Growth - Week 4

ECON1910 - Poverty and distribution in developing countries

Readings: Ray chapter 3

25. January 2011

Thinking About Development

Rates of growth of real per-capita income are . . . diverse, even over sustained periods . . . I do not see how one can look at figures like those without seeing them as representing possibilities. . .

The consequences for human welfare involved in [questions related to development] are simply staggering: Once one starts thinking about them, it is hard to think about anything else.

– Robert Lucas

Road map of today's lecture

- The Harrod-Domar model
- The Solow model

Rate of Growth

How long would it take for a quantity to double if it grows at a compounded rate of growth of 7 percent?

. . . of 10 percent?

Rule of 70

Simple formula: Divide 70 by the rate of growth

At 7 percent compounded rate of growth, the doubling time is 10 years, and vice versa.

The Harrod-Domar model

- Developed independently by Sir Roy Harrod in 1939 and Evsey Domar in 1946
- Explains growth in terms of the level of saving and productivity of capital.

- Production = Consumption goods + Capital goods
- Investment $\Longrightarrow$ Capital formation
- Saving means delaying present consumption
- Growth depends on investing savings in increasing the capital stock

Y represents income (same as output or production)
K represents capital stock
δ represents depreciation rate of the capital stock
S is total savings
s is the savings rate
I is investment
C is consumption

Notation

- I use slightly different notation than what is used in the book.
- Instead of writing $X(t)$, I write X_t

$$X(t) \equiv X_t$$

- Output (or income) is consumption plus savings

$$Y_t = C_t + S_t \tag{1}$$

- The product of the savings rate and output equals saving, which equals investment

$$sY_t = S_t = I_t \tag{2}$$

- We can then write:

$$Y_t = C_t + I_t \tag{3}$$

- Next periods capital stock equals investment less the depreciation of the capital stock

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{4}$$

The Harrod-Domar model

Definitions

Savings rate is s

$$s = \frac{S}{Y}$$

Capital-output ratio is θ = Amount of capital required to produce one unit of output

$$\theta = \frac{K}{Y}$$

$$K = \theta Y$$

$$Y = \frac{K}{\theta}$$

Rate of growth g

$$g = \frac{Y_{t+1} - Y_t}{Y_t}$$

- Lets go back to equation 4

$$K_{t+1} = (1-\delta)K_t + I_t$$

- Replace $K = \theta Y$ and $I_t = S_t = sY_t$

$$\theta Y_{t+1} = (1-\delta)\theta Y_t + sY_t$$

- We can then write

$$\theta Y_{t+1} = \theta Y_t - \delta\theta Y_t + sY_t$$

The Harrod-Domar model

Deriving the Harrod-Domar Equation

- From last slide

$$\theta Y_{t+1} = \theta Y_t - \delta\theta Y_t + sY_t$$

- Subtract θY_t from both sides

$$\theta Y_{t+1} - \theta Y_t = sY_t - \delta\theta Y_t$$

- Divide by Y_t on both sides

$$\frac{\theta Y_{t+1} - \theta Y_t}{Y_t} = s - \delta\theta$$

- From last slide:

$$\frac{\theta Y_{t+1} - \theta Y_t}{Y_t} = s - \delta\theta$$

- Divide by θ on both sides

$$\frac{Y_{t+1} - Y_t}{Y_t} = \frac{s}{\theta} - \delta$$

- Replace $g = \frac{Y_{t+1} - Y_t}{Y_t}$

$$g = \frac{s}{\theta} - \delta$$

- From last slide

$$g = \frac{s}{\theta} - \delta$$

- Rearrange

$$\frac{s}{\theta} = g + \delta \tag{5}$$

- Equation 5 is the Harrod-Domar Equation

What the H-D equation means

$$g = \frac{s}{\theta} - \delta$$

- It links the growth rate to two other rates

1. The savings rate s
2. The capital-output ratio θ

Policy implications

- Capital-uotput ratio is seen as exogenous, but technology-driven.
- Savings rates can be affected by policy.
- It links the growth rate of the economy to two fundamental variables:

1. The ability of the economy to save
2. Capital-output ratio

Policy implications

- By pushing up the rate of savings, it would be possible to accelerate the rate of growth.
- Likewise, by increasing the rate at which capital produces output (a lower θ), growth would be enhanced.

