obj.cv.bs<-cv.k(data=training.data,k=k.in.cv,ind=ind.cv,
                    lm.method="best.subset",
                    tuning.par=seq(0,p,1))
    obj.bs<-lm.best.subset(x=training.data$X,y=training.data$y,
                           k.vec=obj.cv.bs$opt.tuning.par)
    beta0.hat<-obj.bs$coef[1]
    beta.hat<-obj.bs$coef[-1]
    err[[met.no]]<-calc.err(err[[met.no]],
                            beta0,beta,y.test,Ey.test,
                            beta0.hat,beta.hat,X.test)

### Estimate the model by ridge regression and k-fold cross validation
    met.no<-3
    lambda.vec<-c(1e20,
                  1000,500,100,50,10,5,1,
                  0.5,0.1,0.05,0.01,0.005,0.001,
                  0)
    obj.cv.ridge<-cv.k(data=training.data,k=k.in.cv,ind=ind.cv,
                       lm.method="ridge",
                       tuning.par=lambda.vec)
    obj.ridge<-lm.ridge(y~X,data=training.data,
                        lambda=obj.cv.ridge$opt.tuning.par)
    beta0.hat<-coef(obj.ridge)[1]
    beta.hat<-coef(obj.ridge)[-1]
    err[[met.no]]<-calc.err(err[[met.no]],
                            beta0,beta,y.test,Ey.test,
                            beta0.hat,beta.hat,X.test)


  }

  for (met.no in 1:n.met) {
    res[[j]][[met.no]]<-calc.res(err[[met.no]])
  }

}



### divides results into
### bias, standard deviation and root mean squared errors

rowlen<-p+1+2

tmp<-rep(NA,p+1)
for (i in 1:(p+1)) {
  tmp[i]<-paste("beta",i-1,sep="")
}

BIAS<-list()
```

```
SD<-list()
RMSE<-list()
for (j in 1:3) {
  BIAS[[j]]<-matrix(NA,rowlen,n.met)
  SD[[j]]<-matrix(NA,rowlen,n.met)
  RMSE[[j]]<-matrix(NA,rowlen,n.met)

  colnames(BIAS[[j]])<-c("OLS","Best-ss","Ridge")
  colnames(SD[[j]])<-c("OLS","Best-ss","Ridge")
  colnames(RMSE[[j]])<-c("OLS","Best-ss","Ridge")

  rownames(BIAS[[j]])<-c(tmp,"pred","f")
  rownames(SD[[j]])<-c(tmp,"pred","f")
  rownames(RMSE[[j]])<-c(tmp,"pred","f")

  for (met.no in 1:n.met) {
    BIAS[[j]][,met.no]<-res[[j]][[met.no]][,"bias"]
    SD[[j]][,met.no]<-sqrt(res[[j]][[met.no]][,"var"])
    RMSE[[j]][,met.no]<-sqrt(res[[j]][[met.no]][,"MSE"])
  }
}


### Print results on screen

round(BIAS[[1]],1)
round(SD[[1]],1)
round(RMSE[[1]],1)

round(BIAS[[2]],1)
round(SD[[2]],1)
round(RMSE[[2]],1)

round(BIAS[[3]],1)
round(SD[[3]],1)
round(RMSE[[3]],1)
```

```
n.train=20
----------
round(BIAS[[1]],1)
        OLS Best-ss Ridge
beta0   0.0     0.0   0.0
beta1  -0.2     0.4   0.7
beta2   0.1     0.5   0.7
beta3   0.2     0.6   0.7
beta4   0.0     0.4   0.7
beta5  -0.2     0.6   0.6
beta6   0.2     0.1   0.0
beta7   0.0     0.1   0.0
beta8   0.5     0.1   0.1
beta9  -0.3    -0.1   0.0
beta10  0.0     0.0   0.0
beta11  0.2    -0.3  -0.6
beta12 -0.2    -0.4  -0.6
beta13  0.1    -0.5  -0.6
beta14 -0.2    -0.5  -0.6
beta15 -0.1    -0.4  -0.6
pred    0.0     0.0   0.0
f       0.0     0.0   0.0
> round(SD[[1]],1)
        OLS Best-ss Ridge
beta0   2.9     1.9   1.6
beta1   6.6     3.7   2.0
beta2   6.3     3.6   1.7
beta3   6.3     4.0   2.7
beta4   6.3     3.5   1.5
beta5   6.2     3.2   1.5
beta6   6.0     3.0   1.4
beta7   6.4     2.9   1.5
beta8   6.8     3.2   2.0
beta9   6.4     3.6   2.3
beta10  6.3     3.3   1.7
beta11  6.0     2.6   1.5
beta12  6.5     2.9   1.9
beta13  6.5     3.7   2.9
beta14  6.0     2.6   1.4
beta15  6.5     2.8   1.8
pred   12.7     8.1   6.5
f      11.7     6.3   4.1
```

