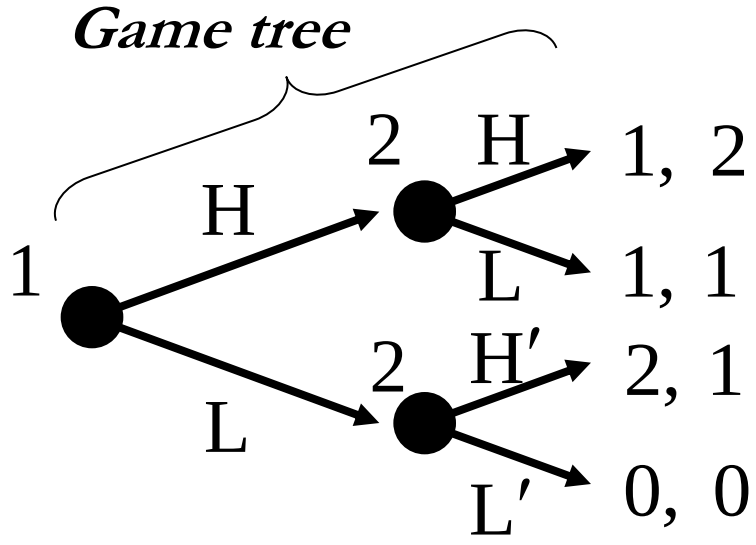
Dynamic games: Backward induction and subgame perfection

Lectures in Game Theory

Fall 2012, Lecture 3

Recall the extensive form: It specifies

- *Players*: {1, ... , *i*, ... , *n*}
- What actions an acting player can choose among, what an acting player knows.
- Payoff for each of the players as a function of the actions that are realized.