The Harrod-Domar model

Adding population growth

- Population P grows at rate n

$$P_{t+1} = P_t(1 + n)$$

- Per capita income is y_t

$$y_t = \frac{Y_t}{P_t}$$

- Per capita income growth rate is g^*

$$y_{t+1} = y_t(1 + g^*)$$

- Lets go back to

$$\theta Y_{t+1} = (1 - \delta)\theta Y_t + sY_t$$

- Replace $Y = yP$

$$\theta y_{t+1} P_{t+1} = (1 - \delta)\theta Y_t + sY_t$$

- From last slide

$$\theta y_{t+1} P_{t+1} = (1-\delta)\theta Y_t + sY_t$$

- Divide both sides by P_t

$$\theta y_{t+1} \frac{P_{t+1}}{P_t} = (1-\delta)\theta y_t + sy_t$$

- Divide both sides by $y_t \theta$

$$\frac{y_{t+1}}{y_t} \frac{P_{t+1}}{P_t} = (1-\delta) + \frac{s}{\theta}$$

The Harrod-Domar model

Adding population growth

- From last slide

$$\frac{y_{t+1}}{y_t}\frac{P_{t+1}}{P_t} = (1-\delta) + \frac{s}{\theta}$$

- Note that $\frac{y_{t+1}}{y_t} = g^* + 1$ and $\frac{P_{t+1}}{P_t} = n + 1$

- We then get:

$$(g^* + 1)(n + 1) = (1-\delta) + \frac{s}{\theta}$$

- Rearrange:

$$\frac{s}{\theta} = (1+g^*)(n+1) - (1-\delta) \tag{6}$$

- From last slide

$$\frac{s}{\theta} = (1 + g^*)(n + 1) - (1 - \delta)$$

- Write out:

$$\frac{s}{\theta} = g^* + n + \delta - g^* n$$

Both g^* and n are small numbers, so their product is very small relative to the other terms and can be ignored as an approximation.

- $$\frac{s}{\theta} \simeq g^* + n + \delta \quad (7)$$

  or

- $$g^* = \frac{s}{\theta} - n - \delta$$

1. Per capita growth rate is reduced by the population growth rate and by the capital depreciation rate
2. Per capita growth rate is increased by the savings rate and by more efficient use of capital

Are the variables exogenous?

- In the H-D model; s, n and θ are treated as constants, and not affected by the growth of the economy
- In the H-D model; s, n and θ are treated as exogenous
- What if the savings rate is a function of per capita income?
    - Poor people cannot save at the same rate as those who are rich
    - Distribution of income – and not just per capita income – affects the saving rate
    - Therefore the savings rate may rise with rising incomes

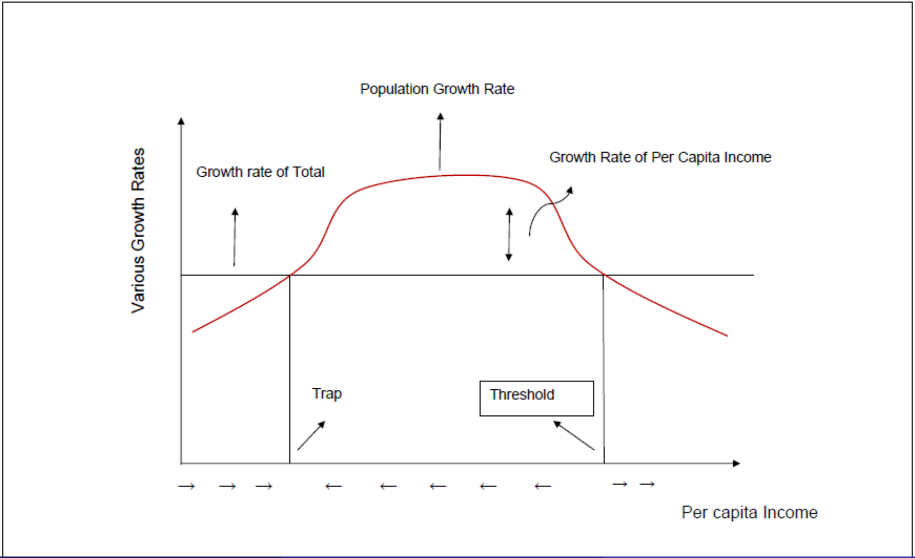
Are the variables exogenous?

