COMPULSORY ASSIGNMENT IN STK4510, AUTUMN 2016

DEADLINE OCTOBER 11, 2016, AT 14.30
DELIVER AT THE DEPARTMENT OF MATHEMATICS, 7TH FLOOR IN NHA BUILDING

In order to qualify for the written exam in STK4510, you must have passed this compulsory assignment. In order to pass, *all* exercises must be fully answered. Everyone must hand in their individual assignment on paper (electronically delivered assignments will not be accepted).

Exercise 1. In this exercise you are going to analyze the statistical properties of financial data. Download a time series of daily prices of some financial asset (you can find historical price series of stocks etc..from yahoo finance, or at the web page of Oslo Børs).

a) Describe your data set, and plot the time series of price data you have downloaded.

b) Derive time series of daily logreturns and returns, and plot these time series. What are the mean and standard deviations of the time series?

c) Plot the empirical density of the logreturns along with the fitted normal distribution. Comment.

d) Compute the autocorrelation function of the logreturns and the squared logreturns, and plot these. Comment.

e) Assume that you want to model the price dynamics by a geometric Brownian motion

$$S_t = S_0 \exp(\mu t + \sigma B_t)$$

where μ and $\sigma > 0$ are constants and B is a Brownian motion. Find μ and σ for your data set. Simulate from the geometric Brownian motion prices 100 days into the future, starting with your last observed price. Redo this 10 times to get 10 price scenarios, and plot these together with the observed data. Compare visually the paths, and comment.

Exercise 2. In this exercise you will study futures prices on commodities. Assume B_t is a Brownian motion.

a) Let $g : \mathbb{R} \to \mathbb{R}$ be a deterministic function. State conditions ensuring that g is Ito integrable on an interval $[0, T]$, and argue using the definition of the Ito integral that $\int_0^t g(s)\, dB_s$ is a Gaussian random variable for each $t \in [0, T]$. Find the mean and the variance of $\int_0^t g(s)\, dB_s$.

b) Define the stochastic process

$$Y_t = Y_0 e^{-\alpha t} + \sigma e^{-\alpha t} \int_0^t e^{\alpha s}\, dB_s$$

for α, σ two positive constants, and Y_0 is some constant. Show that Y_t is normally distributed random variable, and find the mean and the variance of it. What happens when $t \to \infty$?

c) Use the Ito Formula to show that

$$dY_t = -\alpha Y_t\, dt + \sigma dB_t$$

Frequently, Y_t is called the Ornstein-Uhlenbeck process, or a mean-reverting process, or (typically in finance) the Vasicek model.

d) The so-called Schwartz model for the price dynamics of a commodity is defined as

$$S_t = \exp(Y_t)$$

where Y_t is as above. Use the Ito Formula to find the dynamics dS_t of the price.

e) Compute the forward price of a contract promising to deliver the commodity at time T, that is, compute

$$F_t(T) := \mathbb{E}[S_T \mid S_t]$$

where $0 \le t \le T$. Hint: first find S_T expressed by S_t

f) Use Ito's Formula to find dF_t, the dynamics of F_t. What is the volatility of F_t?

g) The process Y_t is frequently used to model temperature variations as well. Let the temperature at time t be defined as

$$\mathcal{T}_t = \Lambda(t) + Y_t$$

for some deterministic seasonality function Λ (typically given by some sine-function, but that does not really matter here). At the Chicago Mercentile Exchange, one can trade in HDD- and CDD-futures. For example, a CDD-futures will deliver the cooling-degree day (=CDD) index translated into Dollars on a given time T, in return for the CDD-futures price $F_{CDD}(t,T)$ agreed when entering the contract. The CDD index at time T is defined to be $CDD(T) = \max(\mathcal{T}_T - 18, 0)$. Compute the CDD-futures price a time $t \le T$ defined as

$$F_{CDD}(t,T) = \mathbb{E}[\max(\mathcal{T}_T - 18, 9) \mid \mathcal{T}_t]$$

Hint: Express $\mathcal{T}_T$ in terms of $\mathcal{T}_t$.

Exercise 3. In this exercise you will use the Black & Scholes pricing formula for call options. As we will show later in the lectures, the price at time $t \ge 0$ of a call option written on an asset with price S_t with strike $K > 0$ and exercise time $T \ge t$ is given in Theorem 4.6 on page 66 in Benth. Here, r is the risk-free interest rate and σ is the volatility of the asset (typically, the standard deviation of the logreturns of the asset).

a) Implement the Black & Scholes formula in matlab, R, Excel or some convenient mathematical software. Let $r = 0$, $K = 100$ and $\sigma = 30\%$ annually. Use your price calculator to find the option price when $S_t = K$, $S_t = K \times 0.9$ and $S_t = K \times 1.1$, with exercise time in one month, two months and 6 months.

b) At Oslo Børs web-page, you can find prices for call and put options written on many companies (and with many different strikes and exercise times). Pick one company, and use your price calculator to find the *implied volatilities* for the options on that company. The implied volatility is defined as the σ that you must use in the Black & Scholes formula in order to get the same option price as the one observed in the market. Hence, you must specify K, T (which will be the time left until exercise, as $t = 0$) and the current stock price S_0 of the company for the option in question. You can pick an interest rate r from the NIBOR yield curve ("rentekurve", in Norwegian), also observable on the Oslo Børs web-page. Go through several of the quoted options, and find the corresponding implied volatilities. For some fixed T, plot the implied volatilities as a function of K. How does your results compare with the *historical volatility*, that is, the volatility derived from the observed logreturns of the company?

Notice: options are often "American", but you can ignore this in the exercise and view the options as "European".