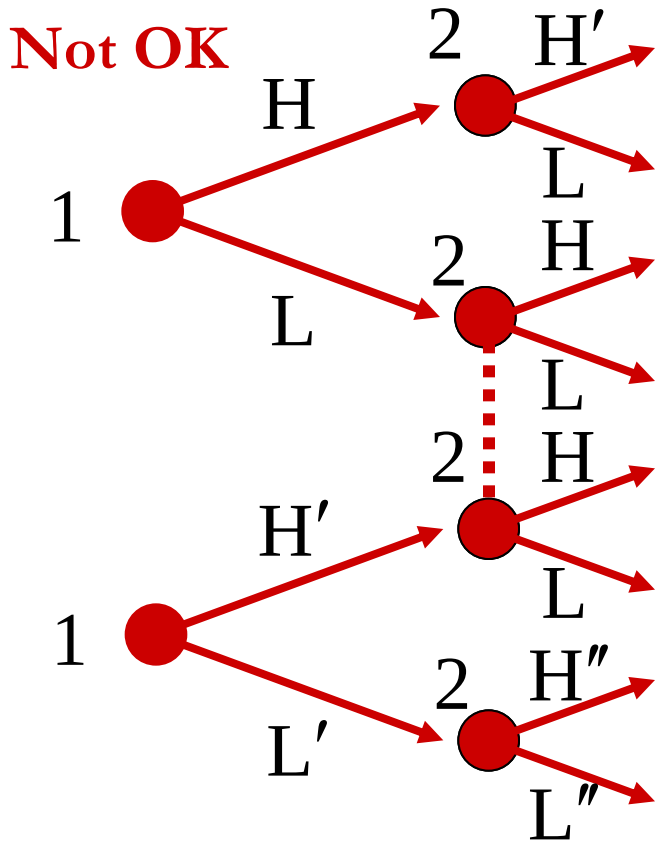


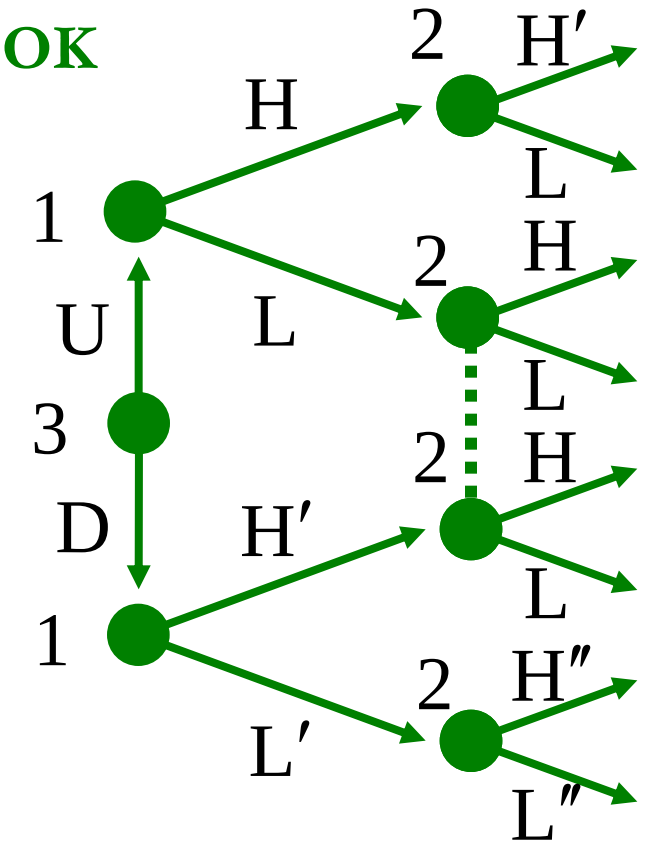
Some terms:

- Successor
- Predecessor
- Immediate sucessor
- Immediate predecessor

Tree rule 1

- Every node is a successor of the initial node, and the initial node is the only one having this property.





Tree rule 2

- Each node except the initial node has exactly one immediate predecessor. The initial node has no predecessor.

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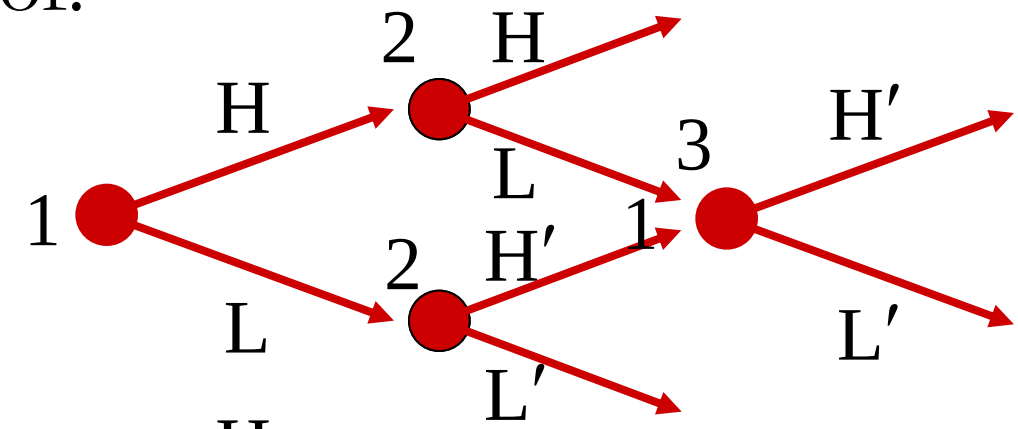

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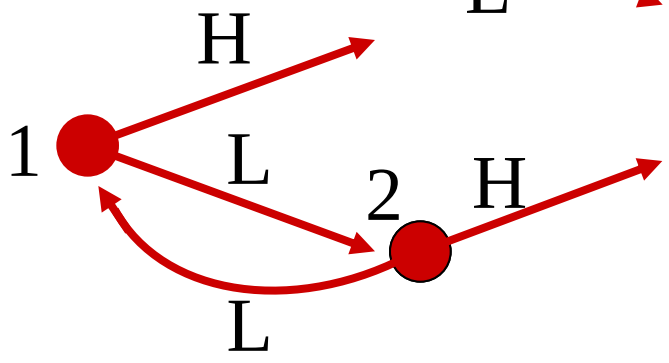

Tree rule 3

- Multiple branches extending from the same node have different action labels.

Tree rule 4

- Each information set contains decision nodes for only one of the players.

