Growth - Week 5

ECON1911 - Poverty and distribution in developing countries

Readings: Ray chapter 3

1. February 2011

Road map of today's lecture

- Introduce Technical Progress
- Convergence
- Empirical Evidence

The Solow model

- Solow model: In the absence of technical progress, a country cannot sustain per capita income growth indefinitely.

- For this to happen, capital must grow faster than population, but then diminishing return implies that the marginal contribution of capital to output must decline, which forces a decline in the growth rate of output and, therefore, of capital.

Technical Progress

- The foregoing argument loses its force if there is continuing technical progress; that is, if the production function shifts upward over time as new knowledge is gained and applied.
- If the shift in the production function outweighs the doom of diminishing returns, there is no reason why per capita growth cannot be sustained indefinitely.

The Solow Model with Technical Progress

- We now make a distinction between the working population P_t and the amount of labour in "efficiency unit" L_t used in production - the effective labour.

$$L_t = E_t P_t$$

- We can think of E_t as the efficiency or productivity of an individual at time t.

- We assume that productivity grows at a constant rate π - technical progress.

$$\frac{E_{t+1} - E_t}{E_t} = \pi$$

The Solow Model with Technical Progress

- We can now write the production function:

$$Y = F(K, EP)$$

- We now define $\hat{k}$ as capital per effective labour.

$$\hat{k} = \frac{K}{EP}$$

$$\frac{\Delta \hat{k}}{\hat{k}} = \frac{\Delta K}{K} - \frac{\Delta P}{P} - \frac{\Delta E}{E}$$

The Solow Model with Technical Progress

Using the same algebra as for the Solow Model without technology, we get the equation

$$\Delta k^{\hat{}} = sf(k^{\hat{}}) - (\delta + n + \pi)k^{\hat{}}$$

$$\Delta k^{\hat{}} = sf(k^{\hat{}}) - (\delta + n + \pi)k^{\hat{}}$$

Tells us how capital per productive labour changes.

If $sf(k^{\hat{}}) > (\delta + n + \pi)k^{\hat{}} \quad -> \Delta k^{\hat{}} > 0$
If $sf(k^{\hat{}}) < (\delta + n + \pi)k^{\hat{}} \quad -> \Delta k^{\hat{}} < 0$
If $sf(k^{\hat{}}) = (\delta + n + \pi)k^{\hat{}} \quad -> \Delta k^{\hat{}} = 0$

The Solow Model with Technical Progress

Steady State

- At the point where both $(\hat{k})$ and $(\hat{y})$ are constant it must be the case that

$$\Delta k = sf(\hat{k}^*) - (\delta + n + \pi)\hat{k}^* = 0$$

or

$$sf(\hat{k}^*) = (\delta + n + \pi)\hat{k}^*$$

- This occurs at our equilibrium point $\hat{k}^*$

The Solow Model with Technical Progress

- The "steady-state" level of $\hat{k}^*$ now denotes a situation where production grows by π.
- Why?
- Observe that in the steady state $\Delta \hat{y}$ is zero.
- This means that the growth of production measured per "effective worker" is zero.
- But the "effective worker" becomes more and more productive.
- Therefore, output per person is steadily increasing.

The Solow Model with Technical Progress

$$y^{\hat{}} = \frac{Y}{EP}$$

$$y^{\hat{}} E = \frac{Y}{P}$$

- We now see that even though $y^{\hat{}}$ is constant in the long run, $(\frac{Y}{P})$ per capita income is growing at the same rate as E.

The Solow Model with Technical Progress

- Even though capital per effective worker converges to a stationary steady state, the amount of capital per member of the working population increases.

- The long-run increase in per capita income takes place precisely at the rate of technical progress.

The Solow Model with Technical Progress

- In the steady state, all variables growth at constant rates:
  - Capital per unit of effective labor, $k\hat{}$: constant;
  - Labor and technology grow at rates n and π, respectively;
  - Capital, $K = EPk$ grows at rate $(n + \pi)$
  - Because of CRTS, output, Y, also grows at rate $(n + \pi)$
  - Capital per worker, $\frac{K}{P}$ and output per worker, $\frac{Y}{P}$ grow at rate π.
- Hence, the equilibrium (steady state) rate of growth of output per capita is determined by the rate of technological progress only.

Convergence?