Population growth

- There is an enormous body of evidence that suggests that population growth rates systematically change with income.

- Demographic transition:
  - In poor countries the net population growth rate is low.
  - With an increase in living standards, death rates starts to fall.
  - Birth rates adjust relatively slowly to this transformation in death rates.
  - This causes the population growth rate to initially shoot up.
  - In the longer run, and with further development, birth rates starts to go down, and the population growth rate falls to a low level.

The endogeneity of population growth



The endogeneity of population growth

- The growth rate of per capita income is the growth rate of income (net of depreciation) minus the rate of population growth.
- This is the vertical distance between the two curves.
- The rate of growth of per capita income turns out to depend on the current income level.
- The growth rate is positive at low levels of per capita income (up to the level marked "Trap")
- The growth rate is then negative (up to per capita income marked "Threshold")
- The growth rate is again positive at income per capita levels above "Threshold".

The endogeneity of population growth

1. If we start from a low level of per capita income, left of "Trap", growth is positive and per capita income will rise over time toward the point marked "Trap".

2. If we start at medium per capita income, between "Trap" and "Threshold", growth is negative and per capita income will fall over time to point marked "Trap".

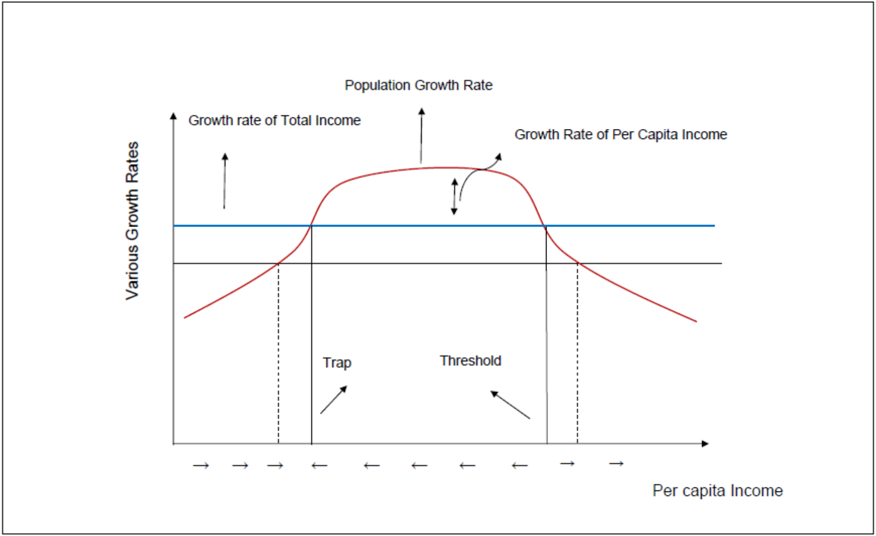
3. If we start at a high per capita income, left of "Threshold", growth is positive and per capita income will rise over time and the economy will be in a phase of sustained growth.

The endogeneity of population growth

- In the absence of some policy that pushes the economy to the right of the threshold, the economy will be caught in the trap.
- The diagram suggests that there are situations in which a temporary boost to certain economic parameters, perhaps through government policy, may have sustained long-run effects.

The endogeneity of population growth

A jump in the savings rate



The endogeneity of population growth

A jump in the savings rate

- The policy that boosts savings does not have to be permanent.
- Once the economy crosses a certain level of per capita income, the old savings rate will suffice to keep it from sliding back, because population growth rates are lower.

The endogeneity of population growth

Strong family planning

- A strong family planning or the provision of incentives to have less children can pull down the population curve, converting a seemingly hopeless situation into one that can permit long-run growth.
- As the economy becomes richer, population growth rates will endogenously induce to fall, so that policy becomes superfluous.