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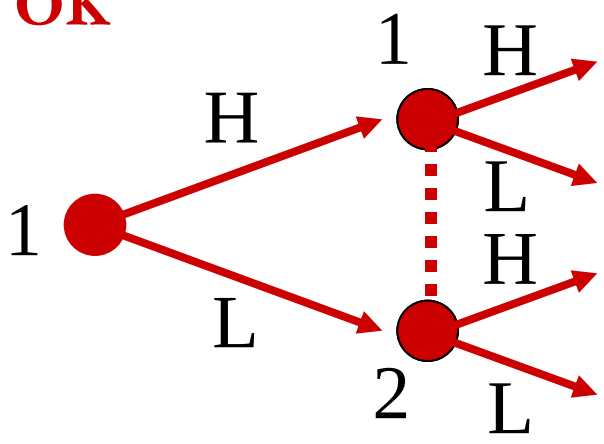

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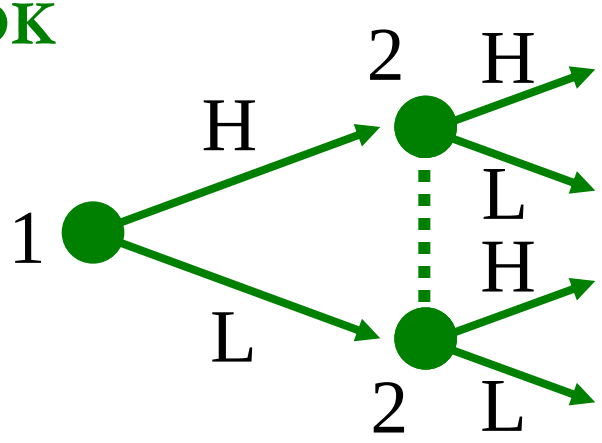
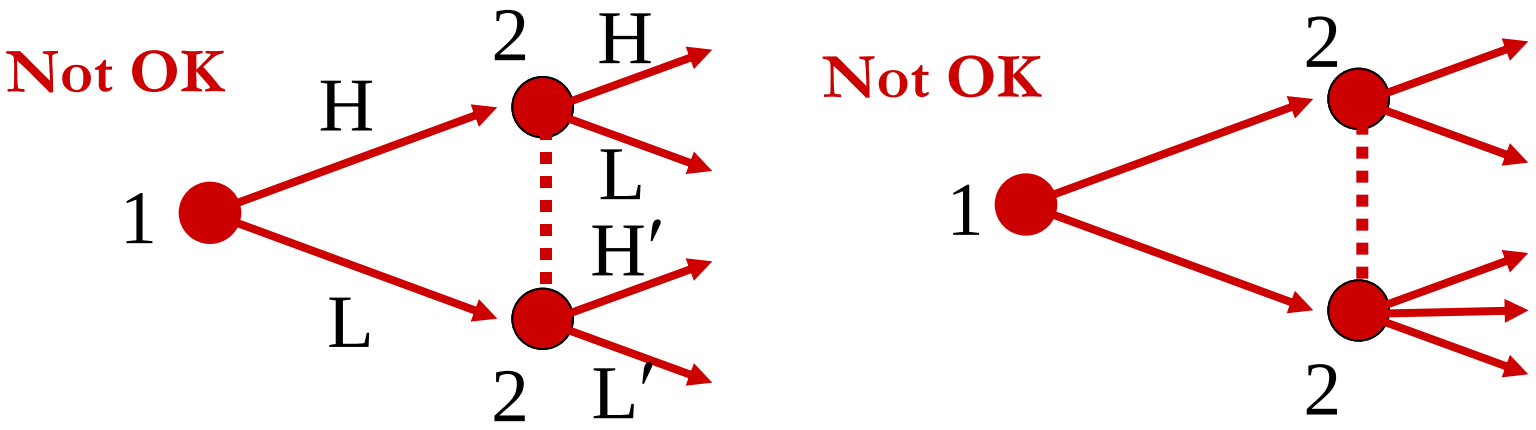