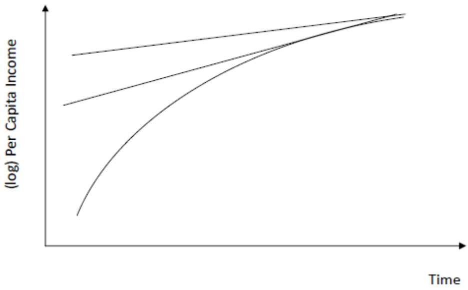
- At the heart of the Solow model is the prediction of convergence.
- But convergence comes in several flavors.
  - "Unconditional convergence" occurs when the income gap between two countries decreases irrespective of countries' “characteristics” (e.g., institutions, policies, technology or even investments).
  - "Conditional convergence" occurs when the economic gap between two countries that are similar in observable characteristics is becoming narrower over time.

Convergence?

Unconditional convergence

- If we think that, in the long run, countries tend to have the same rate of technical progress, savings, population growth, and capital depreciation.
- In such a case, the Solow model predicts that in all countries, capital per efficiency unit of labor converges to the common value $\hat{k}^*$, and this will happen irrespective of the initial state of each of these of these economies.

Convergence?
Unconditional convergence



Unconditional convergence: Evidence or lack of thereof

- The systematic collection of data in the developing economies started only recently, and it is hard to find examples of reliable data that stretch back a century or more.
- Two choices:
  - Cover a small number of countries over a long period of time
  - Cover a large number of countries over a short period of time

A small set of countries over a long time horizon

- Baumol (1986) examined the growth rates of sixteen countries that are among the richest in the world.
- Baumol's idea: Plot 1870 per capita income on the horizontal axis and plot the growth rate of per capita income over the period 1870-1979 on the vertical axis.
- If the hypothesis of unconditional convergence is correct, the observations should approximately lie on a downward-sloping line.

A small set of countries over a long time horizon

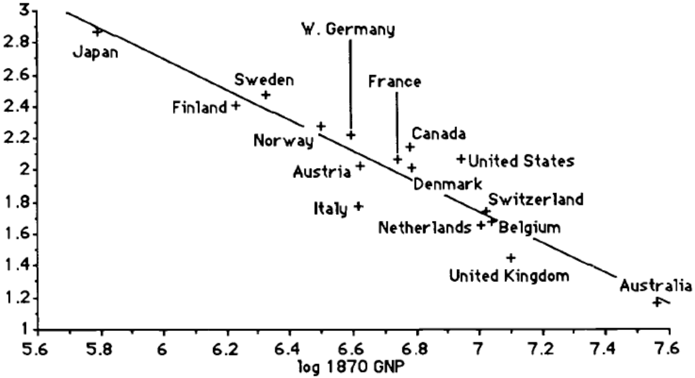
Baumol (1986) set his empirical equation as

$$g = \alpha + \beta y_0 + \varepsilon$$

If unconditional convergence is correct, we would expect β to be negative.

A small set of countries over a long time horizon

PER CAPITA GNP REGRESSION FOR MADDISON'S SIXTEEN



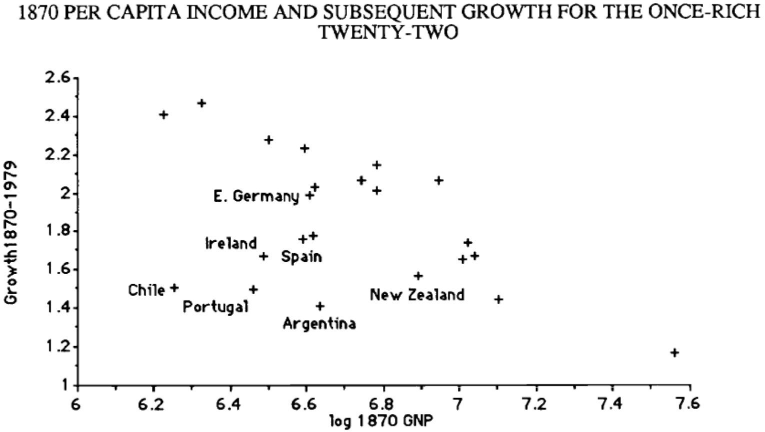
A small set of countries over a long time horizon

- Baumol's study - A classical case of statistical pitfall.
- The sample suffers from selection bias, because any nations relatively rich in 1870 that have not converged fail to make it into Maddison's sample.
- Only countries that are success stories where selected to study convergence.
- This is using wisdom after the event.
- Includes Norway but not Spain, Canada but not Argentina, and Italy but not Ireland.
- A fair test of convergence requires not an ex post sample of countries that have converged but an ex ante sample of countries that in 1870 looked likely to converge.

A small set of countries over a long time horizon

- Does the evidence on convergence hold up in a statistical if we broaden the set of countries?
- De Long (1988) address this question by adding seven other countries, which in 1870 had as much claim to membership in the "convergence club" as many of the other countries included in Baumol's original dataset, and dropping Japan.

