Exercises, STK4040, week 44

October 26, 2007

Exercise 1

Do exercise 6.5.5 in the book.

Exercise 2

Let X be an $n \times p$ matrix of rank r, and $\boldsymbol{y}$ an n vector. Assume that both X an $\boldsymbol{y}$ are centered. Let $\mathrm{X}^T\mathrm{X} = \mathrm{VLV}^T$ be the Eigenvalue decomposition of $\mathrm{X}^T\mathrm{X}$, with Eigenvalues $l_1 \geq l_2 \geq \cdots \geq l_r > l_{r+1} = \cdots = 0$. (If $r = p$, all Eigenvalues are positive.) Let L^- be a $p \times p$ diagonal matrix with $1/l_1$, $1/l_2$, ..., $1/l_r$, 0, ..., 0 along the diagonal.

Show that whether $r = p$ or $r < p$ (for instance if $p > n$),

$$\boldsymbol{b} = \mathrm{VL}^-\mathrm{V}^T\mathrm{X}^T\boldsymbol{y} \tag{1}$$

is a solution to the normal equations $\mathrm{X}^T\mathrm{X}\beta = \mathrm{X}^T\boldsymbol{y}$.