Static games: Coordination and Nash equilibrium

Lectures in Game Theory

Fall 2012, Lecture 2

Rationalizability is about …

- Narrowing down the beliefs I have and the other players may have.
- By discarding those beliefs (mine and other's) that are not rational to have.
- By keeping those beliefs could be rational to have.
- The only requirement is that my strategy is internally consistent with the beliefs I have of the other's strategy and beliefs.
- No guarantee that the other player actually has the strategy I believe (s)he has.

Rationalizability is appropriate …

- when players are strategically sophisticated
- in a situation which does not recur often.
- with no communication or outside coordination.
- i.e. when I do not **know** what the other players believe and strategize.

Two concepts

Consider a set of strategy profiles $X = X_1 \times \cdots \times X_n$, where $X_i \subseteq S_i$ for all i.

- **1) Best response property** (**weak congruity**):

  The set X contains only best responses.

  X has the *best response property* if, for each i and each $s_i \in X_i$, there is $\mu_{-i} \in \Delta X_{-i}$ s.t. $s_i \in BR_i(\mu_{-i})$.

- **2) Best response completeness**:

  The set X contains all best responses

  X is *best response complete* if, for each i and each $\mu_{-i} \in \Delta X_{-i}$, $BR_i(\mu_{-i}) \subseteq X_i$.

- **Loosely:** 1) My strategy is to only do what's best given what I think others plan and 2) my strategy contains all the possible best responses given what I think others plan.

- Up until now, it's all been about beliefs, but without a guarantee that the beliefs are coordinated/right.

What if …

- the game recurs often (even though the opponents change from time to time).
- the players can communicate.
- there is outside coordination.

Nash equilibrium

Are there strategies for the two players so that no player will regret his own choice when being told of the other player's choice? If yes, then such a strategy profile is a *Nash equilibrium*.

- Some examples:

Definition : $(s_1, \ldots, s_n)$ is a *Nash equilibrium* if it, for each player i, holds that $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ for all s_i'. I.e., for each player i, $s_i \in BR(s_{-i})$. If, for each player i, $\{s_i\} = BR(s_{-i})$, then $(s_1, \ldots, s_n)$ is a *strict Nash equilibrium*.

Observations

Result : If $(s_1,\ldots,s_n)$ is a Nash equilibrium, then $(s_1,\ldots,s_n)$ survives iterated strict elimination.

Result : If $(s_1,\ldots,s_n)$ is the only strategy profile which survives iterated strict elimination, then $(s_1,\ldots,s_n)$ is a Nash equilibrium.

Some games have no Nash equilibrium

- Some examples:

Such games have a Nash equilibrium in mixed strategies. Interpretation as a steady state.

Definition : Consider a strategy profile $(\sigma_1, \ldots, \sigma_n)$, where $\sigma_i \in \Delta S_i$ for each player i. The profile $(\sigma_1, \ldots, \sigma_n)$ is a *mixed - strategy Nash equilibrium* if it, for each player i, holds that $u_i(\sigma_i, \sigma_{-i}) \geq u_i(s_i, \sigma_{-i})$ for all s_i.

Result (Nash, 1950) : Every finite game has a mixed - strategy Nash equilibrium.

Example: Volunteer's dilemma

Steady-state interpretation of Nash equilibr.

- The game is a model designed to explain some regularity in a family of similar situations.
- Each participant "knows" the equilibrium and evaluates whether it's worthwhile to use another strategy.
- The interpretation requires that the players meet different opponents each time.

In games with multiple equilibria, will the players coordinate, and if yes, on which equilibrium?

- Some examples:
- Generally game theory does not say how coordination has emerged, it merely assumes it.
- *Third tension:* Coordination on an inefficient NE.

The concept of efficiency

Definition : A strategy profile $s=(s_1,\ldots,s_i,\ldots,s_n)$ is *(Pareto) efficient* if there is no other strategy profile $s'=(s'_1,\ldots,s'_i,\ldots,s'_n)$ such that $u_i(s')\geq u_i(s)$ for every player i and $u_j(s')>u_j(s)$ for some player j.

English: The combined strategies of the players in the game are *efficient* if there isn't another combination of strategies which makes someone better off without making the others worse off.

Implication: Without efficiency there is room for coordination/negotiation/contracts to improve for all

Can Nash equilibrium be used as a solution concept if the game is only played once?

Yes, if each player can predict what each opponent will do.

- For each player, only one strategy survives iterative elimination of strictly dominated strategies.
- Through communication before the game starts, the players make a self-enforcing agreement (co-ordinate on an equilibrium).
- Given a common background, the players are able to co-ordinate on an equilibrium without communication before the game starts (Schelling, 1960, *focal point*).
- A unique Nash equilibrium is not sufficient.

The difference between congruous sets and Nash equilibria

- Nash: My beliefs of the other's strategy is right and the other's believes on my strategies are right.
- Congruity and rationalizability: Our beliefs are rational but they may be wrong.

Behavioral game theory

- Standard game theory provides discipline for our analysis of the relation between the outcome of strategic interaction and our assumptions about behavior.
- But does the theory accurately describe and predict real behavior? To test game theory, one can …
  — Gather data about behavior in real strategic situations.
  — Perform laboratory experiments with monetary payoffs.
- Behavioral game theory seeks to learn about real behavior through laboratory experiments. Problems:
  — Lab. settings may not resemble real strategic settings.
  — May be difficult to control the subjects' payoffs.