Exercises for STK4130.

Exercise 1

a) Show the Tsjebysjeff inequality: If $\mu = \mathrm{E}[Y]$ and then

$$\mathrm{P}(|Y - \mu| > \varepsilon) \leq \frac{\mathrm{Var}[Y]}{\varepsilon^2}$$

b) Show the Markov inequality:

$$\mathrm{P}(|Y| > \varepsilon) \leq \frac{\mathrm{E}[|Y|^p]}{\varepsilon^p}$$

Exercise 2 The weak law of large numbers state that the average $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ converges in probability to $\mu = \mathrm{E}[Y_i]$ (when $n \to \infty$) if the Y_i are iid with existing expectation, i.e. $\mathrm{E}[|Y_i|] < \infty$. The simple proof of this result using the Tsjebysjeff inequality also requires that $\mathrm{Var}[|Y_i|] < \infty$.

For the Student t-distribution ν degrees of freedom the expectation exist (and equals 0) for $\nu > 1$ and the variance exists (and equals $\frac{\nu}{\nu-2}$) for $\nu > 2$.

a) Simulate $n = 100000$ iid Y_i from the t-distribution with $\nu = 3$ (using for instance `R`. Calculate and plot $\bar{Y}_m = \frac{1}{m}\sum_{i=1}^{m} Y_i$ for $m = 1, 2, \ldots, n$. In this case the variance exists and the law of large numbers is covered by the proof using Tsjebysjeff.

b) Repeat the simulation for $\nu = 2, 1.5$ and 1.1 (in which case the weak law of large numbers hold, but requires a more complicated proof).

c) Do also a simulation for $\nu = 1$. Then the weak law of large number no longer hold. Compare with the simulations in questions a) and b).