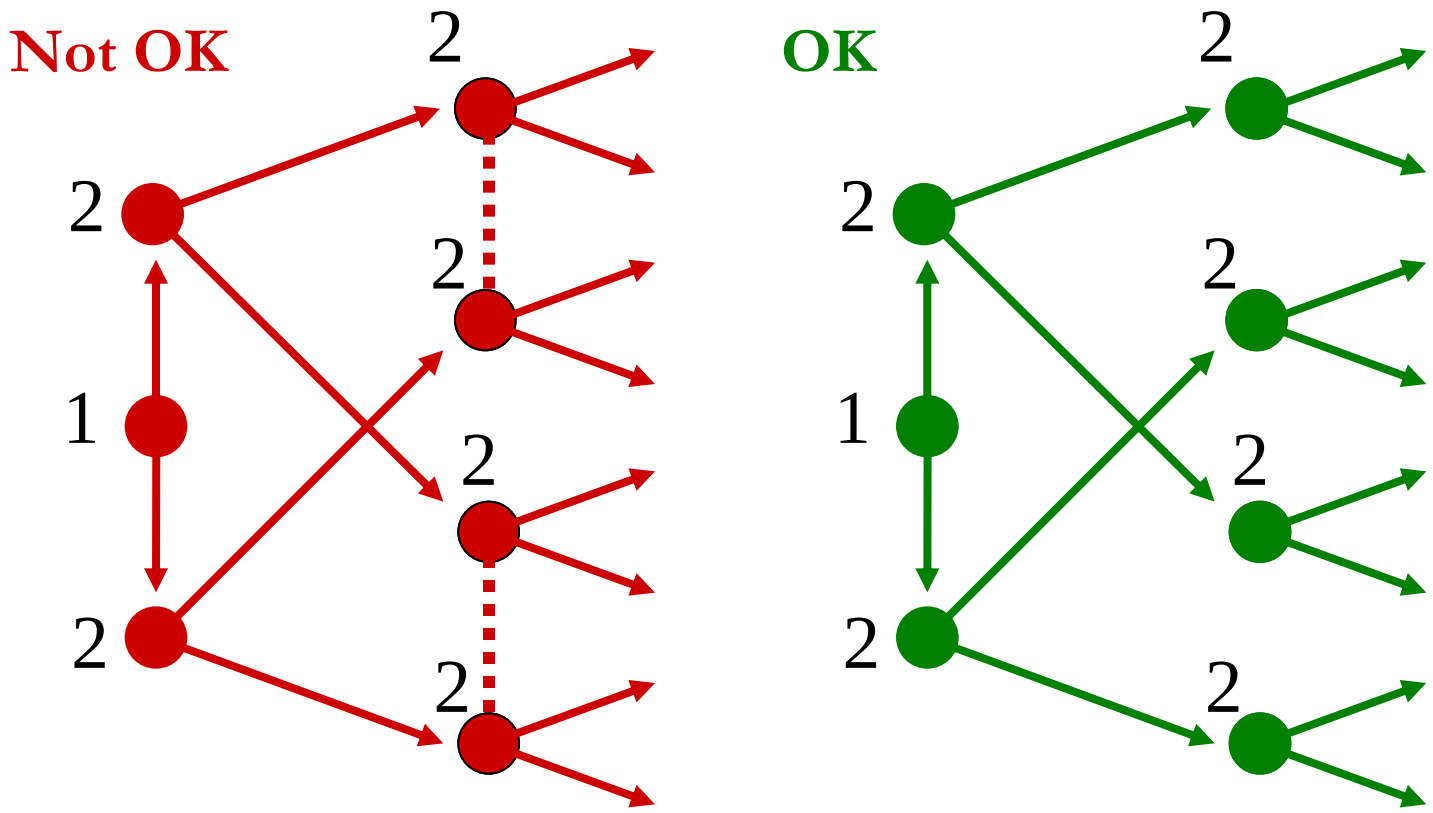
Tree rule 5

- All nodes in a given information set must have the same number of immediate successors and they must have the same set of action labels on the branches leading to these successors.