```
> round(RMSE[[1]],1)
        OLS Best-ss Ridge
beta0   2.9     1.9   1.6
beta1   6.6     3.7   2.1
beta2   6.3     3.6   1.8
beta3   6.3     4.1   2.8
beta4   6.3     3.5   1.7
beta5   6.2     3.3   1.6
beta6   6.0     3.0   1.4
beta7   6.4     2.9   1.5
beta8   6.8     3.2   2.0
beta9   6.4     3.6   2.3
beta10  6.3     3.3   1.7
beta11  6.0     2.6   1.6
beta12  6.5     3.0   2.0
beta13  6.5     3.8   3.0
beta14  6.0     2.6   1.5
beta15  6.5     2.8   1.9
pred   12.7     8.1   6.5
f      11.7     6.3   4.1
```

```
n.train=100
-----------
round(BIAS[[2]],1)
        OLS Best-ss Ridge
beta0   0.0     0.0   0.0
beta1   0.0    -0.1   0.6
beta2   0.0     0.0   0.6
beta3  -0.1     0.0   0.6
beta4   0.0     0.0   0.6
beta5   0.0     0.1   0.6
beta6   0.0     0.2   0.0
beta7   0.0     0.1   0.0
beta8   0.0     0.1   0.0
beta9   0.0     0.0   0.0
beta10  0.0     0.1   0.0
beta11  0.1     0.0  -0.5
beta12  0.0     0.0  -0.5
beta13  0.0    -0.1  -0.5
beta14 -0.2    -0.2  -0.6
beta15  0.0    -0.1  -0.5
pred    0.0     0.0   0.0
f       0.0     0.0   0.0
> round(SD[[2]],1)
       OLS Best-ss Ridge
beta0  0.5     0.5   0.5
beta1  1.2     1.5   0.6
beta2  1.2     1.4   0.6
beta3  1.1     1.4   0.6
beta4  1.2     1.5   0.6
beta5  1.2     1.5   0.6
beta6  1.2     1.2   0.6
beta7  1.2     1.3   0.6
beta8  1.2     1.2   0.6
beta9  1.2     1.3   0.6
beta10 1.2     1.3   0.6
beta11 1.2     1.0   0.6
beta12 1.2     1.1   0.6
beta13 1.2     1.0   0.6
beta14 1.2     1.1   0.6
beta15 1.2     1.1   0.6
pred   5.5     5.5   5.2
f      2.2     2.3   1.5
```

```
> round(RMSE[[2]],1)
       OLS Best-ss Ridge
beta0  0.5     0.5   0.5
beta1  1.2     1.5   0.8
beta2  1.2     1.4   0.8
beta3  1.1     1.4   0.8
beta4  1.2     1.5   0.9
beta5  1.2     1.5   0.9
beta6  1.2     1.3   0.6
beta7  1.2     1.3   0.6
beta8  1.2     1.2   0.6
beta9  1.2     1.3   0.6
beta10 1.2     1.3   0.6
beta11 1.2     1.0   0.8
beta12 1.2     1.1   0.8
beta13 1.2     1.0   0.8
beta14 1.2     1.1   0.8
beta15 1.2     1.1   0.8
pred   5.5     5.5   5.2
f      2.2     2.3   1.5
```

```
n.train=1000
------------
round(BIAS[[3]],1)
        OLS Best-ss Ridge
beta0   0.0     0.0   0.0
beta1   0.0     0.0   0.2
beta2   0.0     0.0   0.2
beta3   0.0     0.0   0.1
beta4   0.0     0.0   0.1
beta5   0.1     0.0   0.2
beta6   0.0     0.0   0.0
beta7   0.0     0.0   0.0
beta8  -0.1    -0.1   0.0
beta9   0.0     0.0   0.0
beta10  0.1     0.1   0.1
beta11  0.0     0.0  -0.1
beta12  0.0     0.0  -0.1
beta13  0.0     0.0  -0.2
beta14  0.0     0.0  -0.2
beta15  0.0     0.0  -0.1
pred    0.0     0.0   0.0
f       0.0     0.0   0.0
> round(SD[[3]],1)
       OLS Best-ss Ridge
beta0  0.1     0.1   0.1
beta1  0.3     0.3   0.3
beta2  0.4     0.4   0.3
beta3  0.3     0.3   0.3
beta4  0.3     0.3   0.3
beta5  0.3     0.3   0.3
beta6  0.3     0.4   0.3
beta7  0.4     0.4   0.3
beta8  0.3     0.4   0.3
beta9  0.4     0.5   0.3
beta10 0.3     0.4   0.3
beta11 0.4     0.3   0.3
beta12 0.3     0.2   0.3
beta13 0.3     0.2   0.3
beta14 0.3     0.2   0.3
beta15 0.3     0.2   0.3
pred   5.0     5.0   5.0
f      0.6     0.6   0.6
```

```
round(RMSE[[3]],1)
       OLS Best-ss Ridge
beta0  0.1     0.1   0.1
beta1  0.3     0.3   0.3
beta2  0.4     0.4   0.4
beta3  0.3     0.3   0.4
beta4  0.3     0.3   0.3
beta5  0.3     0.3   0.4
beta6  0.3     0.4   0.3
beta7  0.4     0.4   0.3
beta8  0.3     0.4   0.3
beta9  0.4     0.5   0.3
beta10 0.4     0.5   0.3
beta11 0.4     0.3   0.3
beta12 0.3     0.2   0.3
beta13 0.3     0.2   0.3
beta14 0.3     0.2   0.3
beta15 0.3     0.2   0.3
pred   5.0     5.0   5.0
f      0.6     0.6   0.6
```