The endogeneity of the capital-output ratio.

- Endogeneity may fundamentally alter the way we think about the economy.
- We have seen how this might happen in the case of endogenous population growth.
- The most startling and influential example of all is the endogeneity of the capital-output ratio –> The Solow model.
- The Solow model (1956) has had a major impact on the way economist think about economic growth.
- It relies on the possible endogeneity of the capital-output ratio.

The Solow Model
Production Function

- Definitions:

$$y_t = \frac{Y_t}{P_t}$$

$$k_t = \frac{K_t}{P_t}$$

- In the Solow model, production is explicitly a result of two production factors: Labor/Population and Capital

$$Y = F(K, P)$$

- It is assumed that the production function has constant returns to scale.
- By this we mean that if we increase both factors by the same fraction, total output will increase by the same.

$$2Y = F(2K, 2P)$$

- More generally

$$\alpha Y = F(\alpha K, \alpha P)$$

where α is any constant.

- Note that if you increase only one of the factors, production increases by less.
- For our application: It we keep the number of people constant, adding capital will increase production, but with smaller and smaller increases for a given amount of capital.

Setting $\alpha = \frac{1}{P}$

$$\frac{Y}{P} = F(\frac{K}{P}, 1)$$

$$y = F(k, 1) = f(k)$$

As before:

$$sY_t = S_t = I_t$$

$$\Delta K = sY_t - \delta K_t$$

Recall that

$$k = \frac{K}{P}$$

$$\frac{\Delta k}{k} = \frac{\Delta K}{K} - \frac{\Delta P}{P}$$

- Insert $\Delta K = sY(t) - \delta K(t)$

$$\frac{\Delta k}{k} = \frac{sY_t - \delta K_t}{K} - \frac{\Delta P}{P}$$

- Write out:

$$\frac{\Delta k}{k} = s\frac{Y_t}{K_t} - \delta - n$$

- Finally:

$$\Delta k = sy - (\delta + n)k \tag{8}$$

$$\Delta k = sf(k) - (\delta + n)k \tag{9}$$

$$\Delta k = \underbrace{sf(k)}_{\text{Actual Investment}} - \underbrace{(\delta + n)k}_{\text{Break Even investment}}$$

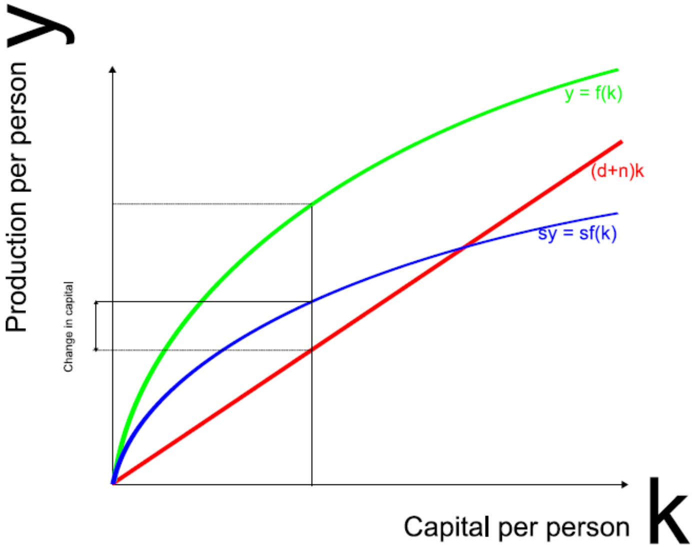
- $(\delta + n)k =$ break-even investment, the amount of investment necessary to keep k constant.