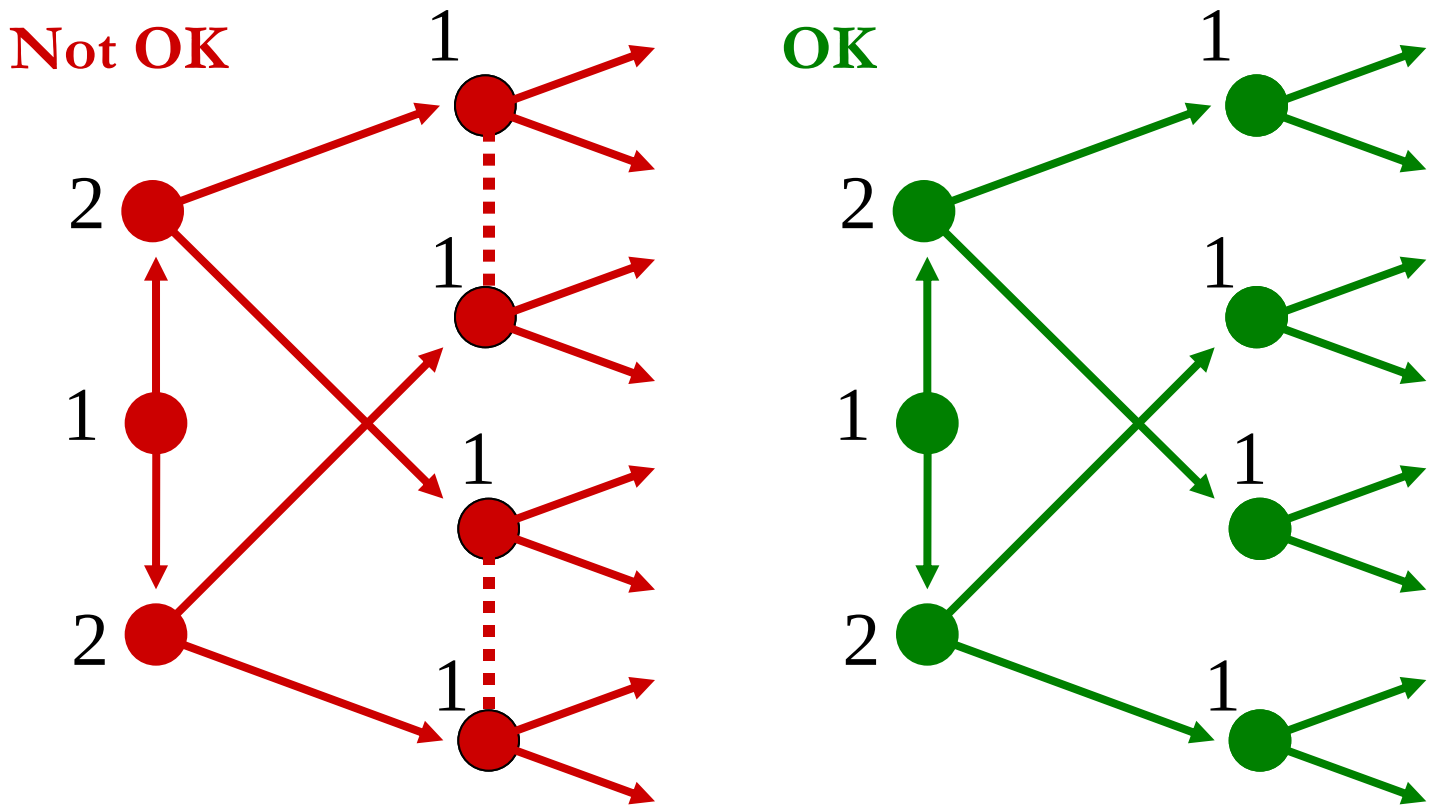
Perfect recall

- A player remembers what he once knew.



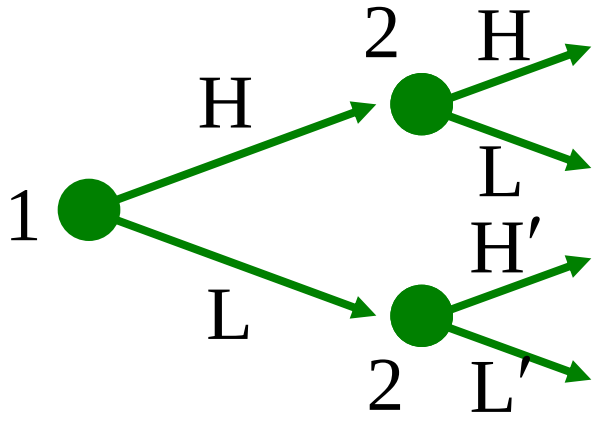
Perfect recall

- A player remembers what he once did.



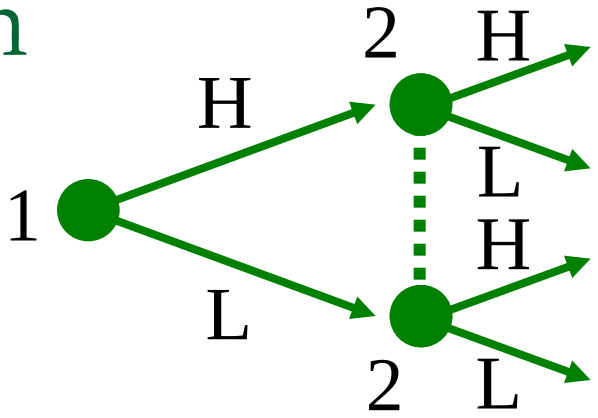
Perfect information

- Each information set contains only one decision node.



