Exercises, STK4040, week 41

October 5, 2007

Exercise 1

Assume that the stochastic vector $\boldsymbol{x}$ has dimension 5, and that it can be written as $\boldsymbol{x} = \mathrm{A}\boldsymbol{v} + \boldsymbol{e}$, where $\boldsymbol{v}$ is a stochastic vector with dimension 2, A is an arbitrary matrix, and $\boldsymbol{e}$ is a stochastic vector of the same dimension as $\boldsymbol{x}$, independent of $\boldsymbol{v}$. Let the matrix A be defined as

$$\mathrm{A} = \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 3 & 0 \\ 2 & -2 \\ 1 & 1 \end{pmatrix}.$$

The vector $\boldsymbol{e}$ is assumed to have $\mathsf{E}(\boldsymbol{e}) = \mathbf{0}$, and $\Phi = \mathrm{Cov}(\boldsymbol{e})$ is assumed to be diagonal with all diagonal elements equal to 0.1. The stochastic vector $\boldsymbol{v}$ is also assumed to have expectation $\mathbf{0}$, and $\Theta = \mathrm{Cov}(\boldsymbol{v}) = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.

Write the covariance matrix of $\boldsymbol{x}$ (Σ). Show that the coloumns of A are Eigenvectors of Σ.

Now assume that $\boldsymbol{v}$ and $\boldsymbol{e}$ are Normally distributed. Simulate 15 independent observations of the vector $\boldsymbol{x}$ and place them in a 15 by 5 data matrix X.

Perform a principal component analysis of X. Make a score plot for the two first components, and plot the two first loading vectors. Compare the loading vectors with the matrix A. Are there any similarities? Differences? Comment!

Exercise 2

Assume we have a data matrix X. Let the spectral decomposition of $\mathrm{X}^T\mathrm{X}$ be VLV^T. Show that the inverse of $\mathrm{X}^T\mathrm{X}$ (if it exists) can be written as $\mathrm{VL}^{-1}\mathrm{V}^T$. How does L^{-1} look?

Exercise 3

Let $\boldsymbol{x} \sim N_2(\mu, \Sigma)$ and $\boldsymbol{y} \sim N_2(\nu, \Sigma)$, where $\mu = (0,0)^T$, $\nu = (1,1)^T$ and $\Sigma = \begin{pmatrix} 3 & -1 \\ -1 & 4 \end{pmatrix}$.

Draw $m = 13$ observations of $\boldsymbol{x}$ and $n = 17$ observations of $\boldsymbol{y}$, and perform a (two-sample) Hotelling's T^2 test to test the hypothesis $H_0\colon \mu = \nu$ against the alternative hypothesis $H_A\colon \mu \neq \nu$.