- Break-even investment includes:

1. δk to replace capital as it wears out
2. nk to equip new workers with capital

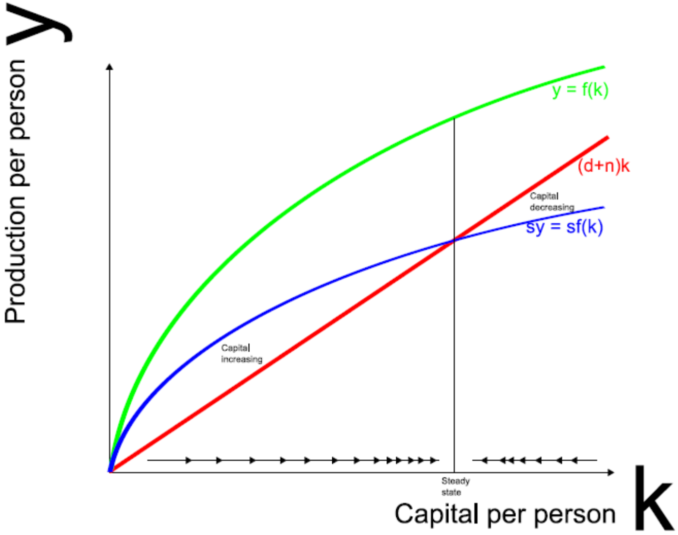
- Equation 9 tells us how capital per population/worker changes.
  - If $sf(k) > (\delta + n)k \quad -> \Delta k > 0$
  - If $sf(k) < (\delta + n)k \quad -> \Delta k < 0$
  - If $sf(k) = (\delta + n)k \quad -> \Delta k = 0$

The Solow Model



The Solow Model

Paths of movement in the Solow model



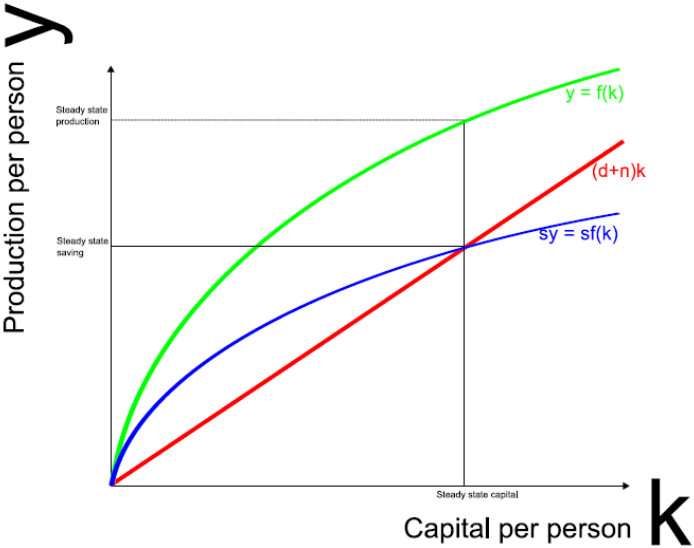
At the point where both (k) and (y) are constant it must be the case that

$$\Delta k = sf(k^*) - (\delta + n)k^* = 0$$

or

$$sf(k^*) = (\delta + n)k^*$$

This occurs at our equilibrium point k^*


y
Production per person
Steady state production
Steady state saving
y = f(k)
(d+n)k
sy = sf(k)
Steady state capital
Capital per person
k

- Suppose population growth increases
- This shifts the line representing population growth and depreciation upward
- At the new steady state capital per worker and output per worker are lower
- The model predicts that economies with higher rates of population growth will have lower levels of capital per worker and lower levels of income.

- Suppose the savings rate increases
- This shifts the curve representing investment/savings upward
- At the new steady state capital per worker and output per worker are higher
- The model predicts that economies with higher rates of savings will have higher levels of capital per worker and higher levels of income.

Higher n –> lower k^*–> and lower y^*
Higher δ –> lower k^*–> and lower y^*
Higher s –> higher k^*–> and higher y^*

- No growth in the steady state - only level effect
- Positive or negative growth along the transition path:

$$\Delta k = sf(k) - (\delta + n)k$$

- Why are some countries rich (have high per worker GDP) and others are poor (have low per worker GDP)?
- Solow model: if all countries are in their steady states, then:
  1. Rich countries have higher saving (investment) rates than poor countries
  2. Rich countries have lower population growth rates than poor countries

The Difference Between H-D and Solow

- In a world with constant returns to scale, the savings rate does have growth effects (The H-D model)
- In a world with diminishing returns to scale, the savings rate does not have growth effects (The Solow model)