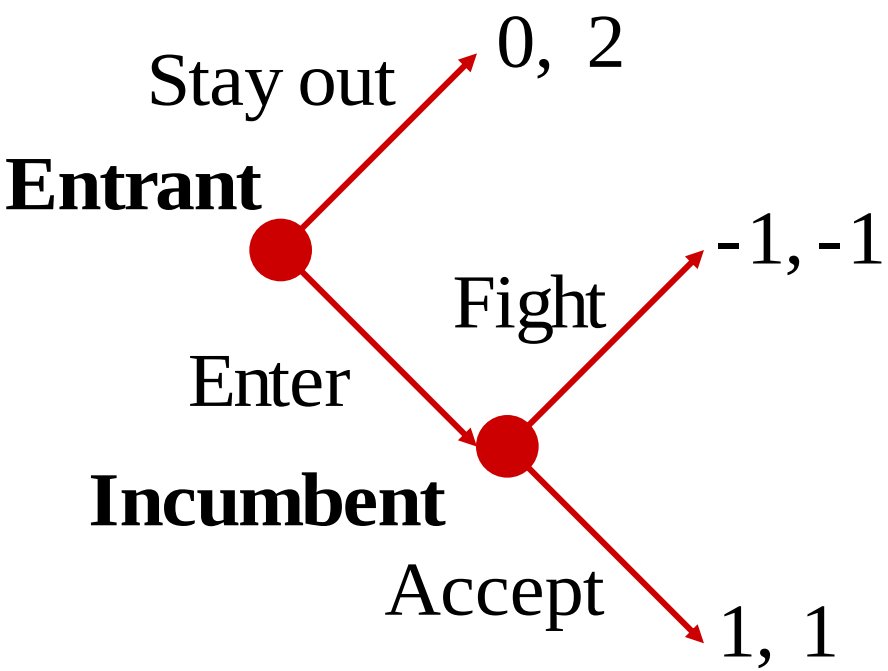
Imperfect information

- There is at least one contingency in which an acting player does not know exactly where he is.



Analyzing dynamic games

A dynamic game can be analyzed both in the ***extensive*** form and the ***normal*** form.

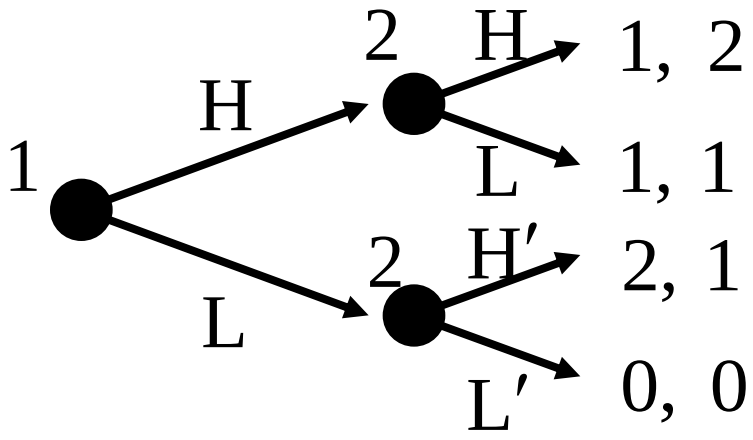


		Incumbent
	Accept	Fight
Enter	1, 1	-1, -1
Entrant		
Stay out	0, 2	0, 2

Are both NEa reasonable?

Is threat of fight credible?