A small set of countries over a long time horizon



A small set of countries over a long time horizon

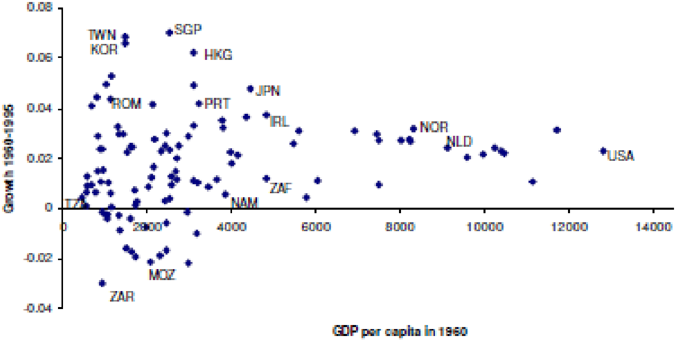
- De Long: very little systematic relationship between a county's growth rate and its per capita GDP, at least in the cross section of the twenty-two countries studied.

- De Long - little evidence of unconditional convergence.

A large set of countries over a short time horizon

- This approach has the advantage of "smoothing out" possible statistical irregularities in looking at small sample.
- The disadvantage is that the time span of analysis must be shortened.
- Regress average per capita growth between 1960 and 1985 on per capita GDP in 1960.
- Barro (1991) - the correlation between the two variable is only 0.09, which amounts to saying that there is no correlation at all.

A large set of countries over a short time horizon



Unconditional convergence?

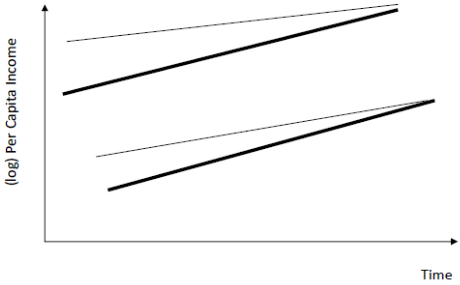
- The data does not support unconditional convergence.
- Is the Harrod-Domar model correct? - No because constant return to physical capital alone is not correct.
- Possible explanation for lack of convergence - the structural characteristics (savings, population growth, institutions) are different across countries.
- The missing convergence in larger samples could reflect that we should test for conditional convergence instead.

Convergence?

Conditional convergence

- Per capita incomes of countries that are identical in their structural characteristics (e.g. preferences, technologies, rates of population growth, government policies, etc.) converge to one another in the long-run, independently of their initial conditions.
    - Converge to its own steady-state
    - The steady state can now be different from country to country
    - No need for countries to converge to each other
- Account for conditionality of steady state
- Convergence in growth rates - not necessarily convergence in income.
- The idea of controlling for the position of steady states amounts to factoring out the effect of parameters that might differ across countries, and then examining whether convergence occurs.

Convergence?
Conditional convergence



- Mankiw, Romer and Weil (QJE, 1992) derive testable predictions of the Solow growth model and put them to an empirical test.
- Cobb-Douglas production function:

$$y^{\hat{}} = (k^{\hat{}})^{\alpha}$$

- The steady-state value of capital per unit of effective labor then is

$$k^{\hat{}} = \left(\frac{s}{n+\delta+\pi}\right)^{\frac{1}{1-\alpha}}$$

- The steady-state level of capital responds positively to the savings rate and negatively to population growth.

- Substituting steady-state capital into the production function, we get an expression for steady-state output per effective labor and output per worker

$$\hat{y} = \left(\frac{s}{n + \delta + \pi} \right)^{\frac{\alpha}{1-\alpha}}$$

Taking logarithms of both sides, we get approximately

$$\ln y \backsimeq \text{constant} + \frac{\alpha}{1-\alpha}\ln s - \frac{\alpha}{1-\alpha}\ln(n+\delta+\pi)$$

- The Solow model predicts not only the signs but also the magnitudes of the effects of the savings rate and population growth on output per worker.
- For $\alpha = \frac{1}{3}$, the elasticity of output per worker w.r.t. the savings rate should be 0.5 while that w.r.t. $(n+\delta+\pi)$ should be - 0.5

- MRW estimate this equation, using Summers and Heston (1988) PWT data over 1960-1985.
- The basic sample includes 98 countries (major oil producers excluded).
- Results:
  - Correct signs for the effects of the savings rate and population growth (approximately equal in magnitude), but their sizes are much larger than those predicted by the model.
  - Nonetheless, the standard textbook Solow model explains more than half of the variance in the data on growth.

Next?

Does Human Capital Provide the Missing Explanation