Another example



	HH′	HL′	LH′	LL′
H	1, 2	1, 2	1, 1	1, 1
L	2, 1	0, 0	2, 1	0, 0

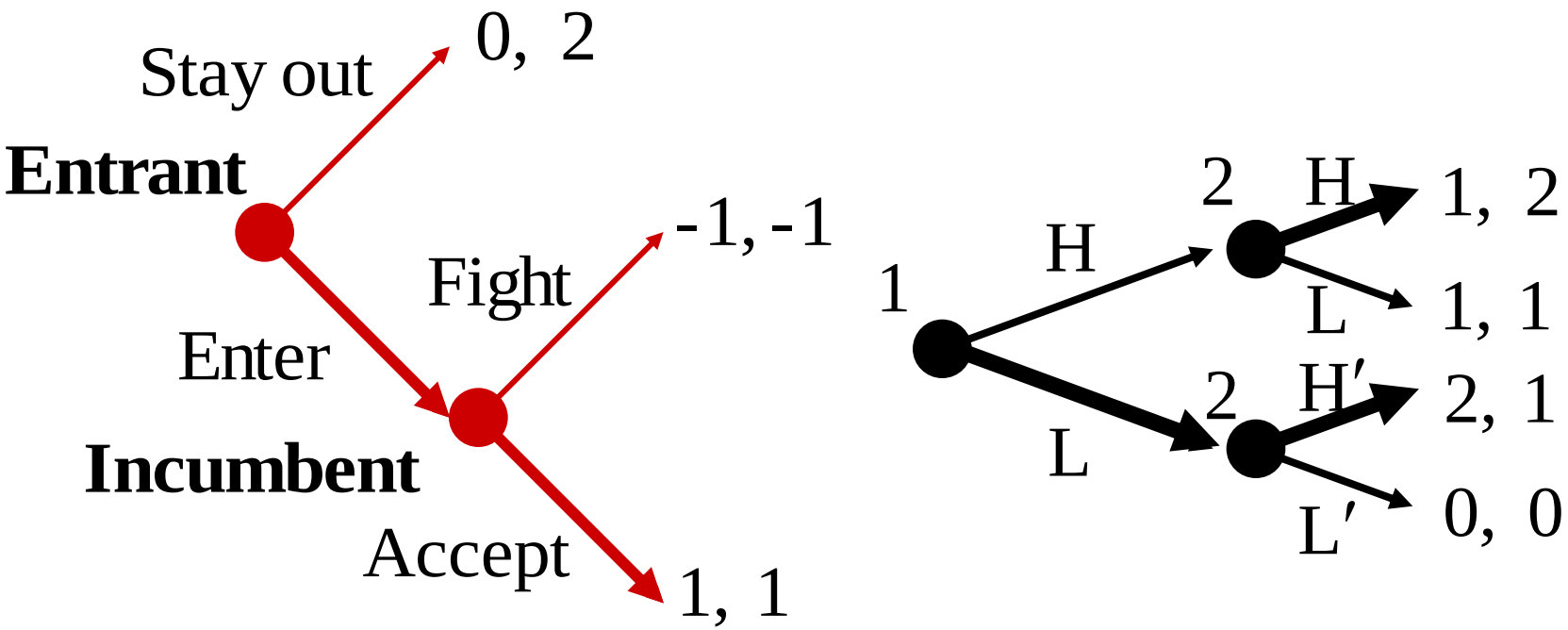
Sequential rationality:

- An optimal strategy for a player should maximize his or her expected payoff, conditional on every information set at which this player has the move. That is, player *i*'s strategy should specify an optimal action from each of player *i*'s information sets, even those that player *i* does not believe (ex ante) will be reached in the game.

Backward induction:

- The process of analyzing a game backwards in time (from information sets at the end of the tree to information sets at the beginning). At each information set, one strikes from consideration actions that are dominated, given the terminal nodes that can be reached.

Illustrating backward induction



Results

- Backward induction identifies a unique strategy profile in a finite perfect information game with no payoff ties.
- Such a strategy profile is a Nash equilibrium.

Observation

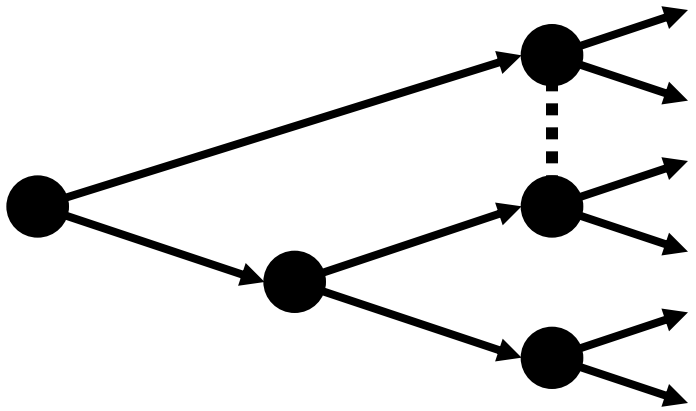
- Backward induction generalizes rationalizability to perfect information games. (Not necessarily true of imperfect info games).

Question

- How to define equilibrium for ext.-form games so that equilibrium implies backw. ind. in perf. info. games.

Subgame

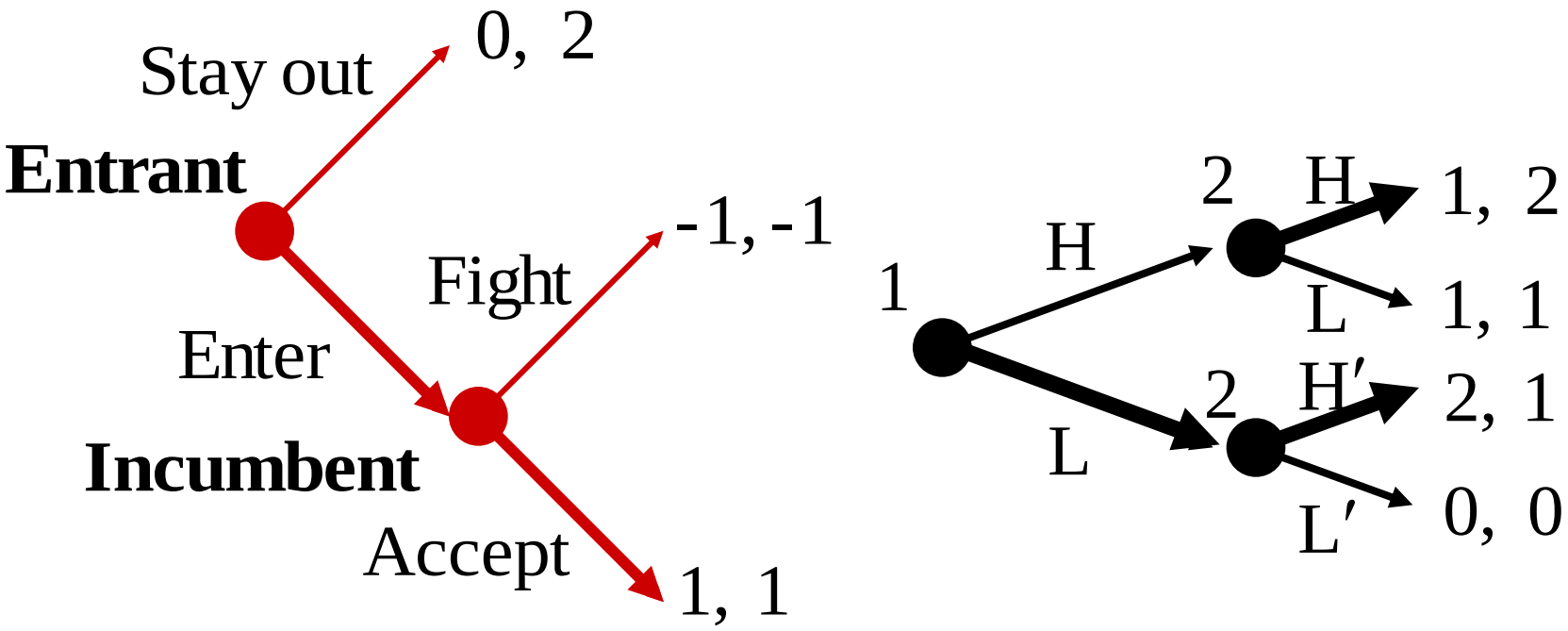
- **Definition:** A *subgame* in an extensive-form game
  a) starts with a decision node (the initiating dec. node)
  b) includes also all successors (decision nodes that can be reached from the initiating decision node).
  c) splits no information set (no included dec. node is in an information set that contains excluded dec. nodes).



What are the subgames?

Subgame perfect Nash equilibrium

- **Definition:** A strategy profile is called a *subgame perfect Nash equilibrium* if it specifies a Nash equilibrium in every subgame of the original game.



Results

- Any finite extensive-form game has a subgame perfect Nash equilibrium.
- In a perfect information game without payoff ties, the unique SPNE coincides with the strategy profile indentified by backward induction.

Algorithm

- Consider the normal forms of all subgames.
- Determine the Nash equilibria of each subgame.
- Find the Nash equilibria of the whole game that are also Nash equilibria of each subgame.

Does actual behavior conform to subgame perfection?

- The centipede game/Game of trust

- The ultimatum game

  - Backward induction: Optimal to accept any positive amount, thus optimal to offer smallest positive amount.
  - Experiments show this not to hold, people reject small offers →no longer rational to offer small amounts.
  - "In search of homo economicus", Henrich et al, 2001.

The paradox of backward induction:

- Why should a player conform to backward induction at decision nodes where he/she knows that an earlier player has deviated from backward induction?